

Internal Pair Conversion in Mg^{24} Nuclei*†

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A gamma-gamma coincidence counting method is used to detect pair emission from an excited state of Mg^{24} . The positrons are stopped in absorbing material surrounding the source and detected by their annihilation radiation. The coincidence rate due to annihilation radiation is distinguished from the rate due to cascaded nuclear gamma-rays by the sharp 180-degree angular correlation of the annihilation radiation. Pair emission from Mg^{24} is distinguished from gamma-ray pair production in the material surrounding the source by using materials of different atomic number Z . The efficiency of the apparatus for detecting positrons is determined by observing the coincidence rate from a Na^{22} source of known disintegration rate. This efficiency, combined with the fraction of the observed coincidence rate that is due to pair emission from Mg^{24} and with the known disintegration rate of the parent Na^{24} source yields a value of $(6.7 \pm 1.0) \times 10^{-4}$ for the internal pair-conversion coefficient of the 2.76-Mev transition in Mg^{24} . This value agrees with that calculated by Rose for electric quadrupole transitions of this energy.

I. INTRODUCTION

ONE of the various processes by which an excited nucleus can lose energy is by the emission of positron-negatron pairs. This process, called internal pair creation or internal pair conversion, provides a

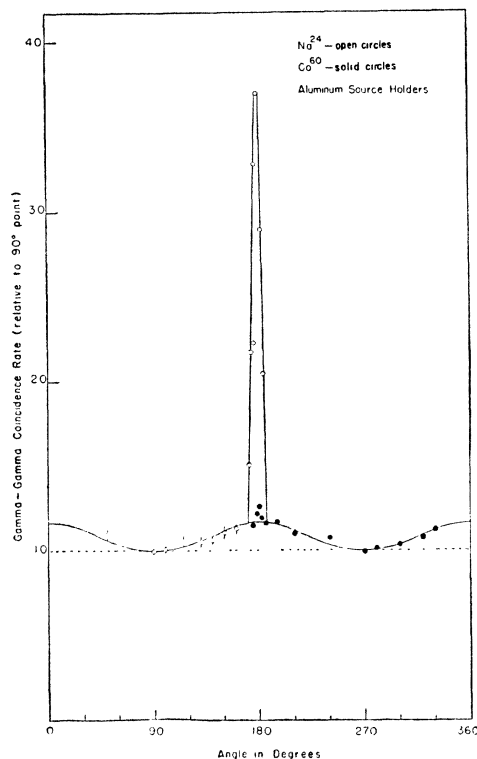


Fig. 1. Gamma-gamma coincidence counting rate as a function of the angle between the two counters. The source is enclosed in an aluminum cylinder thick enough to stop all the beta-rays. The counter efficiency is 30 percent for 0.51-Mev gamma-rays. The counter solid angle as seen from the source is 10^{-3} of 4π .

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means of determining the spin and parity changes involved in the corresponding nuclear transition. It is complementary to the process of internal photoelectric conversion in that the former is large for low multipole order, high energy transitions, while the latter is large for high multipole order, low energy transitions. The internal photoelectric-conversion coefficient would be very difficult to measure for the 2.76-Mev transition in Mg^{24} , while the internal pair-conversion coefficient for this transition should be more readily measured.

II. APPARATUS

Scintillation counters were used in the coincidence work because of their short response time and high gamma-ray efficiency. Each counter consists of an RCA-5819 photomultiplier equipped with a four-inch cylinder of terphenylphenylcyclohexane solution (3 grams/liter) in wet contact with the face of the tube. The gamma-efficiency was about 30 percent in the 0.5- to 3.0-Mev region. The coincidence circuit¹ had a resolving time of 5×10^{-9} second. The source-strength measurements from which the value of the internal pair-conversion coefficient was computed were performed with a proportional counter. The calibration sources were deposited on thin (0.2 mil) Nylon foils and placed entirely inside the counter. With this arrangement every beta-particle above approximately 10-kev energy was counted and the gamma-rays, being in coincidence with their corresponding beta-rays, did not contribute to the counting rate.

III. PROCEDURE

The distinction between annihilation and nuclear gamma-gamma coincidence rates is illustrated in Fig. 1. The Na^{24} source was placed inside an aluminum cylinder thick enough to absorb all the beta-particles. The coincidence rate due to the two cascaded gamma-rays (2.76 and 1.38 Mev) of Mg^{24} is nearly independent of the angle θ between the two counters, and follows the

¹ To be published elsewhere.

angular correlation function,²

$$W = 1 + 0.125 \cos^2\theta + 0.042 \cos^4\theta. \quad (1)$$

Superposed on this distribution is the sharp peak at 180 degrees due to annihilation radiation. The half-width of this peak is equal to the angular width of either of the counters. The coincidence rate caused by annihilation radiation alone can be obtained by subtracting the rate at 160 degrees from the rate at 180 degrees or, more accurately, by subtracting 1.167 times the rate at 90 degrees from the 180-degree rate. Similar data for the Ni^{60} gamma-rays (1.17 and 1.33 Mev) following the Co^{60} disintegration are also presented in Fig. 1 for comparison. Both the external and internal pair-production processes are very weak for these low energies. The relative height of the annihilation peak can be increased indefinitely at the expense of the absolute counting rate by decreasing the solid angles of the counters. This is because the nuclear coincidence rate varies as the second power of the solid angle, while the annihilation coincidence rate varies as the first power of the solid angle. The solid angle used here is about 10^{-3} of 4π .

