The Interaction of π-Mesons with Nuclear Matter

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A number of experiments relating more or less directly to meson scattering and absorption are discussed and compared. Because of the variety of experiments interrelated by such considerations, it is seen that any model to describe meson scattering and absorption will have to meet a corresponding number of conditions. In particular, the absorption experiments of Panofsky and his collaborators permit one to put a lower limit on the absorption cross section for complex nuclei which seems appreciably larger than the expected scattering cross section.

I. INTRODUCTION

INTERACTIONS of π-mesons with nuclear particles have been observed to include those which produce mesons, those which absorb mesons, and those which scatter mesons. It has long been recognized that from an experimental point of view these processes are not independent, since, for instance, a meson once produced may be reabsorbed or scattered before it can be observed. However, because of the observation of competing reactions and by the use of detailed balancing arguments to relate inverse processes it is possible to establish more profound relationships between these fundamental interactions. Of particular interest in this connection are a series of experiments by Panofsky and his collaborators concerning the absorption of mesons by some of the lighter elements. By means of these it is possible to establish relationships between meson scattering phenomena and the production of mesons by nuclear collisions and by γ-rays.

In the course of studying these phenomena, some interesting implications concerning nuclear structure are obtained.

II. THE ABSORPTION OF π-MESONS IN COMPLEX NUCLEI

When a charged π-meson is absorbed by a complex nucleus, A, the most probable process is[1]

$$\pi+A\rightarrow\text{Star}. \tag{S}$$

(When no "star" is observed, presumably only neutrons are emitted from the nucleus.[2] We designate this process also as a star, however.) The absorption of the meson releases on the order of 140 Mev of energy (the meson rest-energy), which must appear in the form of kinetic energy of the absorbing nucleons. To conserve momentum as well as energy the absorption must be accompanied by a high energy scattering of at least two nucleons. However, a hard scattering of only two nucleons seems far more probable than a many particle scattering event, since the energies involved are considerably larger than nuclear binding energies. We thus introduce the hypothesis that the primary absorption event involves a pair of nucleons and is the inverse of meson production in the collision of two nucleons. That is, we have the basic mechanisms

$$\begin{aligned}
&\pi^-+p+n\rightarrow 2n && (\pi^-, pn)\\
&\pi^-+2p\rightarrow n+p && (\pi^-, 2p)\\[6pt]
&\pi^++p+n\rightarrow 2p && (\pi^+, pn)\\
&\pi^++2n\rightarrow n+p && (\pi^+, 2n).
\end{aligned}$$

In each case, the two recoil nucleons are left with a kinetic energy of the order of 70 Mev apiece. Considerable excitation of the residual nucleus is expected as these two particles are torn from their place in the structure of the original nucleus. Further excitation of the residual nucleus is expected as a result of subsequent collisions of the recoil nucleons with others in the nucleus.

The results of Camac *et al.*,[3] charge symmetry considerations, and evidence obtained by Bradner (unpublished, but quoted previously[4]) indicate that the absorption of π^+ and π^- mesons will be similar. We thus confine the arguments of the present section to the absorption of π^- mesons with the understanding that the discussion applies also to π^+ mesons.

The above model suggests that we write the cross section per proton for the absorption of a π^- meson in the nucleus A in the form,

$$(1/Z)\sigma[\pi^-+A\rightarrow\text{Star}]=\Gamma\sigma[\pi^-+\text{D}\rightarrow 2n], \tag{1}$$

where $\sigma[\pi^-+\text{D}\rightarrow 2n]$ is the cross section for the process $\pi^-+\text{D}\rightarrow 2n$, which has been observed.[5] The factor of

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[1] Menon, Muirhead, and Rochat, Phil. Mag. **41**, 583 (1950); F. Adelman, Ph.D. thesis, University of California (1951).
[2] S. Tamor, Phys. Rev. **77**, 412 (1950).
[3] Camac, Corson, Littauer, Shapiro, Silverman, Wilson, and Woodward, Phys. Rev. **82**, 745 (1951), and private communication from Professor R. R. Wilson.
[4] Brueckner, Serber, and Watson, Phys. Rev. **81**, 575 (1951). This paper will hereafter be referred to as I.
[5] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).

proportionality, Γ, is expected to depend on the energy liberated in the absorption and on the relative probabilities for the recoil nucleons to undergo a hard scattering in the nucleus A and in deuterium. To the extent that the kinetic energy of the meson can be neglected compared to its rest energy, we can, and shall, take Γ to be a constant. Some care must be taken in the use of Eq. (1), since both the elementary processes (π^-, pn) and $(\pi^-, 2n)$ contribute to the process (S), while only (π^-, pn) is involved in $\pi^-+\mathrm{D}\rightarrow 2n$. This seems to involve at most an adjustment of parameters and will be discussed further in Sec. VI, where a more complete theory of Eq. (1) will be developed.

By detailed balancing arguments, $\sigma[\pi^-+\mathrm{D}\rightarrow 2n]$ was obtained in I from the inverse reactions:[6]

$$\sigma[\pi^-+\mathrm{D}\rightarrow 2n] = (1/v_\pi)[1.87(10)^{-27}\ \mathrm{cm}^2][1+bv_\pi^2]/(1+\tfrac{1}{4}b), \quad (2)$$

where v_π is the meson velocity in units of c.† The ratio of meson P-state to S-state coupling for the process is represented by b. Because of the short range of interaction implied, it was assumed that higher angular momentum states are not important until the meson kinetic energy becomes of the order of its rest energy. In writing Eq. (1), a small effect due to the center-of-mass motion of the two recoil nucleons is neglected.

