The sum of squared matrix elements $B_{n_3n_4}{}^{(c)}$ is

$$B_{n_3n_4}{}^{(c)}=(\gamma_3{}^{n_3}/n_3!)(\gamma_4{}^{n_4}/n_4!)[(2\hbar\omega_3)^{-1}(P^2/2m)]^{n_3+n_4}\times\exp[c(2\hbar\omega_3)^{-1}(P^2/2m)],$$

where

$$\gamma_3=(1/24)\cos^2\beta,\quad \gamma_4=(1/24)(\omega_3/\omega_4)\sin^2\beta,\quad c=\gamma_3+\gamma_4.$$

The reduced mass of the system (neutron+CH_4) is $\nu=16/17$. Setting $s=2\nu$, one obtains formulas quite analogous to those of the cross section per proton. It must be emphasized, however, that these formulas do not involve any mass tensor approximation. Below the threshold of the first inelastic collision, 0.1788 volt, one gets

$$\sigma^{(c)}/\sigma_f{}^{(c)}=\tfrac{1}{4}(13/12)^2\{[1-\exp(-s^2cy)]/cy+(\Theta/y)s(2-s)(\tfrac{1}{2}-s^2cy)\exp(-s^2cy)+(\Theta^2/y)cs^2(2-s)^2(-\tfrac{3}{8}+\tfrac{3}{2}s^2cy-\tfrac{1}{2}s^4c^2y^2)\exp(-s^2cy)+\cdots\}.$$

The general formula for inelastic scattering is

$$\bar{\sigma}_{n_3n_4}{}^{(c)}/\sigma_f{}^{(c)}=\tfrac{1}{4}(13/12)^2(1/y)(N!/c^{N+1})\times(\gamma_3{}^{n_3}/n_3!)(\gamma_4{}^{n_4}/n_4!)[f_N(v_+)-f_N(v_-)]$$

with

$$N=n_3+n_4,\quad v_\pm=\tfrac{1}{4}cs[(sy)^{\frac{1}{2}}\pm(sy-2y_N)^{\frac{1}{2}}]^2,$$
$$y_N\equiv(2\hbar\omega_3)^{-1}E_N=\tfrac{1}{2}[n_3+(\omega_4/\omega_3)n_4].$$

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The Scattering of Fast Neutrons by Bismuth and Lead

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Using good geometry, homogeneous fast neutrons of energy 4.3 Mev, supplied by the D−D reaction in the Bartol Van de Graaff statitron, have been scattered in bismuth and lead. Using binocular microscopes and Eastman NTA plates, fifty thousand fields of view were examined to obtain 5000 acceptable recoil proton tracks. The energy spectrum of the scattered neutrons was studied to establish the presence or absence of any inelastic groups. From a consideration of gross effects, it is concluded that no inelastic group exists in the bismuth data with an intensity greater than five percent of the elastically scattered neutrons, or that no level in Bi^{209} lying between the ground state and 3.4 Mev is excited by a neutron group of more than five percent of the intensity of the elastic group. Appreciable inelastic scattering was noted in naturally occurring lead, the inelastic scattering cross section being less than half the elastic cross section. The energy distribution of the inelastically scattered neutrons is interpreted to indicate an energy level in lead in the vicinity of 3.3 Mev. This level is consistent with radioactivity measurements which show that most of the beta-disintegrations of $ThC'' \xrightarrow{\beta} {}^*Pb^{208}$ lead to energy levels in the vicinity of 3.2 Mev, and with Q-values of the reaction $Pb^{207}(d,p)Pb^{208}$ which disclose an energy level at 3.45 Mev in the nucleus Pb^{208}.

INTRODUCTION

When neutrons are scattered by nuclei, both elastic and inelastic scattering can occur. The elastic process takes either of two forms, potential scattering or resonance scattering. The former is sometimes referred to as "hard sphere" or "diffraction" scattering, and the latter may be termed "capture elastic scattering." The capture elastic scattering and inelastic scattering are considered to be two phases of the same nuclear process, because an intermediate compound nucleus is actually formed, the product nucleus being left in the ground state after the elastic collision and in an excited state after the inelastic collision. In the case of potential or diffraction scattering, the compound nucleus is not formed. The foregoing terminology has been previously outlined by Feld.[1]

Neutrons from the deuteron-deuterium reaction have been employed in two previous scattering experiments dealing with the elastic and inelastic scattering of fast neutrons by lead.[2,3] These measurements have been interpreted by Feld[1] to give energy levels in naturally occurring lead at ~0.8 Mev,[2] and in the vicinity of 0.92, 1.87, and 2.67 Mev.[3]

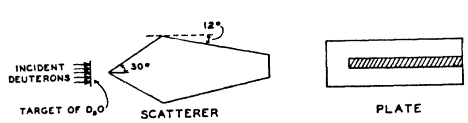

Fig. 1. Geometric arrangement for fast neutron scattering. The average length of the scatterer is 6.1 cm.

It was decided to irradiate both bismuth and lead with fast neutrons to observe the inelastically scattered neutrons, because in both cases magic numbers are involved.[4] All of the naturally occurring isotopes of lead are "magic" for protons, since each of them contains 82 protons; Pb^{208}, in particular, may be regarded as a double closed shell nucleus, because it contains 82 protons and 126 neutrons, both of which are magic numbers. The monoisotopic Bi^{209} contains 83 protons and 126 neutrons and therefore is magic for neutrons.[4] Magic number nuclei should have widely spaced energy levels and should not, therefore, follow the statistical theory of Weisskopf.[5]

The previously mentioned data[2,3] were obtained with the use of "poor geometry," whereas the measurements of the present paper employed "good geometry" for better resolution.