For simplicity in making absolute measurements the flat source holder shown in Fig. 2 was used. Without cylindrical symmetry the nuclear coincidence rate must be determined as close to the 180-degree position as possible. The contribution to the annihilation rate from positrons produced in the absorbing material can be identified by using materials of various atomic number.³ It will be seen that when the Na^{24} source is placed between two aluminum disks 0.5 centimeter thick, the external process accounts for 80 percent of the annihilation radiation. The source was prepared as NaCl and deposited on a 0.2-mil Nylon film. Another Nylon film was cemented over the source with Krylon plastic spray. The resulting film and source weighed about four milligrams per square centimeter. This film was cemented between two 1.0-mil aluminum rings for added strength, placed between the two thick absorbers, and the whole clamped onto the aluminum mounting. This structure was placed directly between the counters with the plane of the source film perpendicular to the line joining the counters. The radiation which is detected must pass through the absorbers in their thinnest dimension.

A 0.1-millicurie source of Na^{22} (positron emitter of 2.6 year half-life), prepared in a separate aluminum source holder similar to that containing the Na^{24} , was used to check the stability of the apparatus. The procedure in taking data was as follows: (1) The Na^{22} source was placed between the counters and its annihilation coincidence rate was determined. (2) The Na^{24} source was placed between two absorbers, and the

² E. L. Brady and M. Deutsch, Phys. Rev. **78**, 558 (1950).

³ The external pair-production cross section varies as Z^2 per atom of absorber (in the Born approximation). See W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, p. 196.

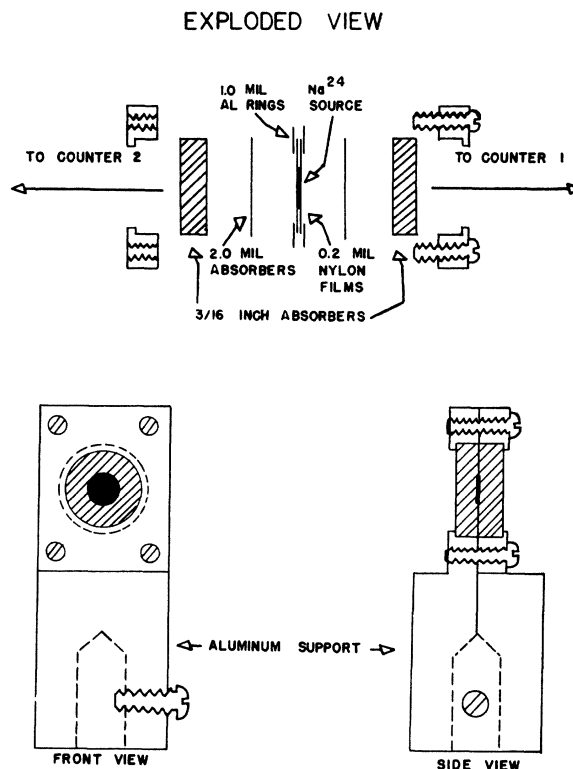


Fig. 2. Design of the source holder used to identify the various processes contributing to the annihilation radiation. Absorbers of beryllium, polystyrene (carbon), aluminum, copper, cadmium, and lead, all $\frac{3}{16}$ in. thick, are used.

single-counter and coincidence rates were determined at 180 degrees and at 160 degrees. (3) The Na^{22} source was replaced and its coincidence rate redetermined. This procedure was repeated for absorbers of beryllium, polystyrene (carbon), aluminum, copper, cadmium, and lead. Various corrections were applied to the coincidence data as follows: (1) The cosmic-ray coincidence rate, measured with source absent, and the random coincidence rate, computed from the single counting rates, were subtracted from the observed coincidence rate. (2) The correction for the 15.0-hour decay⁴ of the source was applied. (3) The corrected coincidence rate at 160 degrees (due to the nuclear gamma-rays) was subtracted from that at 180 degrees, leaving only the annihilation radiation component. (4) This rate was corrected for any variations in counter efficiency as shown by the Na^{22} source data. (5) The rate was further corrected for the absorption of the annihilation radiation in the material surrounding the source. The absorption factor was measured directly for each pair of absorbers by placing them on opposite sides of the Na^{22} source and recording the decrease in its coincidence rate. It is a fortunate circumstance that no matter where the positron stops and annihilates in the absorber, the attenuation factor of the coincidence rate is the same as if one

⁴ J. H. Sreb, Phys. Rev. **81**, 469 (1951).

