Z-Dependence of the Cross Section for Photocapture by Nuclei

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The number of neutrons evaporated by nuclei excited by Li(p, γ) photons is calculated along the compound nucleus formalism. It is found, for light nuclei, to be very small and to exhibit an odd-even alternation. From experimental cross sections for photoneutron production at 17 Mev, the cross section for photocapture by nuclei is deduced, and is found to be proportional to Z within 25 percent. Material is presented for the application of the compound nucleus formalism to light nuclei, in particular a formula for the density of levels.

I. INTRODUCTION

THE purpose of this paper, of which a brief account has been given earlier,[1] is to investigate the charge dependence of the cross section for photon absorption by nuclei, the energy of the photon being around a few tens of Mev. Experiments made with betatron or Li gamma-rays[2–4] show that low Z nuclei have small photoneutron cross sections, with an odd-even alternation. These two facts can be explained in terms of competition of proton emission.

Since competition and cross section for the formation of compound nucleus by the incident gamma-ray depend on energy, and the latter in an unknown manner, it is most convenient to use experiments done with a narrow gamma-ray spectrum. The best experiments for our purpose are then those made with the photons of the Li($p-\gamma$) reaction, in which the cross sections for photo-neutron production are measured directly by BF_3 counters embedded in paraffin.[2]

The competition was calculated along the compound nucleus formalism; if a nucleus can evaporate one particle among particle types i, j, $\cdots$, the probability of evaporating just a particle of type i is given by

$$K_i/(K_i+K_j+\cdots), \tag{1}$$

where K_i is a quantity proportional to the partial width for disintegration with emission of i; this quantity is a function of the available energy W_i which is equal to the excitation energy E minus the binding energy L_i of particle i. Weisskopf and Ewing[5] have shown from detailed balancing that

$$K_i(W_i)=(2s_i+1)(m_i/2M)\int_0^{W_i}\epsilon\sigma_i(\epsilon)\omega_i(W_i-\epsilon)d\epsilon \quad \text{for } W_i>0 \tag{2}$$
$$=0 \quad \text{for } W_i<0.$$

In this formula, σ_i is the cross section for particle i to get into the residual nucleus in the reverse process and ω_i is the level density of the residual nucleus. s_i and m_i are the spin and mass of i, M the mass of a nucleon. Formulas (1) and (2) are valid for the evaporation of the first particle from the compound nucleus; when evaporation of two or more particles is possible, the subsequent evaporations have to be taken into account.

In order to carry out our calculations we see that we have to consider successively: (i) the determination of the binding energies; (ii) the determination of the level densities; (iii) the determination of the penetrability cross sections; (iv) the evaluation of the integrals K.

Once the competition is calculated, the experimental values of the cross section for photoneutron production allow the determination of the cross section for photocapture by nuclei.

Our attention was given to the following nuclei: (i) light nuclei with Z between 8 and 20, owing to their remarkable behavior; (ii) copper and nickel nuclei, as representative of medium weight nuclei; (iii) a few single-isotoped nuclei such as I, Ta, Bi, and U, as representative of heavy nuclei.

II. BINDING ENERGIES

We want to know the energy required to extract from the nucleus one proton or one neutron, and the energy required to extract from this already diminished nucleus another proton or another neutron. These energies will be denoted by L_p, L_n, L_{pp}, L_{pn}, L_{np}, and L_{nn}, respectively. The values of use in this paper are listed in Table I; the letters refer to the origin of the determination. The references a, b, d, i, k, and j refer to review material, c to ($n-\gamma$) experiments, and e and l to ($\gamma-n$) experiments. Reference f indicates use of beta-ray measurements, h use of the Weizsäcker semi-empirical formula; g indicates that the binding energy was obtained through interpolation of experimental values for neighboring nuclei of the same class, i.e., having same A and Z parities and same neutron excess. Reference l indicates use of the relation

$$L_n+L_{np}=L_p+L_{pn}.$$

These methods were used concurrently. In Table I the italicized letter, if any, points out which determina-

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[1] J. Heidmann, Phys. Rev. 83, 237 (1951).
[2] McDaniel, Walker, and Stearns, Phys. Rev. 80, 807 (1950).
[3] G. A. Price and D. W. Kerst, Phys. Rev. 77, 806 (1950).
[4] H. Wäffler and O. Hirzel, Helv. Phys. Acta 21, 200 (1948).
[5] V. F. Weisskopf and D. H. Ewing, Phys. Rev. 57, 472 (1940).

TABLE I. Binding energy of first and second nucleon (in Mev).