The mean free path for absorption in the nucleus A is then

$$\lambda_a = \frac{V_A}{Z(1/Z)\sigma[\pi^-+A\rightarrow\mathrm{Star}]}, \quad (3)$$

where $V_A=(4\pi/3)a_0{}^3A$ is the nuclear volume. Using Eq. (2), we have

$$\lambda_a = a_0\frac{89.6}{\Gamma}\frac{(1+\frac{1}{4}b)v_\pi}{1+bv_\pi{}^2}. \quad (4)$$

In I it was deduced that $b\simeq 8$, although this value is quite approximate. Evidence from meson production experiments[6] suggests that b is probably at least this large. Then choosing $b=8$ and for energies which are not too low, setting $v_\pi\simeq 1$, we have an approximate relation

$$\lambda_a = 30a_0/\Gamma. \quad (4')$$

To determine the value of Γ, we must appeal to experiment; however, from an analysis quoted in I and based on the Chew-Goldberger nuclear momentum distribution, it appears reasonable to expect Γ to be of the order of ten.

It seems not unlikely that the elementary process $(\pi^-, 2p)$ has a cross section of the form (2), with perhaps a different numerical coefficient (although it is known from the meson production cross sections[7] not to differ greatly from that for the (π^-, pn) cross section). We assume that the effect of this is included in the definition of Γ.

III. EVIDENCE DRAWN FROM THE PHOTOMESON PRODUCTION CROSS SECTIONS

As was pointed out by McMillan and Mozley,[8] the nuclear interaction of mesons can be expected to modify the production cross sections from complex nuclei. The most simple example of this is photomeson production.

Let the cross section for the production of a positive meson from a nucleus A be σ_A. Then we write σ_A as

$$\sigma_A/Z = \eta\sigma_P f_a, \quad (5)$$

where Z is the atomic number of A and σ_P is the production cross section for a free proton. η represents the effects of nuclear binding on the cross section and f_a represents the fraction of mesons produced in A which are not reabsorbed before leaving A. We have

$$f_a \leq 1.$$

Also, the effects of nuclear structure can be expected to decrease the cross section (except very near threshold), so we assume

$$\eta \leq 1.$$

On the basis of the model of Fernbach, Serber, and Taylor,[9] f_a can be expressed in terms of λ_a on the assumption that the cross section for absorption is much larger than that for scattering. We have

$$f_a = 1/V_A \int_V e^{-D/\lambda_a} d\tau = 3\{(1/2)(1/x)-(1/x^3)+(1/x^3)(1+x)e^{-x}\}, \quad (6)$$

where the integration is taken over the nuclear volume and D is the distance the meson travels from the point at which it is produced to that at which it leaves the nucleus. $x\equiv 2R/\lambda_a$, where $R=a_0A^{\frac{1}{3}}$ is the nuclear radius.

When $x\gg 1$, we can write Eq. (5) as

$$\sigma_A/Z \simeq \eta\sigma_P[\tfrac{3}{4}\lambda_a/a_0]A^{-\frac{1}{3}}. \quad (5')$$

As η is not expected to show a uniform trend with A, we see that σ_A/Z should vary as $A^{-\frac{1}{3}}$ as long as the condition $x\gg 1$ is satisfied. This is, indeed, the variation of σ_A/Z measured by Mozley[8] and by Littauer and Walker.[10] These measurements indicate that the $A^{-\frac{1}{3}}$ dependence is roughly valid for a series of elements from lithium to lead. For a mean free path considerably larger than the nuclear radius, $f_a\simeq 1$ and σ_A/Z should be a constant. The fact that the curve giving σ_A/Z *vs* A did not become constant for the lighter elements permits us to put an upper limit on λ_a. Indeed, from Eq. (6) we estimate

$$\lambda_a \leq 2a_0. \quad (7)$$

[6] Cartwright, Richman, Whitehead, and Wilcox, to be published.

† We have for simplicity replaced $p/\mu c$ (p the meson momentum) by v_π in these formulas.

[7] C. Richman and H. A. Wilcox, Phys. Rev. **78**, 496 (1950).

[8] R. F. Mozley, Phys. Rev. **80**, 493 (1950).

[9] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).

[10] R. M. Littauer and D. Walker, Phys. Rev. **83**, 206 (1951).

Measurements of the absolute cross sections and angular distributions of π^+ mesons from carbon and hydrogen by Steinberger and Bishop[11] (these are in agreement with the cross sections of Mozley[8] and Littauer and Walker[10]) imply that

$$(\sigma_A/Z)/\sigma_P=\eta f_a=\tfrac{1}{3} \tag{8}$$

for carbon. The assumption that $\eta=1$ permits us to put a lower limit on λ_a:

$$\lambda_a\geq a_0. \tag{7$'$}$$

With the assumption that $\lambda_a=2a_0$, we have for carbon $f_a=0.5$ and from Eq. (8) we obtain

$$\eta\simeq\tfrac{2}{3}. \tag{9}$$

If scattering of the mesons within the nucleus is not negligible, we can expect the mean distance traveled by the meson in leaving the nucleus to be increased, which would permit a somewhat longer mean free path for absorption.

Measurements by Panofsky[12] of the photoproduction cross section for π^0-mesons from a number of elements show also the dependence on A given by Eq. (5′). This suggests that the mean free path for absorption of π^0-mesons is approximately the same as that for charged π-mesons.