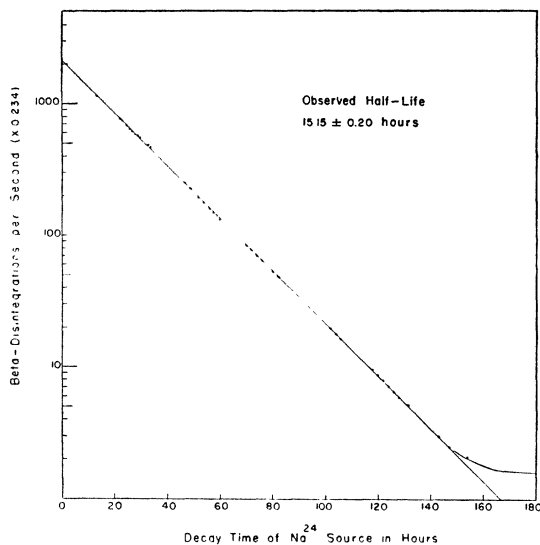


FIG. 3. Decay of the beta-activity of a small sample of the Na^{24} source placed inside the proportional counter.

photon had to traverse both absorbers and the other, neither absorber. This is at once clear if one considers that the attenuation factor for the coincidence rate is given by the product of two exponential factors (one for each annihilation quantum) whose arguments can be added together. This is one of the simplifications afforded by the use of a source with planar symmetry. The 2.76-Mev gamma-ray of Mg^{24} has a much smaller absorption coefficient than the 0.511-Mev annihilation radiation, so that, when the external pair-production component is computed, no corrections need be applied for the intensity of the gamma-ray flux bathing the absorbers. The lead absorber is an exception in that the high energy gamma intensity is down by 20 percent at the outside edge of the absorber, causing a 10 percent decrease in the external process in the lead absorbers.

The coincidence rates thus corrected should fit the formula,

$$C = K_1 + K_2 Z^2 M, \quad (2)$$

where C is the corrected coincidence rate, K_1 is a number proportional to the internal pair-conversion coefficient, K_2 is a number proportional to the external pair-production cross section per atom for hydrogen ($Z=1$), Z is the atomic number of the atoms in the absorbing material, and M is the molar weight of the absorber (proportional to the number of atoms per square centimeter in the absorber). A plot of C against Z yields values for K_1 and K_2 which, when combined with the source strength measurements, can be used to evaluate both the external pair-production cross section and the internal pair-conversion coefficient.

IV. EXPERIMENTAL RESULTS

The sharp peak in the gamma-gamma coincidence rate at 180 degrees is characteristic of the positron-

negatron annihilation radiation⁵ and definitely indicates the birth and death of positrons in the region of the Na^{24} source. These positrons could come from any of four processes: (1) positron decay of the radioactive nucleus; (2) positron decay of some unknown contaminant in the source; (3) external pair-production the source assembly caused by the radiations from the Na^{24} source; (4) internal pair-conversion in the source (pair-production in the field of the disintegrating nucleus). It will be shown that there is no contribution from the first two processes in this experiment.

The first process, that of positron decay of the Na^{24} nucleus, is not energetically possible, since the mass of the product nucleus Ne^{24} exceeds that of Na^{24} . The mass of Ne^{24} can be estimated from the known mass of Ne^{23} (23.0016 amu),^{6,7} the mass of the neutron (1.00898 amu),⁸ and the average neutron binding energy (0.0085 amu).⁹ The probable mass of Ne^{24} is therefore 24.002 amu, while the mass of Na^{24} is 23.998 amu.^{9,10} The difference of 0.004 amu represents about 4-Mev energy

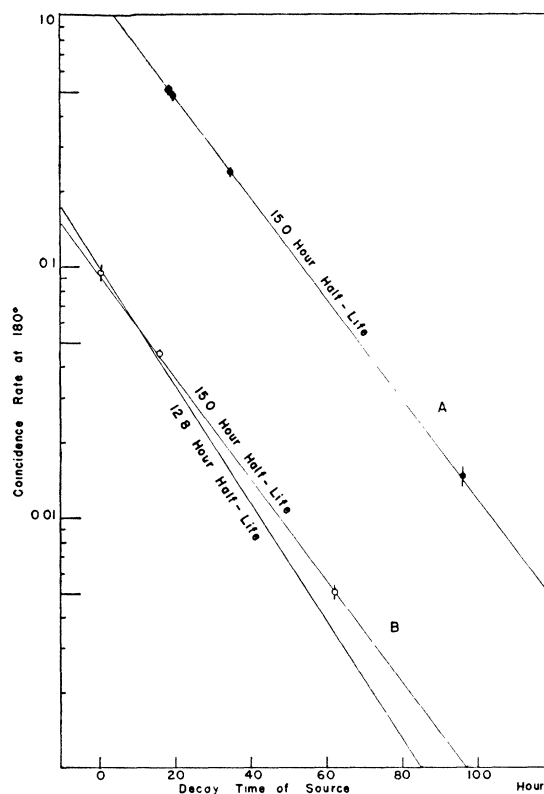


FIG. 4. Decay of the 180-degree coincidence rate with the Na^{24} source enclosed in beryllium absorbers.