	Z	A	L_p	L_n	L_{pp}	L_{pn}	L_{np}	L_{nn}
N	7	14	7.6[a] 7.5[n]	10.5[a] 10.6[e] 10.5[n]				
O	8	16	12.1[a] 12.1[n]	15.6[a] 15.5[n]	10.2[a]	10.8[a]	7.3[a]	
O	8	17		4.1[a] 4.1[k]				
F	9	19	7.9[a]	10.3[a]				
F	9	20		6.4[a] 6.6[c] 6.5[k]				
Ne	10	20	12.8[a] 13.0[m]	16.8[a]				
Na	11	23	8.9[a] 9.6[k]	11.3[a] 11.8[k]		10.3[a]	7.9[a]	11.2[a]
Na	11	24		7.5[a] 7.0[m]				
Mg	12	24	12.3[a] 11.7[m]	16.8[a] 16.2[f]	8.9[a]	11.3[a]	6.7[a]	11[j]
Mg	12	25	10.6[a] 11.5[f]	6.8[a] 7.2[m]				
Mg	12	26	14.2[a] 14.0[f]	11.2[a] 13.5[p]				
Mg	12	27		7.3[a] 6.4[m]				
Al	13	27	8.3[a] 7.5[b] 8.6[m] 8.3[j]	13.1[a] 11.1[b] 14.0[f]	14.2[a] 14.0[f]	11.2[a] 13.5[p]	6.5[a] 7.8[m]	11[i]
Al	13	28		7.7[a] 7.9[b] 7.7[c] 7.7[j]				
Si	14	28	11.5[a] 10.6[b] 11.7[m]	16.9[a] 16.1[b] 16.8[f]	8.3[a] 8.3[j]	13.1[a] 14.0[f]	7.7[a] 8.9[p]	
Si	14	31		7.0[a] 6.6[m]				
P	15	31	8.0[a] 6.5[b] 7.5[m]	12.8[a] 11.3[b] 12.4[f]	11.5[a]	9.8[a] 10.6[m]	5.0[a] 5.6[m]	10.5[a]
S	16	32	9.1[a] 9.0[d] 9.9[j]	14.8[a] 14.6[d] 14.8[e]	8.0[a] 7.5[m]	12.8[a] 12.4[f]	7.2[a]	11[h]
S	16	33	10.0[a]	9.0[a] 8.7[c] 8.7[j]				
S	16	34		11.3[a] 10.9[m]				
Cl	17	35	5.8[a] 6.5[d]	12.7[a] 13.4[d]	11[g]	11.3[a]	4.4[a]	10.3[a]
Cl	17	37	7.9[a] 9.4[d]	9.2[a] 10.7[d]	13.5[h]	9.2[a]	7.9[a]	8.5[a]
Cl	17	38		6.1[c] 6.2[a] 5.1[b] 6.3[j]				
A	18	36	8.4[a] 8.2[d]	14.6[a] 14.3[d]	5.8[a] 6.5[d]	12.7[a]	6.6[a]	
A	18	41		6.6[a] 7.0[b] 6.1[m]				
K	19	39	7.2[a] 4.8[d]	13.5[a] 11.0[d] 13.2[e]	10.4[a] 10.3[m]	12.5[h]	6.3[l]	10[g]
K	19	41	8.3[a] 7.8[m]	9.2[a]				
K	19	42		7.6[a] 7.4[c] 7.4[j]				
Ca	20	40	8.4[a] 8.2[d]	14.7[d] 15.9[e]	7.2[a]	13.5[a]	7.0[l]	11[h]
Ca	20	41		8.5[a] 8.3[m]				
Ca	20	42	10.3[a] 10.2[m]	10.4[a]				
V	23	52		7.3[c]				
Mn	25	56		7.3[c]				
Fe	26	57		7.6[c]				
Co	27	60		7.7[c]				
Ni	28	58	7.0[h]	12.4[h]				
Ni	28	59		9.0[c]				
Ni	28	60	7.8[h]	11.6[h]				
Ni	28	61	8.6[h]	7.7[h] 8.6[c]				
Ni	28	62	9.7[h]	10.1[h]				
Ni	28	64	10.2[h]	9.2[h]				
Cu	29	63	8.3[a]	10.9[e]	12.8[a]	12.4[a]	9.8[eh]	12.8[eh]
Cu	29	64		7.9[c]				
Cu	29	65	8.2[a]	10.6[a] 10.2[f]	11.4[h]	9.6[fa]	7.2[fa]	7.9[a]
I	53	127	6.9[h]	9.3[e]	8.7[h]	9.3[h]	6.9[eh]	7.5[eh]
Ta	73	181	5.8[h]	7.7[e]	7.2[h]	7.7[h]	5.8[eh]	6.3[eh]
Bi	83	209		7.4[e]				6.1[h]
U	92	238	5.7[h]	5.6[h]	4.5[h]	5.6[h]	5.7[h]	4.4[h]

[a] L. Rosenfeld, *Nuclear Forces* (1948).
[b] E. H. Segrè, chart (1948).
[c] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **78**, 481 (1950).
[d] W. Low and C. H. Townes, Phys. Rev. **80**, 609 (1950).
[e] Johns, Katz, Douglas, and Haslam, Phys. Rev. **80**, 1067 (1950). McElhinney, Hanson, Becker, Duffield, and Diven, Phys. Rev. **75**, 542 (1949).
[f] Beta-spectra.
[g] Semi-empirical formula, interpolated.
[h] Semi-empirical formula, calculated.
[i] H. A. Bethe, *Elementary Nuclear Theory* (John Wiley and Sons, Inc., New York, 1947).
[j] D. E. Alburger and E. M. Hafner, Revs. Modern Phys. **22**, 383 (1950).
[k] C. W. Li, private communication.
[l] From the relation $L_n+L_{np}=L_p+L_{pn}$.
[m] B. C. Diven and G. M. Almy, Phys. Rev. **80**, 407 (1950).

tion was believed to be the most reliable, if any choice existed at all. The binding energies of alpha-particles will be taken from reference a of Table I.

III. LEVEL DENSITIES

We use the semi-empirical formula for the level density of a nucleus in function of the excitation energy:

$$\omega(E)=a\exp(b\sqrt{E}), \tag{3}$$

where a and b are two parameters which have to be determined from the scant experimental data. Blatt and Weisskopf[6] have given estimates for a and b for mass numbers around 27, 55, 115, and 201. We shall use these for medium and heavy nuclei. For light nuclei we shall obtain a by interpolation of Blatt and Weisskopf values; in the determination of b we shall use all of the experimental data known to us. Three kinds of experiments are available:

(i) *Nucleon resonances:* Study of the cross section for capture of a nucleon by a nucleus shows resonances, thus permitting to evaluate the level density of the

[6] J. M. Blatt and V. F. Weisskopf, ONR Technical Report No. 42 (1950).