IV. ANALYSIS OF THE CAPTURE OF STOPPED MESONS IN CARBON

Panofsky, Aamodt, and Hadley[5] and more recently Panofsky, Aamodt, Crowe, and Phillips[13] have searched for the high energy γ-rays originating from the absorption of stopped mesons in carbon (the experiments have been repeated with aluminum and give similar results) by the reaction

$$\pi^-+A\rightarrow\text{high energy }\gamma\text{-ray}. \tag{A$^\gamma$}$$

We denote the transition rates for processes (S) (see Sec. II) and (A$^\gamma$) by T_A and $T_A{}^\gamma$, respectively. The observed ratio of transition rates was found to be

$$T_A{}^\gamma/T_A\leq 0.015. \tag{10}$$

(γ-rays of about 140-Mev energy were counted but an unknown background made it necessary to quote only the inequality (10).) This is to be contrasted with the absorption in deuterium,[5] which led to

$$T_{\mathrm{D}}{}^\gamma/T_{\mathrm{D}}=3/7. \tag{11}$$

Here T_{D} and $T_{\mathrm{D}}{}^\gamma$ are the transition rates for the respective processes,

$$\pi^-+\mathrm{D}\rightarrow 2n \tag{D}$$
$$\pi^-+\mathrm{D}\rightarrow 2n+\gamma. \tag{D$^\gamma$}$$

[11] J. Steinberger and A. Bishop, Phys. Rev. **78**, 494 (1950).
[12] Panofsky, Steinberger, and Steller, to be published.
[13] Panofsky, Aamodt, Crowe, and Phillips, private communication.

To analyze the meaning of relations (10) and (11), it is necessary to consider in some detail the mechanism of the capture and subsequent absorption of the meson. For the case of deuterium this was treated in I. The capture process in carbon is more complicated, but the initial stages have been studied by Fermi and Teller.[14] Since the meson almost certainly will not be absorbed by the nucleus until it is well within the electronic K-shell orbit, we can confine our attention to the final stages of the capture process considered by these authors. According to their analysis, for the meson radial quantum number, n, greater than three, the meson will give up its energy to atomic electrons by dipole transitions. For $n\leq 3$, radiative transitions will predominate. This estimate was based on circular orbits for the meson (high angular momentum), so the actual values of n for which radiative transitions predominate may be somewhat larger. In any case, the transitions being dipole transitions imply the angular momentum selection rule $\Delta l=\pm 1$. Thus for the meson to reach an S-state it must pass through a P-state. However, once it reaches a P-state, electronic excitation can no longer compete with radiation to an S-state (statistical considerations would seem to imply that it is unlikely for the meson to reach a P-state until it is well within the electronic K-shell orbit). We also note that arguments of the sort made in I imply that we need not consider absorption as a probable process until the meson reaches a P-state.

We thus picture the meson as eventually reaching one of the lower P-states (say, $n\leq 6$). At this point the following processes are most probable: absorption to give a star [process (S)], absorption to give a γ-ray [process (A$^\gamma$)], or a radiative transition to an S-state. If the latter event occurs, then from the S-state either of the processes (S) and (A$^\gamma$) will take place.

In deuterium the absorption rate from a P-state is too small to compete effectively with radiation. In carbon this is not the case, since the radiative transitions vary as Z^4 and P-state absorptions as Z^6. Indeed, other things being equal, reference to the table of absorption rates in I indicates that the absorption rate from a P-state for carbon should be about twice the radiation rate (i.e., after increasing the absorption rate in I by a factor of $Z^2=36$).

We proceed to evaluate the ratio, $T_A{}^\gamma/T_A$ of Eq. (10) in terms of the parameters, Γ and b, of Eqs. (1) and (2). For this purpose, let us suppose the meson reaches a P-state with radial quantum number n_1. Let $T_p{}^\gamma$ and T_p be the respective absorption transition rates for processes (A$^\gamma$) and (S) from this P-state. Let A_r be the radiative transition rate to the S-state with theq uantum number n_2. Let $T_s{}^\gamma$ and T_s be the corresponding absorption transition rates from this state. Then the fraction of absorptions by processes (S) and (A$^\gamma$) are,

[14] E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947).

respectively,

$$f=\frac{T_p}{A_r+T_p+T_p^\gamma}+\frac{A_r}{A_r+T_p+T_p^\gamma}\frac{T_s}{T_s+T_s^\gamma},$$
$$f^\gamma=\frac{T_p^\gamma}{A_r+T_p+T_p^\gamma}+\frac{A_r}{A_r+T_p+T_p^\gamma}\frac{T_s}{T_s+T_s^\gamma}. \tag{12}$$

We next evaluate the ratios of transition rates in Eqs. (12) in terms of Γ and b. From Eq. (1), remembering that only S-states are involved, we have

$$\frac{T_s}{T_D}=\frac{|\phi_c^{n_2}(0)|^2}{|\phi_D^{n_3}(0)|^2}Z\Gamma, \tag{13}$$

where T_D is defined in connection with Eq. (11) and $Z=6$ for carbon. $\phi_c^{n_2}(0)$ and $\phi_D^{n_3}(0)$ are the S-state coulomb wave functions evaluated at the position of the nucleus for carbon and deuterium and having respective radial quantum numbers n_2 and n_3.

Again from Eqs. (1) and (2), and recalling that b represents the relative strength of the P-wave and S-wave couplings, we have

$$T_p/T_s=(|(\hbar/\mu c)\nabla\phi_c'^{n_1}(0)|^2)/(|\phi_c^{n_2}(0)|^2)b. \tag{14}$$

$\nabla\phi_c'^{n_1}(0)$ is the gradient of the P-state coulomb wave function for carbon evaluated at the position of the nucleus and having a radial quantum number n_1.

For the transition rates for absorption with radiation, we have

$$T_s^\gamma/T_D^\gamma=(|\phi_c^{n_2}(0)|^2)/(|\phi_D^{n_3}(0)|^2)Z\eta', \tag{15}$$

where η' is a factor giving the dependence of this ratio on nuclear structure. Since the process is similar to photomeson production, we take η' to be the ratio of the η [see Eq. (5)] for carbon to that for deuterium (deduced to be $\frac{2}{3}$ in I). From Eq. (9), we then estimate η', to be about unity.