⁵ de Benedetti, Cowan, Konneker, and Primakoff, *Phys. Rev.* **77**, 205 (1950).

⁶ J. Ambrosen and K. M. Bisgaard, *Nature* **165**, 888 (1950).

⁷ A. Zucker and W. W. Watson, *Phys. Rev.* **78**, 14 (1950).

⁸ Orear, Rosenfeld, and Schluter, *Nuclear Physics* (University of Chicago Press, Chicago, 1950), revised edition.

⁹ K. Siegbahn, *Phys. Rev.* **70**, 127 (1946).

¹⁰ A. C. G. Mitchell, *Revs. Modern Phys.* **22**, 36 (1950).

difference in favor of negatron decay of Ne^{24} into Na^{24} and excludes the inverse decay.

The second process, that of positron decay of some contaminant, is ruled out by the fact that both the beta-radiation and the annihilation radiation from the source diminish with the characteristic 15-hour half-life of Na^{24} . Figure 3 shows the decay of the beta-radiation from a small sample of the source. This sample was deposited on a thin (0.2 mil) Nylon foil which was mounted inside the proportional counter. The counter was sensitive to beta-particles of energy above about 10 kev. The data show that no more than 0.1 percent of the beta-radiation of the initial source can come from a long-lived contaminant. The deviation from the 15-hour period after 180 hours may not be real as the counting rate from the source was down to about one-tenth the background rate of the counter. The purity of the source is remarkable, considering the fact that no chemical separation was performed. This source was prepared by a one-hour bombardment with 10-Mev deuterons from the Washington University cyclotron of a single NaCl crystal mounted on an aluminum target plate. The activity was taken from the face of the crystal by quickly depositing and then removing a drop of water with an eye-dropper.

A long-lived positron contaminant such as Na^{22} with an intensity of 0.1 percent of that of the source could still account for the observed intensity of the annihilation radiation, since this figure is of the same order of magnitude as the internal pair-production coefficient. In this case the decay of the annihilation radiation itself is more critical in identifying the source. The decay of the 180-degree coincidence rate obtained with the source enclosed in beryllium absorbers is shown in Fig. 4. The solid circles of line A represent data obtained from the source described above. No more than 1.0 percent of the initial annihilation radiation can come from a long-lived contaminant, and thus no more than one part in 10^6 of the initial source can be a long-lived positron emitter. The open circles of line B represent data obtained in an earlier exploratory experiment with a Na^{24} source prepared by a one-hour deuteron bombardment of metallic sodium which was deposited on a copper target plate. There is a slight possibility that Cu^{64} (12.8-hour positron emitter) formed in the copper target plate might contaminate the sodium and increase the positron activity of the source. The observed decay rate shows no evidence of this, however.

It thus appears that the annihilation radiation is definitely related to the Na^{24} source. One can expect some internal and external pair production from the nuclear beta-rays,^{11,12} but the cross sections are about 100 times smaller than those for gamma-rays of about the same energy. The external pair-production cross-section of the 2.76-Mev gamma-ray of Mg^{24} is about

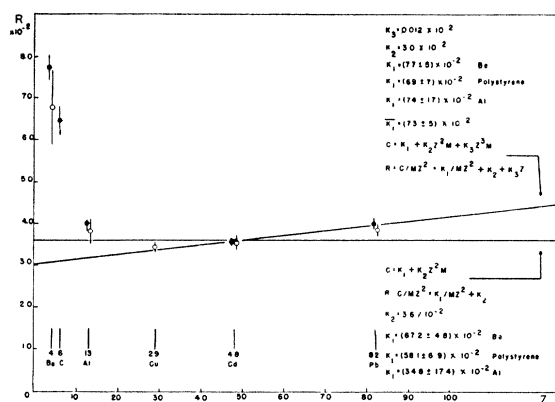


FIG. 5. Plot of the intensity of the annihilation radiation *versus* the Z of the absorber. The vertical axis represents the corrected annihilation coincidence rate per mole $\times Z^2$ of absorber. The high points for low values of Z indicates a component of the annihilation radiation which is independent of the absorbing material and hence caused by pair emission from the source itself.