TABLE II. Data from nucleon-resonance experiments.

Target nucleus	Incident nucleon nature	Incident nucleon energy in Mev	Number of resonances	D in kev	Reference
N^{15}	p	0.8 –1.4	3	160	7
F^{19}	p	0.18–2.2	18	110	8
Al^{27}	p	0.45–2.59	30	36	9
O^{16}	n	0–1.45	3	500	12
F^{19}	n	0–0.70	7	100	12
Na^{23}	n	0.03–1.0	8	120	13
Al^{27}	n	0.01–1.0	12	83	10
Al^{27}	n	0.13–0.50	10	37	11
Ca^{40}	n	0.03–0.52	6	82	13

compound nucleus at an excitation equal to the binding plus the kinetic energies of the oncoming nucleon. The energy resolution of the incident beam has to be small compared to the level spacing D. The data[7–13] are summarized in Table II.

Measurements have also been made for heavier elements but in this case the spacing is too small compared to the resolution for them to be reliable.[14,15] Another technique of resolution is then necessary; the best is to use a velocity spectrometer for neutrons of low energy. Most measurements of this type have been made for heavy nuclei.[15–17] A few, however, give information for a group of nuclei around $Z=25$, which is interesting for us: Cr has a resonance at 4200 ev, Mn two at 345 and 2400 ev,[15] Co one at 115 ev,[18] V one at 2700 ev,[19] and Ni one at 3600 ev (and others at 15 and 70 kev[6]).

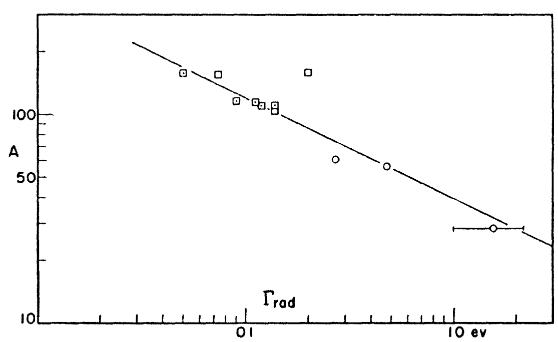

FIG. 1. Radiation width *vs* atomic weight. The squares are from reference 6; the circles are determined in Sec. III, iii.

[7] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **58**, 192 (1940).
[8] Bernet, Herb, and Parkinson, Phys. Rev. **54**, 400 (1938).
[9] Plain, Herb, Hudson, and Warren, Phys. Rev. **57**, 188 (1940).
[10] L. W. Seagondollar, Phys. Rev. **72**, 442 (1947).
[11] R. L. Henkel and H. H. Barschall, Phys. Rev. **80**, 145 (1950).
[12] C. K. Bockelman, Phys. Rev. **80**, 1011 (1950).
[13] Adair, Barschall, Bockelman, and Sala, Phys. Rev. **75**, 1124 (1949).
[14] Barschall, Bockelman, and Seagondollar, Phys. Rev. **73**, 659 (1948).
[15] Rainwater, Havens, Wu, and Dunning, Phys. Rev. **71**, 65 (1947).
[16] W. W. Havens and J. Rainwater, Phys. Rev. **70**, 154 (1946).
[17] Havens, Wu, Rainwater, and Meaker, Phys. Rev. **71**, 165 (1947).
[18] Wu, Rainwater, and Havens, Phys. Rev. **71**, 174 (1947).
[19] M. Hamermesh and C. O. Muehlhause, Phys. Rev. **78**, 175 (1950).

So we see that these nuclei have one or two levels in a range of the order of 5 kev and we can take for the level spacing a value around 4 kev.

(ii) *Neutron width:* The theory of Weisskopf *et al.* of the compound nucleus has yielded a relation between the neutron width Γ_n and the spacing of the levels.[6,20] Supposing that the levels are regularly spaced by an amount D^* we have

$$D^*=\pi K_0\Gamma_n/2k, \tag{4}$$

where k is the wave number of the neutron outside the nucleus and $K_0\sim 1\times 10^{13}$ cm^{-1} its wave number inside. In our case the neutron width is taken directly from the experimental width of the resonance in the total neutron cross section, as the only other component of the total width is the radiation width, which, for the light nuclei of interest to us, is smaller than a tenth of the total width.

In Table III we summarize the data.[6,13,19,21]

TABLE III. Data from neutron width experiments.

Target nucleus	Resonance energy in kev	Neutron width in kev	D^* in kev	Reference
Na^{23}	3	0.17	26	22
	60	3	100	13
Mg^{24}	2540	150	670	6
Al^{27}	155	10	200	6
S^{32}	115	25	520	6
V^{51}	2.7	$\sim$0.78	$\sim$107	20
Mn^{55}	0.345	0.018	4	
Co^{59}	0.115	0.003	2.4	6
Ni?	15	3	190	6
Ni?	70	5	130	6

(iii) *Radiation width and radiative capture cross section:* It has been shown by Bethe[22] that the average radiative neutron capture cross section is related to the radiation width $\Gamma_{\rm rad}$ and to the level spacing D in the following way:

$$\langle\sigma_{n\gamma}\rangle=2\pi^2(R+\lambda\!\!\!^{-})^2\Gamma_{\rm rad}/D, \tag{5}$$

where R is the radius of the nucleus and $2\pi\lambda\!\!\!^{-}$ the wavelength of the incoming neutron. $\langle\sigma_{n\gamma}\rangle$ has been measured by Hughes *et al.*[23,24] for 1-Mev neutrons on about 50 isotopes, of which 11 will be of use to us.