To calculate the P-state absorption rate, T_p^γ, a knowledge of the relative strength of S- and P-state couplings of the meson to individual nucleons would be desirable. Because, however, of the finite size of the nucleus this is not a very important point. We further note that the evidence from the inverse process of photomeson production suggests that the S-state couplings are most important. We thus assume a coupling entirely to S-states. (This should not overestimate the transition rate. In virtue of the inequality (10), a lower limit is all that is really needed.) Thus T_p^γ will be nonvanishing because of the finite size of the nucleus. Writing the wave function as

$$\phi_c'^{n_1}(r)\simeq\mathbf{r}\cdot\nabla\phi_c'^{n_1}(0)$$

for small r, we have T_p^γ proportional to

$$(1/V_c)\int_{V_c}|\mathbf{r}\cdot\nabla\phi_c'^{n_1}(0)|^2d^3r=\tfrac{1}{5}|\nabla\phi_c'^{n_1}(0)|^2R_c^2, \tag{16}$$

where V_c is the nuclear volume and R_c is its radius. Then we have

$$\frac{T_p^\gamma}{T_s^\gamma}=\tfrac{1}{5}R_c^2\frac{|\nabla\phi_c'^{n_1}(0)|^2}{|\phi_c^{n_2}(0)|^2}. \tag{17}$$

Referring to Eqs. (1) and (2), we write

$$T_p=T_p^0[b/(4+b)]\Gamma \tag{18}$$

Here T_p^0 is the value that T_p has when $b=\infty$, $\Gamma=1$. We can thus identify T_p^0 with the transition rates of the table in I, corrected by a factor of $Z^2=36$. Reference to this table implies

$$T_p^0/A_r\equiv r\simeq 2. \tag{19}$$

Combining relations (13), (14), (15), (17), and (19) with (11), we obtain from Eqs. (12) a result independent of the radial states n_1, n_2, and n_3 and thus valid for the total transition rates:

$$\frac{T_A}{T_A^\gamma}=\frac{f}{f^\gamma}$$
$$=\frac{7}{3}\frac{\Gamma}{\eta'}\frac{[r(b/4+b)[\Gamma+(3/7)\eta']+1]}{(1.05r/4+b)[\Gamma+(3/7)\eta']+1}\geq 67.0. \tag{20}$$

Taking $\eta'=1$, $r=2$, $b=8$, we are led to

$$\Gamma\geq 6.4. \tag{21}$$

Combined with Eq. (4′) for the mean free path, we have

$$\lambda_a\leq 4.7a_0, \tag{22}$$

which is quite consistent with Eqs. (7) and (7′). This upper limit is not entirely rigorous, however, owing to some uncertainty in η' and b. However, it seems that the value of λ_a cannot be much greater than that given by Eq. (22).

The importance of the present experiment is in its separation of the effects of meson absorption from scattering. On the basis of meson theory [Eq. (26), following], the scattering cross section is expected to be considerably less than the lower limit on the absorption cross section given by Eq. (22). It would thus appear that multiple scatter of a meson within a nucleus is improbable since the meson is more likely to be absorbed.

Combining the evidence obtained in the present section with that obtained from photomeson production in the preceding section, it seems reasonable to expect the value of λ_a to be approximately $2a_0$ to $3a_0$.

V. DISCUSSION OF FURTHER EXPERIMENTAL RESULTS

There is evidence that mesons are scattered in collisions with individual nucleons as well as absorbed. This suggests that we introduce a mean free path, λ_s, for the scattering of a meson in nuclear matter. The mean free path, λ, for a nuclear interaction is then

$$1/\lambda=(1/\lambda_s)+(1/\lambda_a). \tag{23}$$

The scattering of mesons by nucleons is known to be of two types, simple and charge exchange, which are illustrated by the respective processes:

$$\pi^- + p \rightarrow p + \pi^-,$$
$$\pi^- + p \rightarrow n + \pi^0.$$

The only available evidence on the magnitude of the charge exchange scattering cross section is obtained from the measured absorption of π^- mesons in hydrogen, as done by Panofsky, Aamodt, and Hadley.[5] They found that the ratio of transition rates for the processes

$$\pi^- + p \rightarrow n + \gamma \quad (p^\gamma),$$
$$\pi^- + p \rightarrow n + \pi^0 \quad (p^{\pi^0})$$

is

$$T_{p^\gamma}/T_{p^{\pi^0}} \simeq 1.$$

The process (p^γ) is the inverse of photomeson production, so the transition rate T_{p^γ} can be calculated from detailed balancing arguments. From this we can obtain the charge-exchange scattering cross section for low-energy mesons:

$$\sigma[\pi^- + p \rightarrow n + \pi^0] = \left[1 + \frac{(\Delta M)c^2}{\epsilon_{\pi^-}}\right]^{\frac{1}{2}} 1.4(10)^{-27} \text{ cm}^2,$$

$$\sigma[\pi^0 + n \rightarrow \pi^- + p] = \left[1 - \frac{(\Delta M)c^2}{\epsilon_{\pi^0}}\right]^{\frac{1}{2}} 1.4(10)^{-27} \text{ cm}^2. \quad (24)$$

Here ΔM is the $\pi^- - \pi^0$ minus the neutron-proton mass difference, ϵ_{π^-} and ϵ_{π^0} are the respective π^- and π^0 kinetic energies. The necessary numerical detail to deduce Eqs. (24) has been given elsewhere.[4] These expressions are valid only for low energy mesons.