8.9 times larger than that for the 1.38-Mev ray,¹³ while the internal pair-conversion coefficient of the 2.76-Mev ray is about 50 to 70 times that for the 1.38-Mev ray.¹⁴ Thus, the effect is due almost entirely to the high energy gamma-ray.

The relative intensities of the external and internal processes are determined by fitting the corrected coincidence data to formula (2). For convenience in plotting the data, the coincidence rates were divided by the quantity Z^2M appropriate for each absorber. The rates should then fit the formula,

$$R = C/Z^2M = (K_1/Z^2M) + K_2. \quad (3)$$

A plot of the values of R is shown in Fig. 5. The solid circles represent data obtained with the source prepared by bombardment of a NaCl crystal. The open circles represent data obtained with the source prepared by bombardment of metallic sodium. The latter data have been multiplied by 11.5, the ratio of the source strengths, to make them directly comparable with the former. The term K_1/Z^2M is very small for the copper, cadmium, and lead absorbers because of the large value of Z^2 . For these three points the curve should approximate a horizontal line if the Z^2 law for external pair production holds. The points for beryllium, carbon and aluminum should show the presence of the K_1/Z^2M term and rise above this line. That they do is plain from the diagram, indicating that there is indeed a component of the annihilation radiation which is independent of the absorbing materials. The three points for large Z appear to lie along a sloping line that suggests a Z^3 term in the formula for the coincidence rates, i.e.,

$$C = K_1 + K_2Z^2M + K_3Z^3M, \quad (4)$$

$$R = C/Z^2M = (K_1/Z^2M) + K_2 + K_3Z. \quad (5)$$

¹¹ L. Landau and E. Lifshitz, *Physik. Z. Sowjetunion* **6**, 244 (1934).

¹² H. Bradt, *Helv. Phys. Acta* **17**, 59 (1944).

¹³ J. G. Jaeger and J. Hulme, *Proc. Roy. Soc. (London)* **153**, 443 (1935).

¹⁴ M. E. Rose, *Phys. Rev.* **76**, 678 (1949).

A horizontal line and a sloping line are drawn through the three high Z points of the diagram. It is not possible to draw a horizontal line without violating the probable error limits of at least one of the points, but it is possible to draw a sloping line to lie within each of the probable error limits. The values of the constants K_1 , K_2 , and K_3 , determined from these two lines and from the beryllium, carbon, and aluminum data (solid circles only), are shown on the diagram. The value of the constant K_1 , which represents the internal contribution of the source independent of the absorbers, has been determined separately for each of the three low Z points. These three determinations do not agree when based on the horizontal line, but do agree when based on the sloping line. This fact, together with the fact that the sloping line fits the high Z points better, suggests that it is appropriate to include the Z^3 term.

It is not clear whether this apparent higher power of Z is a result of some peculiar geometrical effect or represents a deviation of the external pair-production cross-section from the Z^2 law. There is some theoretical justification for a higher power of Z in the formula for the cross section (see Appendix I). The average value of K_1 , determined from the beryllium, carbon, and aluminum solid circles by using the sloping line, is 0.73 ± 0.05 coincidence per second due to internal processes in the source. This contribution from the source is due both to internal pair conversion and to the action of the gamma-rays on neighboring nuclei in the source material and its supporting films. The light weight of the source (4 mg/cm^2) makes the probability of interaction with neighboring nuclei small. A quantitative measure of this effect can be obtained by observing the increase in the annihilation radiation caused by placing thin foils of materials on each side of the source to increase its effective weight. One-mil foils of aluminum (approximate Z of source material) cause such a slight increase in the radiation that in practice it is necessary to use thin foils of higher Z and extrapolate back to aluminum. Figure 6 shows a plot of the increased annihilation radiation caused by foils of lead,

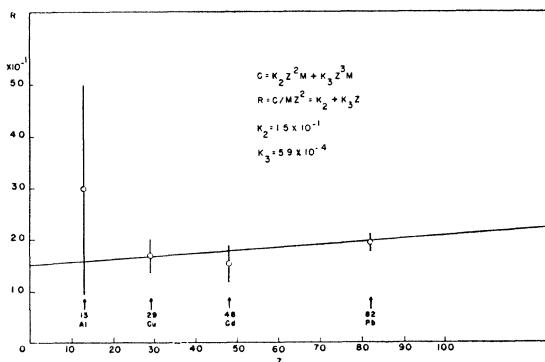


FIG. 6. Increased annihilation radiation caused by placing one-mil foils of various materials between the source and the thick beryllium absorbers. The vertical axis represents the increase in the coincidence rate per mole $\times Z^2$ of the thin foils.