As for the radiation width, values have been given for heavy nuclei by Teichmann;[25] we shall now derive the radiation width values for three lighter nuclei from three experiments:

(a) Harris and Muehlhause have measured the resonance absorption and scattering integrals[26] Σ_a and Σ_s of the 115-ev neutron resonance of Co. From these $\Gamma_{\rm rad}$ may be deduced. A better way is to use concur-

[20] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).
[21] Hibdon, Muehlhause, and Woolf, Phys. Rev. **77**, 730 (1950).
[22] H. A. Bethe, Phys. Rev. **57**, 1125 (1940).
[23] Hughes, Spatz, and Goldstein, Phys. Rev. **75**, 1781 (1949).
[24] D. J. Hughes and D. Sherman, Phys. Rev. **78**, 632 (1950).
[25] T. Teichmann, Princeton University thesis (1949).
[26] Harris, Muehlhause, and Thomas, Phys. Rev. **79**, 11 (1950).

rently the thermal cross section value:

$$\sigma_{th}=\pi\lambda_{th}\lambda_0\Gamma_n\Gamma_{rad}/E_0^2, \quad (6)$$

where E_0 is the resonance energy. Using the one-level Breit-Wigner formula one gets[26]

$$\Sigma_a+\Sigma_s=\pi^2\lambda_0^2\Gamma_n/E_0, \quad (7)$$

where, as the spin of Co is high, we put the statistical spin factor equal to $\frac{1}{2}$. These two equations yield: $\Gamma_{rad}=0.27$ ev. Co is the only light nucleus for which experimental data can be treated in this way. Mn has two resonances and Al a dozen and it would be necessary to disentangle them.

(b) Mn has two neutron levels at 345 and 2400 ev whose resonance amplitudes and angular momenta have been measured by Harris *et al.*[27] The thermal neutron cross section can be written:

$$\sigma_{th}=2\pi\lambda_{th}[(g_1a_1\Gamma_{rad1}/E_1)+g_2a_2\Gamma_{rad2}/E_2], \quad (8)$$

where the g's are the statistical weights of the two levels E. Assuming Γ_{rad2} to be the same as Γ_{rad1}, which is permissible as $g_1a_1/E_1 \gg g_2a_2/E_2$, we get $\Gamma_{rad1}=0.48$ ev.

TABLE IV. Data from radiative capture experiments.

Target nucleus	D in kev
Na^{23}	150
Mg^{26}	60
Al^{27}	80
Si^{30}	20
Cl^{37}	20
A^{40}	20
K^{41}	6
V^{51}	5
Mn^{55}	2.5
Co^{59}	1.0
Cu^{63}	0.8

(c) In order to have a little more information about the radiation widths as a function of A we shall for a moment reverse the argument: from the neutron resonance experiments we know already that D for Al is around 40–80 kev. Taking the value of the radiative capture cross section as given in reference 24, we have, using formula (5),

$$\Gamma_{rad}=1.0-2.2 \text{ ev}.$$

Results for radiation widths are summarized in Fig. 1. The straight line represents the values we adopted for the calculation of D through formula (5). Table IV gives the obtained D values.

From the values of D we got in sections i, ii, and iii, the parameter b of formula (3) can be calculated. The results are shown on Fig. 2. On the basis of our evidence, a fair value of b is, for $15<A<70$,[28]

$$b^2=0.14(A-12) \text{ Mev}^{-1}. \quad (9)$$

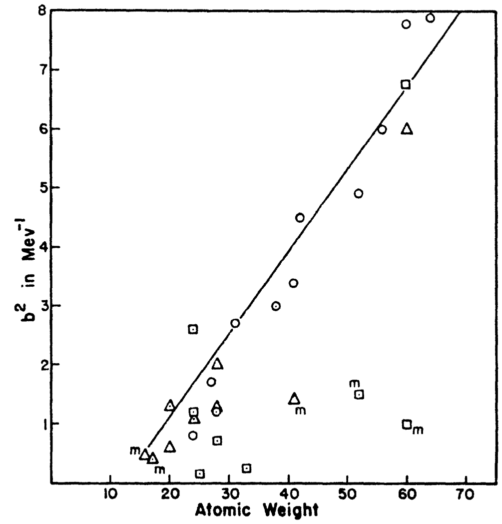

FIG. 2. b^2 in Mev^{-1} *vs* atomic weight. b is the coefficient entering in the level density formula: $\omega(E)=a\exp(b\sqrt{E})$. The triangles refer to nucleon-resonance data, the squares to neutron width data and the circles to radiative capture data.

The magic nuclei are labeled "*m*" on Fig. 2; the scant data we have show that they might have a smaller b-value.

Practically all the data of Fig. 2 refer to odd A initial nuclei; probably even nuclei have bigger level densities; unfortunately not much is known on that matter.

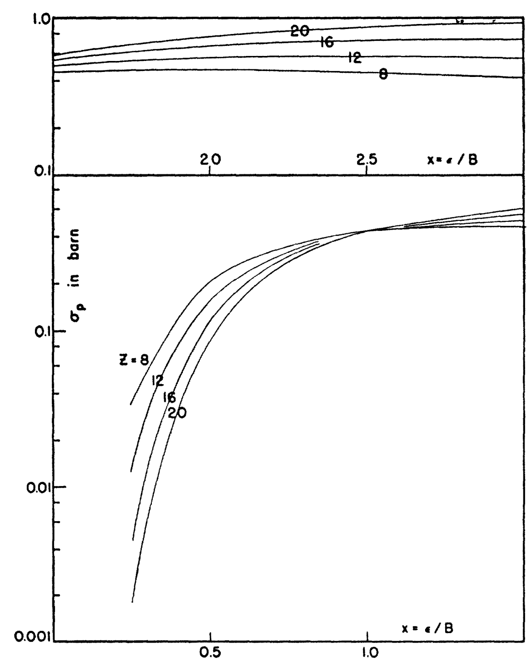

FIG. 3. Penetrability cross section of protons, in barns, *vs* proton energy for nuclei of $Z=8$, 12, 16, 20. x is the ratio of the proton energy to the coulomb potential barrier.