A direct measurement of the total cross section for scattering of 85-Mev π^- mesons by protons has been made by Chedester, Isaacs, Sachs, and Steinberger.[15] They found

$$\sigma[\pi^- + p] = (1.33 \pm 0.11)(10)^{-26} \text{ cm}^2. \quad (25)$$

Comparison with Eq. (24) suggests that the scattering cross section may increase with energy between low energies and 85 Mev. Such a conclusion is in agreement with the conclusions drawn from pseudoscalar meson theory with pseudovector coupling.[16] The cross section is

$$\sigma = 4\pi g^4(\hbar/\mu c)^2(q^4/E_q^2\mu^2), \quad (26)$$

where q is the meson momentum, E_q is its total energy, and μ is its rest mass. The value of $g^2 \simeq 0.15$ deduced from the photoproduction of π^+ mesons in hydrogen,[11] leads to a cross section only $\frac{1}{4}$ as large as that given by Eq. (25), however.

A study of experiments by Camac *et al.*[3] and by Shapiro[17] concerning the scattering of mesons by carbon has been made by Bethe and Wilson.[18] They deduce that[19]

$$\lambda \simeq 2.4 a_0. \quad (27)$$

[see Eq. (23)].

If we accept the value (25), we can estimate the scattering mean free path, λ_S, to be[20]

$$\lambda_S \simeq V_A/A\sigma[\pi^- + p] \simeq 6.3 a_0. \quad (28)$$

From Eqs. (23) and (27) we would then have

$$\lambda_a \simeq 4 a_0. \quad (29)$$

This value is not in disagreement with the absorption experiments [Eq. (22)], but appears a little too large to account for the photomeson production experiments [Eq. (7)]. The discrepancy probably arises from making a comparison of experiments performed at different energies.

VI. THE MECHANISM OF MESONIC ABSORPTION IN COMPLEX NUCLEI

In accordance with the model proposed in Sec. II, we shall examine the consequences of the hypothesis that meson absorption in nuclear matter takes place by a mechanism that is the inverse of meson production in free nucleon-nucleon collisions. We suppose a pair of nucleons to participate directly in the absorption event. These nucleons are expected to recoil with an energy of relative motion which is of the order of the meson rest energy (i.e., of the order of 70 Mev apiece). As these particles are ejected from their place in the structure of the initial nucleus, we may expect considerable excitation of the residual nucleus. There will, in general, be further excitation of the residual nucleus due to subsequent scatterings of the fast particles with others in the nucleus. We shall not concern ourselves with these latter events, as we are interested only in the total absorption rate.

To describe the absorption we shall employ the R-matrix formalism used by Watson and Brueckner[21] to describe meson production. That is, the transition

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[15] Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. **82**, 958 (1951).

[16] We do not consider the pseudoscalar form of coupling, since this is in disagreement with the observed production of π^+-mesons in $p-p$ collision (reference 6) and with the observed small cross section for producing π^0-mesons in $p-p$ collision [Bjorkland, Crandall, Moyer, and York, Phys. Rev. **77**, 213 (1950)]. The details are given by K. A. Brueckner, Phys. Rev. **82**, 598 (1951).

[17] A. Shapiro, Phys. Rev. **83**, 874(A) (1951).

[18] H. A. Bethe and R. R. Wilson, Phys. Rev. **83**, 690 (1951).

[19] *Note added in proof:*—This measurement was made at 45 Mev. Experiments by Bernardini, Booth, and Lederman (to be published) are in essential agreement with this result.

[20] We can possibly expect somewhat different characteristics in the scattering against free and bound nucleons. With a field theoretic model, the amplitude of the scattered wave is proportional to

$$\sum_I \frac{(F|H'|I)(I|H'|A)}{E\mu + \mu c^2 - (\epsilon_I - \epsilon_A)},$$

where ϵ_μ is the meson kinetic energy and $\epsilon_{A,I}$ are states of excitation of the nucleus. A large probability for true absorption suggests that states I for which the denominator in the above expression is small may contribute appreciably to the cross section. This would imply large nuclear excitation and meson energy loss.

[21] K. Watson and K. Brueckner, Phys. Rev. **83**, 1 (1951).

amplitude for the absorption of a meson by two nucleons (all described by plane waves) is

$$R=(\mathbf{p}|R^0|\mathbf{p}',\mathbf{q}_r')\delta(\mathbf{G}'+\mathbf{q}'-\mathbf{G}). \quad (30)$$

Here $p\simeq(M\mu)^{\frac{1}{2}}c$ is the relative momentum of the nucleons after absorbing the meson, $\mathbf{p}'$ is their relative momentum before absorbing the meson, $\mathbf{q}_r'$ is the relative momentum of the meson and the center-of-mass of the two nucleons. $\mathbf{G}'$ and $\mathbf{G}$ represent the total momentum of the two nucleons before and after the meson is absorbed and $\mathbf{q}'$ is the meson momentum. We assume the kinetic energy of the meson to be neglected in comparison with its rest-mass energy and the initial nucleons to be slow. To within terms of relative order $(\mu/2M)$, $\mathbf{q}_r'=\mathbf{q}'-\mu/2M\mathbf{G}$.

Transforming R to coordinate space, we have

$$R=(\mathbf{r}'|R^0|\mathbf{r},\mathbf{z}-\mathbf{x})\delta[\mathbf{x}'-\mathbf{x}-\mu/2M(\mathbf{z}-\mathbf{x})], \quad (31)$$

where $\mathbf{r}$ and $\mathbf{r}'$ are the relative coordinates of the two nucleons before and after the absorption, $\mathbf{x}$ and $\mathbf{x}'$ are the center-of-mass coordinates of the nucleons before and after the absorption, and $\mathbf{z}$ is the meson coordinate.