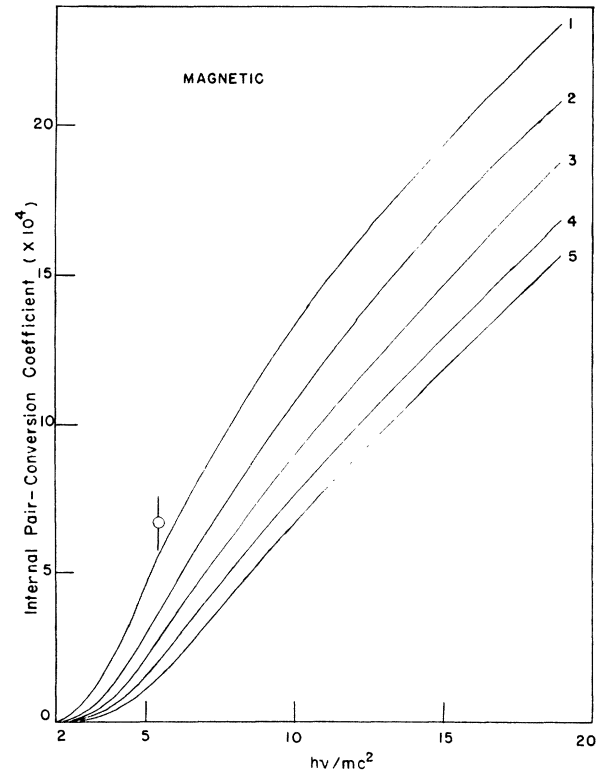


FIG. 7. Theoretical curves of the internal pair-conversion coefficient for magnetic multipoles given by Rose. The horizontal axis represents the transition energy in units of mc^2 . The numbers affixed to the curves show the multipole order L of the transition. The plotted point shows the value obtained in this experiment for the 2.76-Mev transition in Mg^{24} .

cadmium, copper, and aluminum placed between the source and the thick beryllium absorbers. The line has been drawn through the points with the same slope as the line of Fig. 5, although the error limits of the points do not justify such care. The extrapolated value for aluminum, when reduced to foils of the same weight per square centimeter as the source material, indicates a contribution of 0.02 coincidence per second due to action of the gamma-rays on neighboring nuclei in the source. The corrected annihilation coincidence rate, 0.71 ± 0.05 count per second, must be attributed entirely to internal pair conversion. Since the calculated coefficient of the 1.38-Mev transition is about 1/70 of that of the 2.76-Mev transition,¹⁴ it follows that 0.70 ± 0.05 coincidence per second are due to internal pair-conversion of the 2.76-Mev transition alone.

The internal pair-conversion coefficient is defined as the ratio of the number of nuclear transitions resulting in the emission of pairs to the total number of transitions. In order to obtain a value for this coefficient it is necessary to know the total disintegration rate of the Na^{24} parent source as well as its pair-emission rate. The total disintegration rate of the Na^{24} source was obtained by comparison with a smaller Na^{24} source of known disintegration rate. The pair-emission rate was obtained

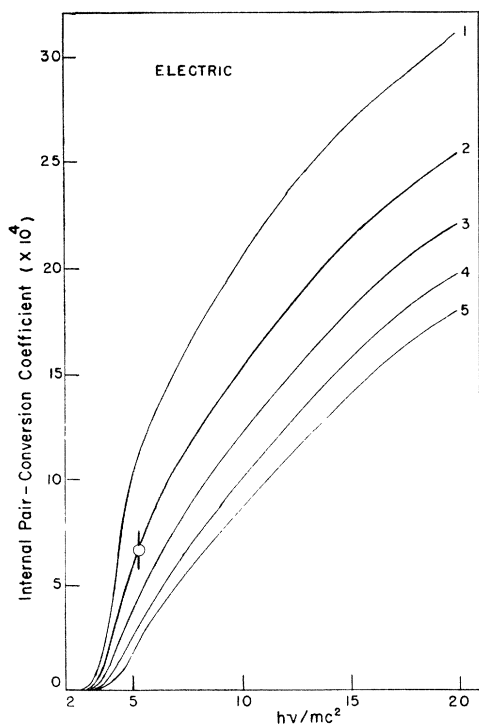


FIG. 8. Theoretical curves of the internal pair-conversion coefficient for electric multipoles given by Rose. The horizontal axis represents the transition energy in units of mc^2 . The numbers affixed to the curves show the multipole order L of the transition. The plotted point shows the value obtained in this experiment for the 2.76-Mev transition in Mg^{24} .