[27] Harris, Hibdon, and Muehlhause, Phys. Rev. **80**, 1014 (1950).

[28] At an earlier stage of this work, the value $b^2=0.17(A-15)$ Mev^{-1} was determined on lesser experimental evidence and is used through the rest of this paper; the difference is small, especially for $A=20$ to 40.

TABLE V. Values of σ_α in barns.

Z compound \ x	0.50	0.75	1.0
8	0.011	0.14	0.34
12	0.0047	0.10	0.44

IV. PENETRABILITY CROSS SECTIONS

Coulomb and centrifugal barriers will prevent nuclear particles, in the reverse process, from getting into the nucleus. Moreover, the nuclear surface has a certain coefficient of reflection for incident wave functions. All these factors are important at the energy we are interested in, that is evaporation energies.

(i) *Penetrability of protons:* It has been calculated by Weisskopf and Ewing[5] for atomic number Z larger than 20 and by Blatt and Weisskopf[6] for Z larger than 30. Here we just extended calculations to lower Z, following the more modern procedure introduced by Blatt and Weisskopf.[6] The cross section for formation of the compound nucleus by a proton of kinetic energy ϵ and wavelength $2\pi\lambda\!\!\!^{-}$ is accordingly written

$$\sigma_p(\epsilon)=\pi\lambda\!\!\!^{-2}\Sigma_l(2l+1)T_l(\epsilon), \qquad (10)$$

where T_l is the transmission factor for the lth wave. T_l was calculated through use of the WKB method for values of $x\equiv\epsilon/B$, B being the coulomb barrier, equal to 0.25, 0.50, 0.75, 1.0, 1.5, 2.0, and 3.0 and for Z equal to 12 and 20. The calculation had to be pushed till $l=5$. The values of T_l were then plotted and smoothed out around the value $x=B_l/B$, B_l being the barrier for the lth wave, where the WKB method is defective. For the nuclear radius we took

$$R=1.40\times10^{-13}\times A^{\frac{1}{3}}\ \mathrm{cm}.$$

The resulting values of σ_p are shown in Fig. 3. For $Z=16$ we simply interpolated arithmetically on the log scale of Fig. 3, and for $Z=8$ we extrapolated in the same manner.

(ii) *Penetrability of neutrons:* It has been calculated by Feshbach and Weisskopf[29] for Z larger than 16. We shall simply refer to these data. In the case of lighter nuclei for which the extrapolation is unsafe we shall use, for slow neutrons, the formula[6]

$$\sigma_n\sim4\pi\lambda\!\!\!^{-}/K_0. \qquad (11)$$

When the value of σ_n as given by (11) is smaller than the geometrical cross section, we shall take for σ_n the geometrical cross section value.

(iii) *Penetrability of alpha-particles:* We need only to consider the case of the lighter nuclei, as for heavier nuclei the potential barrier for alpha-particles becomes prohibitively high, the excitation energy in our case being around 17 Mev. We further simplified matters by supposing that σ_α is equal to the geometrical cross section when the energy of the alpha is above the potential barrier. Taking the nuclear radius as in reference 6, i.e., $1.30\times A^{\frac{1}{3}}\times10^{-13}$ cm, we got the σ_α-values as shown in Table V.

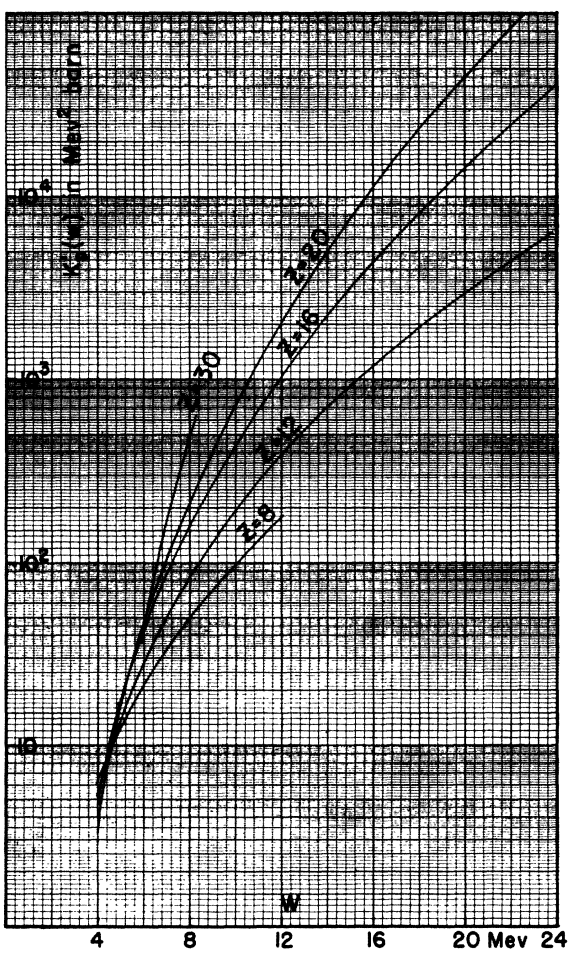

FIG. 4. Integral $K_p'(W)$ in Mev^2 barn *vs* W in Mev for $Z=8$, 12, 16, 20, 30.