As described in I, the momentum transferred to the nucleons, of order p, suggests that the absorption takes place with the particles separated by a distance of order $\hbar/p$, which is considerably less than the range of nuclear forces. This suggests a zero range approximation, which was used by Watson and Brueckner.[21] If the nuclear forces are singular at small distances, the zero range approximation is inapplicable, so instead we follow the arguments of Brueckner, Chew, and Hart.[22] That is, for R operating on a bound state wave function, $\psi(r)$, we write

$$\int R^0\psi(r)d^3r=(\mathbf{r}'|R^0|,\mathbf{z}-\mathbf{x})\psi(\mathbf{r}_{\mathrm{Av}}). \quad (32)$$

Here $(\mathbf{r}'|R^0|,z-\mathbf{x})$ is independent of $\mathbf{r}$ and $\mathbf{r}_{\mathrm{Av}}$ is a distance of order $\hbar/p$. For a nonsingular potential, r_{Av} can be set equal to zero.

In accordance with the notions developed in I, we split R^0 into two parts, representing S- and P-state interactions with the meson.

$$R^0=(\mathbf{r}'|R_1|\mathbf{r},|\mathbf{z}-\mathbf{x}|)-i(\hbar/\mu c)\nabla_z\cdot(\mathbf{r}'|\mathbf{R}_2|\mathbf{r},|\mathbf{z}-\mathbf{x}|). \quad (33)$$

Because of the short ranges involved, we can assume only S-state interactions with the nucleons in the initial state (from the analysis of Watson and Brueckner[21] this seems substantiated to a good approximation).

To calculate the absorption rate, we assume that R is the absorbing mechanism in the nucleus A. For simplicity, we shall assume the meson to be bound in a coulomb S-state and that A is small enough that the meson wave function has a constant value, ϕ_0, over the volume of the nucleus. Then the transition amplitude for the absorption is

$$H_{FA}=\phi_0(\psi_F,R\psi_A), \quad (34)$$

where ψ_A and ψ_F are the initial and final nuclear wave functions, respectively.

We describe the two recoil nucleons by plane waves and because of their high energy neglect exchange effects between them and those in the residual nucleus. Then, if the coordinates of the particles in the initial nucleus are $\mathbf{x}_1,\mathbf{x}_2,\cdots\mathbf{x}_A$, we shall suppose for the moment that the nucleons with coordinates $\mathbf{x}_1$ and $\mathbf{x}_2$ absorb the meson and recoil. In terms of these coordinates we introduce the following:

$$\begin{aligned}
&\mathbf{X}''=\frac{1}{A}\sum_{i=1}^{A}\mathbf{x}_i,\quad \mathbf{X}'=\frac{1}{A-2}\sum_{i=3}^{A}\mathbf{x}_i,\\
&\mathbf{z}_i=\mathbf{x}_i-\mathbf{X}''\quad(i=1,2,\cdots,A-1),\\
&\mathbf{z}_i'=\mathbf{x}_i-\mathbf{X}'\quad(i=3,4,\cdots,A-1),\\
&\mathbf{r}'=\mathbf{x}_1-\mathbf{x}_2,\quad \mathbf{x}'=\tfrac{1}{2}(\mathbf{x}_1+\mathbf{x}_2).
\end{aligned} \quad (35)$$

Then the wave functions become

$$\begin{aligned}
\psi_A&=\frac{1}{J_A^{\frac{1}{2}}(2\pi)^{3/2}}\psi_A'(\mathbf{z}_1,\mathbf{z}_2,\cdots,\mathbf{z}_{A-1}),\\
\psi_F&=\frac{1}{J_{A-2}^{\frac{1}{2}}(2\pi)^{9/2}}\exp[i\mathbf{p}\cdot\mathbf{r}'+i\mathbf{G}\cdot\mathbf{x}'+i\mathbf{K}\cdot\mathbf{X}']\\
&\quad\times\psi_F'(\mathbf{z}_3',\mathbf{z}_4',\cdots,\mathbf{z}_{A-1}'),
\end{aligned} \quad (36)$$

where $\mathbf{p}$ is the relative momentum of the two recoil nucleons, $\mathbf{G}$ is their total momentum, and $\mathbf{K}$ is the momentum of the recoil nucleus. ψ_A' and ψ_F' are the wave functions of the initial and residual nuclei, respectively, normalized with respect to integrations over the $\mathbf{z}$ and $\mathbf{z}'$ coordinates. ψ_A and ψ_F are normalized with respect to a volume element $d^3x_1\cdots d^3x_A$, and J_A, J_{A-2} are defined by the transformations

$$\begin{aligned}
d^3x_1\cdots d^3x_A&=J_Ad^3X''d^3z_1\cdots dz_{A-1},\\
d^3x_1\cdots d^3x_A&=J_{A-2}d^3x_1d^3x_2d^3X'd^3z_3\cdots d^3z_{A-1}.
\end{aligned} \quad (37)$$

We have $J_A=A^3$; $J_{A-2}=(A-2)^3$.

We can neglect the small contribution of the term involving R_2 in Eq. (33) to Eq. (34).

Let us define

$$(p|R_1|r)\equiv(2\pi)^{-3}\int\exp[-i\mathbf{p}\cdot\mathbf{r}'](\mathbf{r}'|R_1|\mathbf{r},|\mathbf{z}-\mathbf{x}|)\times\exp[-i\mathbf{G}\cdot(\mathbf{z}-\mathbf{x})\mu/2M]d^3r'd^3z, \quad (38)$$

where the dependence on G is neglected because of the assumed short range of the interaction and the smallness of $\mu/2M$.