from the corrected coincidence rate by calibrating the apparatus with a small positron-emitting source (Na^{22}) of known disintegration rate. These calibration sources were deposited on 0.2-mil Nylon foils and measured by their beta-counting rates when placed completely inside the proportional counter. An accuracy of 10 percent is assigned to these measurements to allow for possible failure of the assumptions of 4π solid angle and 100 percent counting efficiency. The conversion coefficient depends upon the ratio of the two calibration sources. Since any failure of the assumptions would effect both measurements in the same direction, this ratio is probably quite reliable. The numerical results follow. The calibrated Na^{22} source having a disintegration rate of 500 ± 50 positrons per second gives a coincidence rate (annihilation peak at 180 degrees corrected for attenuation by the beryllium absorbers holding the source) of 0.0945 ± 0.001 count per second. The over-all efficiency of the apparatus for detecting positrons is thus $(1.9 \pm 0.2) \times 10^{-4}$ coincidence per positron. The corrected coincidence rate from internal pair-conversion in Mg^{24} (0.70 ± 0.05 count per second) indicates a pair-emission rate of 3700 ± 450 pairs per second. The Na^{24} source calibration shows a total activity of $(55.5 \pm 0.60) \times 10^6$ disintegrations per second. The ratio of these last two numbers gives a value of $(6.7 \pm 1.0) \times 10^{-4}$ for the in-

ternal pair-conversion coefficient of the 2.76-Mev transition in Mg^{24} (see Appendix II).

V. DISCUSSION

Numerical values of this coefficient have been calculated by Rose¹⁴ for electric and magnetic transitions with multipole orders L from 1 to 5, and for gamma-ray energies from 1.02 to 10 Mev. The Born approximation was used in the calculations, limiting the applicability of the results to $Z < 40$ and energy > 2.5 Mev. Both of these criteria are met by Mg^{24} . The curves of Figs. 7 and 8 are taken from reference 14 and show the variation with energy of the coefficient for each type of transition. The plotted point shows the value obtained in this experiment with its assigned probable error limits. It clearly rules out all of the magnetic transitions (Fig. 7) except possibly the case $L=1$ (magnetic dipole). Among the electric transitions (Fig. 8), the point selects the case $L=2$ (electric quadrupole) without ambiguity. The electric dipole value differs from the electric quadrupole value by four times the assigned error, while the electric octupole value differs by twice

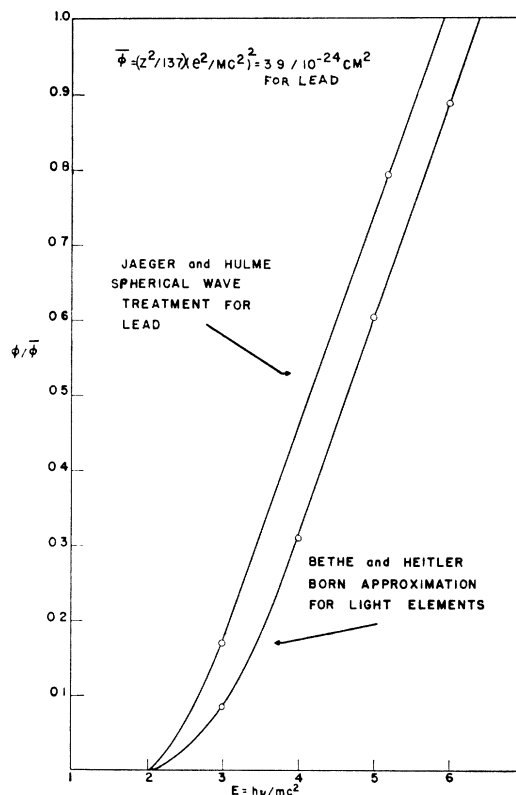


FIG. 9. Theoretical cross sections for external pair production by gamma-rays. The vertical axis represents the cross section per atom ϕ in units of ϕ_0 . The Born-approximation values are applicable for elements of low Z and for high energy gamma-rays. The spherical-wave values are applicable to gamma-rays of all energies but for lead only. The difference between the two curves indicates the error of the Born approximation when applied to lead. The spherical-wave values are higher by a factor of 2 for 1.5-Mev gamma-rays and higher by a factor of 1.15 for 3.0-Mev gamma-rays.

the assigned error. A magnetic dipole assignment ($L=1$) would conflict with the 0-2-4 spin assignment of the Mg^{24} levels suggested by the angular correlation of the two nuclear gamma-rays (reference 2 and Fig. 1) which restricts L to lie between 2 and 6. An electric quadrupole assignment ($L=2$) would be quite consistent with the spin assignments and would mean that the wave functions of the initial and final states of this transition have the same parity.

The beta-spectrometer measurements of Rae¹⁵ using a thick source (36 mg/cm² of sodium carbonate) have given a value of $(11.6 \pm 1.0) \times 10^{-4}$ for the internal pair-conversion coefficient of Mg^{24} . It appears likely that this result is high by a factor of 1.3 to 1.5 because of external pair-production processes in the material of the source and that if correction for this is applied, the result can be considered as further evidence for the electric quadrupole assignment.