V. INTEGRALS K

We can now evaluate the integrals of formula (2); we performed the integrations by use of Simpson's rule. The result is shown on Fig. 4 for particle i being a proton and on Fig. 5 for particle i being a neutron, for $Z=8$, 12, 16, 20, 30 and for W in the range 0–24 Mev. The ordinate represents, instead of K, the quantity

$$K_i'=\frac{K_i}{a_i}=\int_0^W\frac{(2s_i+1)m_i}{2M}\epsilon\sigma_i(\epsilon)\times\exp[b_i(W-\epsilon)^{\frac{1}{2}}]d\epsilon \qquad (12)$$

in Mev$^2\cdot$barn units. For alpha-particles only two values have been calculated:

$Z=8$, $W=10$ Mev : $K'=86$ Mev$^2\cdot$barn;

$Z=12$, $W=7$ Mev : $K'=13$ Mev$^2\cdot$barn.

[29] H. Feshbach and V. F. Weisskopf, Phys. Rev. 76, 1550 (1949).

VI. EVAPORATION PROCESS

(i) *Theory:* When a nucleus is excited, it will evaporate particles: nucleons, alpha-particles, deuterons, photons. . . . The most usual particles are: nucleons, alpha-particles, and photons. Photons can, in general, not compete with the others in the evaporation process,[5] except when the excitation energy of the nucleus is smaller than the binding energy of any other particle or similar cases. Therefore, we shall neglect the emission of photons in this paper.

When the excitation of the nucleus is not high enough for the emission of more than one particle, the chance for particle i to be evaporated is given by Eqs. (1) and (2).[5]

We are going to apply our calculations to nuclei excited by Li gamma-rays. These gammas consist of photons of energy around 17 Mev. More exactly we have two lines:[30] one at 14.7 Mev with a half-half width of 1.0 Mev and one at 17.6 Mev; the total intensity of the second one is about twice that of the first; its measured half-half width is 0.5 Mev, but this width is presumably due to the resolving power of the instrument; the true width of the line is presumably about 10 kev (the width of the excited state), since the final state after emission of the 17.5-Mev gamma-ray is the long-lived Be^8 ground-state.

If the binding energies L are substantially less than 14 Mev, it will suffice to assume that the excitation energy E is equal to the mean energy of the gamma-rays, say, 16.6 Mev.

If this is not the case, we shall have to take into account the shape of the spectrum of the gamma-rays, because when W is small, the integrals K are varying very rapidly. If we are interested only in the calculation of the chance for a neutron to be evaporated, we shall have to take that shape into account only when W_n is small. When the only energetically possible particle emission is that of a low energy proton (or α-particle), the γ-emission will of course compete seriously.

For some nuclei it will turn out that the excitation energy is high enough for the emission of more than one particle to be possible. In that case the calculation of the number of neutrons evaporated will be a little more complicated. It will be based on the following formula, which gives the probability for having emission of i then j:

$$P_{ij}=\frac{1}{\sum_k K_k'(W_k)}\cdot\int_0^{W_i-L_{ij}}\frac{K_j'(W_i-\epsilon_i-L_{ij})}{\sum_l K_l'(W_i-\epsilon_i-L_{il})}\cdot I_i'(\epsilon_i)d\epsilon_i, \quad (13)$$

where L_{ij} is the binding of j in the intermediate nucleus which is left after emission of particle i.

In this formula $I_i'(\epsilon_i)$ is the spectrum of the first emitted particle i; it is simply equal to the integrand of formula (12). The fraction in the integrand of (13) is the branching ratio for j emission following i emission.

The sharpness of the 17.6-Mev line of Li may in some instances make trouble with the interpretation of the experiments. For very light nuclei such as O^{16} it may well happen that the levels are still well separated at 17-Mev excitation energy, i.e., that their spacing is larger than their width, and that the spacing is also large compared with the natural width of the 17.6-Mev line (10 kev). Indeed, our formula gives for O^{16} at 17 Mev a spacing of 100 kev. It can then happen that the cross section for Li γ-rays does not represent an average but may be either close or far from a resonance. For Mg^{24} the theoretical level spacing has decreased to 14 kev, so that for this nucleus, and heavier ones, the average cross section is likely to be measured.

(ii) *Calculations:* Following the procedure of the preceding section, we calculated the number N of neutrons evaporated by nuclei when excited by the Li gamma-rays, as used in McDaniel's experiments.[2] The results are shown in Table VI; Fig. 6 exhibits the results for light elements. As a matter of fact, not all