Then Eq. (34) becomes

$$\begin{aligned}
H_{FA}&=\frac{1}{(2\pi)^3}\phi_0\frac{1}{(J_AJ_{A-2})^{\frac{1}{2}}}\int\exp[i\mathbf{G}\cdot\mathbf{x}-i\mathbf{K}\cdot\mathbf{X}']\\
&\quad\times\psi_F'^*(\mathbf{z}_3'\cdots\mathbf{z}_{A-1}')(p|R_1|r)\\
&\quad\times\psi_A'(\mathbf{z}_1\cdots\mathbf{z}_{A-1})d^3x_1\cdots d^3x_A\\
&\equiv\delta(\mathbf{G}+\mathbf{K})H_{FA}^0,
\end{aligned} \quad (39)$$

[22] K. Brueckner, Phys. Rev. 82, 598 (1951).

where

$$H_{FA}{}^0=\phi_0(J_A/J_{A-2})^{\frac{1}{2}}\times\int\exp[-\tfrac{1}{2}i\{A/(A-2)\}\mathbf{G}\cdot(\mathbf{z}_1+\mathbf{z}_2)]\times\psi_F{}'^*(\mathbf{z}_3{}',\cdots,\mathbf{z}_{A-1}{}')(p|R_1|\mathbf{z}_1-\mathbf{z}_2)\times\psi_A{}'(\mathbf{z}_1,\cdots,\mathbf{z}_{A-1})d^3z_1\cdots d^3z_{A-1}. \quad (40)$$

Consistent with our assumption that most of the meson rest energy goes into the relative motion of the two fast nucleons, we shall use a partial closure approximation to evaluate the total absorption rate. That is, we set $p=(M\mu)^{\frac{1}{2}}c$ and sum over the states, F, and the momentum $\mathbf{G}$. After some algebra, we obtain

$$I\equiv\int d^3G\sum_F|H_{FA}{}^0|^2$$
$$=(2\pi)^3\phi_0{}^2\int\psi_A{}'^*(\mathbf{u},\mathbf{Z},\mathbf{z}_3,\cdots,\mathbf{z}_{A-1})R_1{}^*(\mathbf{u})R_1(\mathbf{z})\times\psi_A{}'(\mathbf{z},\mathbf{Z},\mathbf{z}_3,\cdots,\mathbf{z}_{A-1})d^3u\,d^3z\,d^3Z\,d^3z_3\cdots d^3z_{A-1}, \quad (41)$$

where $\mathbf{u}$ and $\mathbf{z}$ are $(\mathbf{z}_1-\mathbf{z}_2)$ and $\mathbf{Z}=\frac{1}{2}(\mathbf{z}_1+\mathbf{z}_2)$. Using the relation (32), this reduces to

$$I=(2\pi)^3\phi_0{}^2|R_1|^2P(\mathbf{z}_{Av}), \quad (42)$$

where $P(\mathbf{z}_{Av})$ is the probability of finding the two absorbing nucleons at a distance $\mathbf{z}_{Av}$ apart in the nucleus A and $|R_1|^2$ is a constant.

For the capture in deuterium, one would have

$$I^D=(2\pi)^3\phi_0{}^2|R_1|^2P_D(\mathbf{z}_{Av}{}^D), \quad (43)$$

where $P_D(\mathbf{z}_{Av}{}^D)$ is the probability of finding the neutron and proton at a distance $z_{Av}{}^D$ in deuterium. Presumably, $z_{Av}{}^D\simeq z_{Av}$ and both can be set equal to zero if the forces between elementary particles are not singular at close distances of approach.

For the capture of a meson in flight there will also be P-state capture [the term R_2 in Eq. (35)]. Then to the approximation that the meson kinetic energy can be neglected compared to its rest-energy, I in Eq. (42) is modified to become

$$I=(2\pi)^3|R_1|^2\left[1+b\frac{q^2}{\mu^2c^2}\right]P(\mathbf{z}_{Av}) \quad (44)$$

where q is the meson momentum and b represents the relative strength of the S- and P-state meson couplings [see Eq. (2)]. Eq. (43) will be similarly modified.

We must now consider the various possible means of absorption. For a π^- meson, we have the processes given in Sec. II:

$$\pi^-+n+p\rightarrow 2n \quad (\pi^-, pn),$$
$$\pi^-+p+p\rightarrow n+p \quad (\pi^-, 2p).$$

As argued above, we can assume the reaction to occur from an initial S-state of the nucleons. Then for the capture of the meson from a P-state we have the transitions (permitted by angular momentum and parity conservation, see reference 21)

$$(\pi^-, pn)\quad {}^3S\rightarrow{}^1S,\ {}^1D,$$
$$(\pi^-, 2p)\quad {}^1S\rightarrow{}^3S,\ {}^3D$$

for the nucleon states. For the capture from an S-state of the meson, we know from the deuterium capture[5] (see I) that a triplet→singlet transition accounts for a considerable fraction of the total transition rate. If we neglect small effects from a possible singlet→triplet absorption from a meson S-state, we can evaluate the total transition rate due to process (π^-, pn) in the nucleus A on the assumption of a statistical distribution of spin and parity states of the neutrons and protons. Since, then, only $\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$ of the neutrons and protons will have triplet spin and even parity, we have, on summing I over neutron-proton pairs,

$$I(\pi^-, pn)=\tfrac{3}{8}Z(A-Z)(2\pi)^3|R_1(np,t\rightarrow s)|^2\times[1+bq^2/\mu^2c^2]P(z_{Av}). \quad (45)$$

Here $R_1(np,t\rightarrow s)$ is the appropriate R_1 for $n-p$ absorption with a triplet→singlet spin transition.

Since the same spin transitions occur for the absorption in deuterium, we have again the same value of $|R_1|^2$ and b.