The experiment of Mims *et al.*¹⁶ is very similar to the one described in this paper, differing chiefly in that the source strengths were measured by the method of gamma-gamma coincidence counting rather than absolute beta-counting. The value obtained for the internal pair-conversion coefficient of Mg^{24} was $(7.6 \pm 1.9) \times 10^{-4}$. On a repeat experiment a value of $(8.25 \pm 1.05) \times 10^{-4}$ was obtained by this group. These values are somewhat higher than that obtained in the present experiment but agree within the assigned error limits. If these two values are averaged with the value obtained here, the result is $(7.5 \pm 0.8) \times 10^{-4}$, which strongly favors the electric quadrupole assignment.

APPENDIX I

It has been pointed out previously that the Z^2 law for the external pair-production cross section will fail in cases where the Born approximation is not valid.^{17,18} The condition for the applicability of this approximation is³

$$2\pi Ze^2/hv \ll 1,$$

where v is the velocity of either electron of the pair. This condition is not well met in the interaction of comparatively low energy gamma-rays with lead.

The cross section for lead has been calculated for two different gamma-ray energies by Jaeger and Hulme¹⁴ using spherical wave functions for both electrons. Their results are presented in Fig. 9 along with the Born-approximation values given by Bethe and Heitler.¹⁹ The smooth curves have been drawn by visual extra-

¹⁵ E. R. Rae, *Phil. Mag.* **40**, 1155 (1949).

¹⁶ Mims, Halban, and Wilson, *Nature* **166**, 1027 (1950).

¹⁷ Nishina, Tomonaga, and Sakata, *Sci. Papers Inst. Phys. Chem. Research (Tokyo)*, **24**, Suppl. 17 (1934).

¹⁸ W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, Appendix II.

¹⁹ H. Bethe and W. Heitler, *Proc. Roy. Soc. (London)* **146**, 83 (1934).

polation, remembering that both cross sections must vanish at the threshold. For elements of sufficiently small Z , the spherical wave values will presumably approach the plane wave values. The separation of the two curves can be viewed as the addition of a higher power of Z to the cross section, although this is a great oversimplification of the problem. Using the above interpretation of the curves, one can estimate how much the external pair production produced by the 2.76- and 1.38-Mev gamma-rays of Mg^{24} will exceed the Born-approximation values in going from $Z=0$ to $Z=82$. The ratio $\phi/\bar{\phi}$ for the high energy ray increases by a factor of 1.2, while $\phi/\bar{\phi}$ for the low energy ray (whose cross section is only $\frac{1}{3}$ as large) increases by a factor of 2.2. The sum of the external effects of these two rays should, therefore, rise to a factor of 1.3 above the Born-approximation value for lead absorbers. This is just the amount of rise shown in Fig. 5. The Z^2 dependence of the cross section for external pair production has been checked experimentally by Heiting²⁰ and by de Benedetti,²¹ both using the 2.62-Mev ray of $Th-C''$; and a 30 percent increase was not observed in either case. It would be interesting to look into this question further using the 1.17- and 1.33-Mev rays of Ni^{60} . The factor by which the annihilation radiation might be expected to rise above the Born-approximation value is 2.2, which would be a much stronger effect than that for Na^{24} . The Ni^{60} gamma-gamma coincidence data of Fig. 1 show a small but definite annihilation radiation component. This could be made to stand out much more clearly above the nuclear gamma-gamma coincidence rate by decreasing the solid angles of the counters.

APPENDIX II

Further confidence in this technique can be gained from an evaluation of the cross section for external pair production in the absorber surrounding the source. The cross section ϕ is calculated from the equation,

$$\phi = P/Nn,$$

where N is the number of gamma-rays traversing the absorber per second, n is the number of atoms per square centimeter in the absorber as seen from the source, and P is the number of pairs created per second in the absorber. The value of P is calculated from the annihilation coincidence rate for lead absorbers (Fig. 5) and from the measured efficiency of the apparatus for detecting positrons. The value of n is calculated from the molar weight and area of the absorbers (0.1363 mole, 1.0-inch diameter disk on each side of the source) and N is just the disintegration rate of the Na^{24} source, since we disregard the low energy gamma-ray. Using these figures, one finds a cross section of 3.8×10^{-24} square centimeter per atom of lead. The theoretical value extrapolated from the calculations of Jaeger and Hulme¹⁴ (Fig. 9) is 3.3×10^{-24} square centimeter. The close agreement must be regarded as fortuitous, considering the fact that the closely-packed geometry of the source and absorbers (Fig. 2) does not lend itself to a precise calculation of the number of atoms per square centimeter in the absorbers as seen from the source. A result of the same order of magnitude as the theoretical cross section would have given adequate confidence in the technique. The measurement of the internal pair-conversion coefficient does not depend on geometrical approximations, since the pairs come directly from the source and the entire apparatus, source geometry, counter efficiencies, and solid angles are calibrated at once by the use of the measured Na^{22} positron source.

²⁰ T. Heiting, *Physik* **87**, 127 (1933).

²¹ S. de Benedetti, *Compt. rend.* **200**, 1389 (1935).