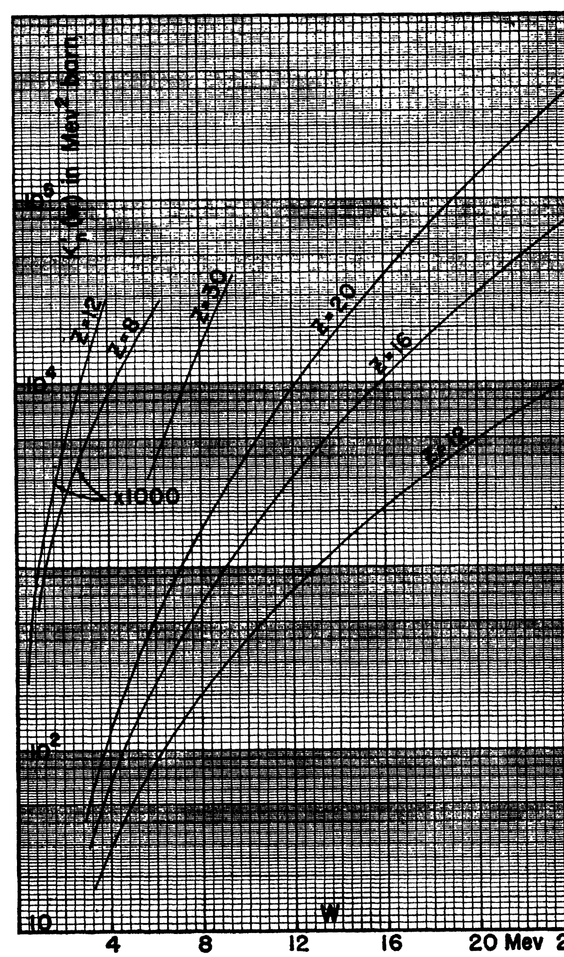

FIG. 5. Integral $K_n'(W)$ in $Mev^2\cdot$barn *vs* W in Mev for $Z=8$, 12, 16, 20, 30.

[30] R. L. Walker and B. D. McDaniel, Phys. Rev. 74, 322 (1948).

TABLE VI. Number of evaporated neutrons.

Target	N	Target	N
O^{16}	0.015	A	1.0
F^{19}	0.30	K^{39}	0.15
Ne^{20}	$\sim$0.03	K^{41}	0.76
Ne	0.12	K	0.19
Na^{23}	0.36	Ca^{40}	0.055
Mg^{24}	0.031	Ca^{42}	0.91
Mg^{25}	0.92	Ca	0.083
Mg^{26}	0.50	Ni^{58}	0.29
Mg	0.17	Ni^{60}	0.62
Al^{27}	0.10	Ni^{61}	0.98
Si^{28}	$\sim$0.02	Ni^{62}	0.97
Si	0.10	Ni^{64}	0.99
P^{31}	0.21	Ni	0.42
S^{32}	0.077	Cu^{63}	0.81
S^{33}	0.90	Cu^{65}	0.87
S	0.12	Cu	0.83
Cl^{35}	0.08	I	1.2
Cl^{37}	0.72	Ta	1.7
Cl	0.24	Bi	1.7
A^{36}	0.073	U	$\sim$2.2

isotopes were calculated for light elements because of lack of information on binding energies; but those which were calculated show that there is practically no proton evaporation once the number of neutrons exceeds the number of protons.

The case of argon is very special, owing to its peculiar isotopic abundance, because of its formation by beta-decay from K^{40}. If only the isotope A^{36} existed, the number of neutrons evaporated would be 0.07.

Uranium has to be examined for fission. The threshold for photofission of U^{238} is about 5.1 Mev.[31] As a matter of fact, several workers[32,33] have shown that the photo-

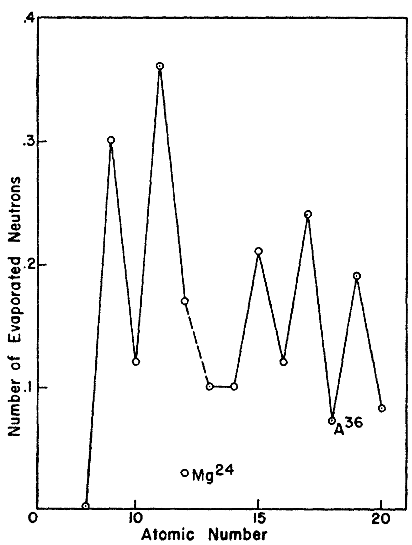

FIG. 6. Calculated number N of neutrons evaporated by light elements excited by Li gamma-rays *vs* atomic number. For argon the isotope 36 value is plotted in place of the natural element value. Mg^{24} value is plotted too.

[31] Koch, McElhinney, and Gasteiger, Phys. Rev. **77**, 329 (1950).
[32] G. C. Baldwin and G. S. Klaiber, Phys. Rev. **71**, 3 (1947).
[33] W. E. Ogle and J. McElhinney, Phys. Rev. **81**, 344 (1951).

fission of U^{238} around 15 Mev is quite important, of the order of 10^{-26} cm^2. Such is not the case for Bi, for which the cross section is estimated to be 1/1000 of that for U, even with an 85-Mev bremsstrahlung spectrum.[34]

Price and Kerst[3] have concluded from their experiment that U will fission in about 50 percent of the cases, assuming that each fission yields $\nu=2.5$ neutrons, the usually assumed value. However, their derivation is wrong because they assumed that when U does not fission it evaporates one neutron only, whereas we have shown that already at least two are emitted at 17-Mev excitation.

Indeed, direct measurements have been made of the cross section for photofission of U by Li gamma-rays.[35] They yield

$$\sigma_f=0.046\pm0.015 \text{ barn},$$

which is much smaller than the $\sigma_{\gamma n}$ section measured by McDaniel: $\sigma_{\gamma n}=0.51$ barn. The total cross section for evaporation and fission is, with our assumptions about the number of neutrons emitted,

$$\sigma_{\text{tot}}=0.51b(\sigma_{ev}+\sigma_f)/(2.2\sigma_{ev}+2.5\sigma_f)\sim0.25b$$

so that the fraction of U atoms which undergo fission is about 20 percent. Thus, on the average U will yield about 2.3 neutrons per photon absorbed.