For process $(\pi^-, 2p)$, we have a singlet→triplet spin transition for both meson S- and P-states. Statistically, $\frac{1}{4}$ of the proton pairs will be in a singlet state. However, a factor of 3 is obtained relative to Eq. (45) in performing the sum over the final triplet substates. Thus, summing over proton pairs, we replace

$$\tfrac{3}{8}Z(A-Z)\text{ by }\tfrac{3}{4}[Z(Z-1)/2]$$

in Eq. (45). We can expect now a different value for $|R_1|^2$, b and $P(z_{Av})$. However, since the corresponding production cross sections seem to be of the same order of magnitude, we can probably choose $|R_1|^2$ and b to be the same for processes (π^-, pn) and $(\pi^-, 2p)$ without being greatly in error. For purposes of argument, we shall also set $P(z_{Av})$ equal for both processes. We can then write the absorption cross section per proton as

$$\frac{\sigma}{Z}[\pi^-+A\rightarrow\text{Star}]=(2\pi)^8\frac{pM}{2v_\pi}[\tfrac{3}{8}(A-Z)+\tfrac{3}{8}(Z-1)]\cdot|R_1|^2\left[1+b\frac{q^2}{\mu^2c^2}\right]P(z_{Av}), \quad (46)$$

where M is the nucleon mass.

Taking the ratio to the absorption cross section for deuterium, we have

$$\frac{(\sigma/Z)[\pi^-+A\rightarrow\text{Star}]}{\sigma[\pi^-+\text{D}\rightarrow 2n]}\equiv\Gamma=[\tfrac{3}{8}(A-Z)+\tfrac{3}{8}(Z-1)]\frac{P(\mathbf{z}_{Av})}{P_D(\mathbf{z}_{Av}{}^D)}. \quad (47)$$

Our choice of statistics in the nucleus is not so arbitrary as might be thought, since it amounts primarily to a choice of normalization of $P(z_{Av})$.

To interpret $P(z_{Av})$ and $P_D(z_{Av}^D)$ further, we set $z_{Av}^D=0$. Then

$$P_D(0)=|\psi_D(0)|^2,$$

where $\psi_D(0)$ is the deuteron wave function for zero separation of the neutron and proton. $P_D(0)$ is just the probability of finding them in contact. We write

$$P(z_{Av})=f/(4\pi/3)a_0^3A \quad (48)$$

or a correlation factor divided by the nuclear volume. $f=1$ would correspond to random spacing of particles in a box of nuclear volume. Using the Chew-Goldberger[23] wave function for the deuteron, we have

$$P(z_{Av})/P_D(0)=0.82f/A. \quad (49)$$

For C^{12}, Eq. (47) yields $\Gamma=0.28f$. Choosing the reasonable value $\Gamma\simeq 10$, we have

$$f\simeq 35.0. \quad (50)$$

This would seem to indicate a reasonably strong degree of correlation in nuclear structure. Such a conclusion appears quite compatible with the evidence presented by several authors from high energy $p-p$ scattering[24,25] for strong nuclear interactions at close distances. It is also compatible with the evidence concerning nuclear structure which was given by Chew and Goldberger[23] on the basis of York's [26] measurement of high energy $(n-d)$ processes (see also the discussion in I on this point).

The analysis of π^+ absorption can be carried through in the same manner. As mentioned in Sec. II, we have reason to expect the absorption of π^+ and π^- mesons to be similar.

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On the Polarization of High Energy Bremsstrahlung and of High Energy Pairs

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The polarization of bremsstrahlung due to electrons with initial energies much larger than $137Z^{-\frac{1}{3}}\ mc^2$ is calculated under relativistic, small angles approximations. The cross section for photons polarized normally to the plane containing the initial direction of the electron and the direction of the photon is found to be larger than for photons polarized in that plane. A similar calculation shows that the plane containing one of a pair produced by a polarized photon together with the direction of that photon tends to lie parallel to the plane of polarization rather than normal to it, except for one special case. The effect of the deviation due to multiple scattering of electrons in the target upon the angular dependence of the polarization is considered.

In this note we shall investigate the polarization of bremsstrahlung due to electrons of energy $E_0 \gg 137Z^{-\frac{1}{3}}\ mc^2$, where Z is the atomic number of the target material. We shall then carry out analogous calculations for pairs produced by high energy, polarized γ-rays and obtain a preferred azimuth of the plane of the pairs relative to the plane of polarization. In the last part of the paper, we shall consider the effect of multiple scattering of electrons in the target upon our results for the polarization of bremsstrahlung. This note has been written in confirmation and in partial extension of previous results[1,2] obtained by using the method of virtual quanta.[3]

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I. BREMSSTRAHLUNG

Let us consider an electron of total energy E_0, momentum $\mathbf{p}_0$, deflected by a nucleus of charge Ze. Let a quantum of momentum $\mathbf{k}$ (we take $c=1$ from here on) be radiated at an angle θ_0 with the initial direction of the electron. (See Fig. 1.) After radiation, let E be the total energy and $\mathbf{p}$ the momentum of the deflected electron, and let its direction make an angle θ with the direction of the emitted quantum. Call ψ the angle between the $\mathbf{p}_0\mathbf{k}$ plane (plane of emission) and the $\boldsymbol{\varepsilon}\mathbf{k}$ plane, where $\boldsymbol{\varepsilon}$ is the polarization vector of the photon; call φ the angle between the $\mathbf{p}_0\mathbf{k}$ and the $\mathbf{pk}$ planes, and φ_0 the angle between the $\mathbf{p}_0\mathbf{k}$ plane and some fixed plane. If $\mathbf{q}$ is the momentum given up to the nucleus, the conservation conditions read:

$$\mathbf{q}=\mathbf{p}_0-\mathbf{p}-\mathbf{k};\quad E_0=E+k.$$