(iii) *Conclusions:* Examination of Table VI shows the following results: (a) Light nuclei evaporate mainly protons. Even for Ni, protons are expected in equal numbers as neutrons. For Cu, neutrons begin to be in the majority, (still only 83 percent) and when one gets to iodine, protons are absent and 2-neutron emission begins to be possible. For the heaviest nuclei, two or more neutrons are in fact always emitted. (b) An odd-even (in Z) alternation in the number of evaporated neutrons is clearly shown for light nuclei as can be seen on Fig. 6. There is only exception for Mg and A; it is due to the fact that Mg and A have a big percentage of heavy isotopes. Such an alternation was beautifully shown in the $(\gamma-n)$ reactions induced by 22-Mev betatron gamma-rays.[3]

VII. PHOTOCAPTURE CROSS SECTION

As explained in the introduction we now apply our calculations to experiments made on the $(\gamma-n)$ reactions induced by Li photons[30] where the neutrons are detected by BF_3 counters.[2] If $\sigma_{\gamma n}$ is the cross section thus measured, the total cross section σ_γ for capture of a Li gamma-ray by the nucleus can be calculated from

$$\sigma_\gamma=\sigma_{\gamma n}/N,$$

where N is the average number of neutrons evaporated as calculated in the preceding section.

The results are plotted on Fig. 7. We shall first make a few remarks:

[34] N. Sugarman, Phys. Rev. **79**, 532 (1950).
[35] Charbonnier, Scherrer, and Wäffler, Helv. Phys. Acta **22**, 385 (1949).

(i) Na shows a particularly large deviation from the general trend. McDaniel[36] pointed out to us that the $\sigma_{\gamma n}$ for this substance was of the order of the background of neutrons in their apparatus and that a factor 3 could be applied to their measurements. That the Na value should be higher appears likely from the experiments of Price and Kerst,[3] which yield for the ratio of the $\sigma_{\gamma n}$ cross section of Na to that of Al for 18 and 22 Mev bremsstrahlung the values 0.9±0.2 and 0.8±0.1, respectively. As there is no strong variation in this ratio, we can say that McDaniel's result for Na should have been about the same as for Al. Then we get for the Na value: $\sigma_\gamma \sim 15$ mb which is plotted on Fig. 7 with the label *PK*.

(ii) O is also very erratic. Fortunately, in this case there are direct experiments which show up the trouble with our theory. Wäffler[37] has measured directly the cross sections for *n*, *p*, and α-production from oxygen

TABLE VII.

	p	n	α	Units
Cross sections	6.8±1.7	5.4±1.4	1.8±0.6	10^{-28} cm^2
K'	17	2.4	87	Mev^2·barn

bombarded by Li gamma-rays; the results (Table VII) do not compare at all with our determination of the K's: The most striking discrepancy is that the α-emission is perhaps 50 times smaller than we calculated. This may perhaps be understood *a posteriori* because (1) the final nucleus, C^{12}, has particularly few levels, (2) the ^1P states which will ordinarily be formed by absorption of dipole radiation in O^{16}, may not disintegrate very easily into C^{12} in the ground state and an α-particle because of the symmetry of the wave function, and (3) C^{12} and an alpha-particle cannot have a dipole moment. That such considerations may be important is shown by the result of Gaerther and Yeater[38] that He^4 will in general not disintegrate into two deuterons. Presumably, similar troubles will not arise with heavier nuclei, especially when the competition is only between neutrons and protons and not with α-particles, which is the rule for nuclei heavier than Al.

[36] B. D. McDaniel, private communication.
[37] H. Wäffler and S. Younis, Helv. Phys. Acta **22**, 614 (1949).
[38] N. L. Yeater and E. R. Gaertner; G. E. Research Lab. Report *RL*-488 (1951).

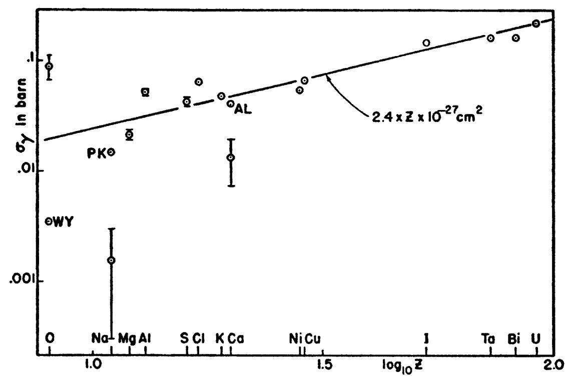

FIG. 7. Nuclear photocapture cross section in barns *vs* the log of the atomic number.

For oxygen, it is most reasonable to use the experimental values of the cross sections in place of the K' in the determination of the number of neutrons evaporated, so we get for that number the value $N=0.39 \pm 0.10$. Hence $\sigma_\gamma=(3.5\pm 2.0)$ mb, which is plotted on Fig. 7 with the label *WY*.

(iii) The only remaining exception is Ca. The main isotope of Ca is not only even-even, but also doubly magic, and as a consequence the cross section for photoelectric absorption may reach substantial values only at higher energies than 17.6 Mev. It is therefore conceivable that the contribution of Ca^{40} to the cross section is negligible. In Table VI it is shown that Ca^{40} contributes 0.055 to the total value of N, which is 0.083 for the element. Thus if Ca^{40} were omitted, N would be reduced to one-third, and the calculated cross section σ_γ would be increased to 40 mb, in good agreement with neighboring elements. It is plotted on Fig. 7 with the label *AL*.

Coming back to Fig. 7 we see that the Z-dependence of the cross section for photocapture of 17-Mev gamma is reasonably well described by

$$\sigma_\gamma(Z)=2.4\times Z\times 10^{-27}\ \text{cm}^2. \tag{14}$$

Leaving off the O, Na, and Ca cases, this relation is verified within 25 percent average deviation.