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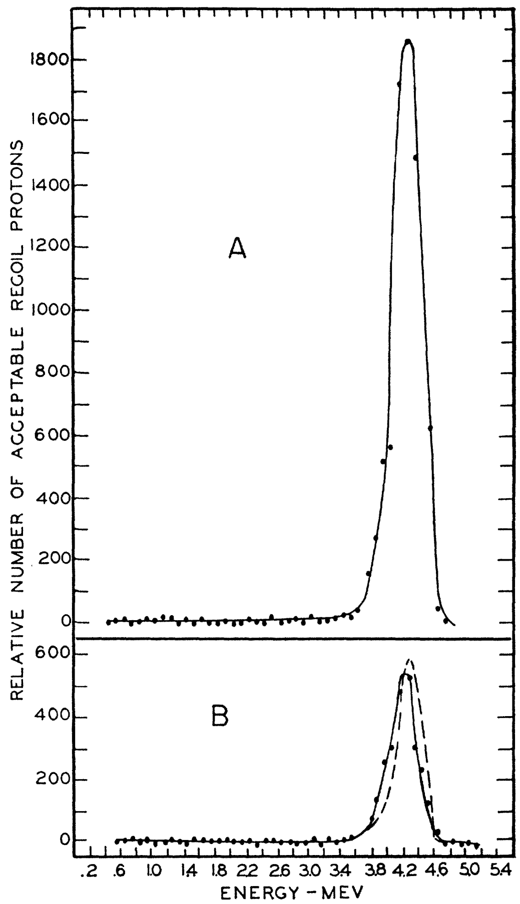

FIG. 2. Curve A is the "background run" or data taken without scatterer so that the data collected with the scatterer in position could be properly corrected for "room scattering" and other effects. Curve B was observed with the bismuth scatterer in position. The two curves were "normalized" to take into account irradiation time and plate area scanned.

GEOMETRIC ARRANGEMENT AND EXPERIMENTAL PROCEDURE

Fast neutrons were prepared in the Bartol Van de Graaff statitron when deuterium targets were bombarded by deuterons of mean energy 1.15 Mev. The targets were of heavy ice calculated to have a thickness of 0.2 Mev for 1.25-Mev deuterons. The scatterer was placed before the target as shown in Fig. 1. As indicated in the figure, an angle of 30° in the forward direction was intercepted by the scatterer at the target, resulting in a mean neutron energy of 4.36 Mev and an energy spread of 0.27 Mev. Since an acceptance angle of 12° in the forward direction was employed in selecting recoil protons in the photographic plates, the half-angle of the truncated cone of the scatterer adjacent to the plates was chosen to be 12°. Recoil proton track lengths in Eastman NTA plates were measured in calibrated binocular microscopes. Only tracks lying in the shaded area of the plates were observed so the acceptance angle would be preserved.

The general procedure was to expose the plates with the scatterer in position, then remove the scatterer, replace the plates, and observe the background or the shape of the neutron spectrum with no scatterer in place. Fresh targets were frequently prepared, and the experimental arrangements with and without scatterer were alternated so that for each target a constant fraction of bombarding time was given to the background run and to the run with scatterer in position. If a newly prepared target was first employed in a background bombardment, the next target was first irradiated with scatterer in position. This procedure was followed in order to compensate for any changes in target thickness or deposition of carbon which might occur in the course of a given bombardment. About one thousand tracks were usually collected over the entire spectrum both for the case of background, and for the case of the scatterer in position. Beyond this point the actual recording of the tracks was limited to those lying below 3.7 Mev in energy. This was done to increase the rapidity of scanning and the statistical accuracy in the region of inelastic scattering. Before the data with and without scatterer could be compared, they had to be normalized to take into account plate area scanned and bombarding time.

In neutron measurements, room scattering is usually a serious problem. To reduce this effect, the beam of the Van de Graaff statitron was brought to a point about 6 ft from the iron beam focusing magnet and 4 ft above the floor. The flooring was lined with about 5 cm of paraffin to further reduce scattering.

BISMUTH

The energy distribution of the fast neutrons scattered from Bi^{209} is shown in Fig. 2. The data without scatterer are shown in Fig. 2(A), and the measurements with scatterer intervening between target and plates is shown in Fig. 2(B). From the areas under the two curves, it is calculated that the principal group is reduced to about 30 percent of its initial value by insertion of the scatterer. Using the peak of Fig. 2(A) as a model, the data of Fig. 2(B) were replotted (indicated by the broken line) to give the shape expected

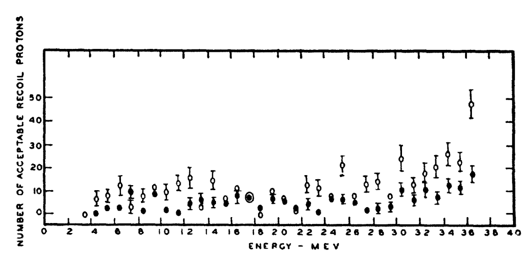

FIG. 3. Distribution of neutron energies less than or equal to 3.7 Mev collected with and without the bismuth scatterer in position. The data are plotted in energy intervals of 0.10 Mev. The open circles refer to the "background run" with no scatterer, and the closed circles to data obtained with scatterer in position. The open circles clearly dominate, showing that the number of neutrons in the inelastic region is reduced with insertion of the scatterer.

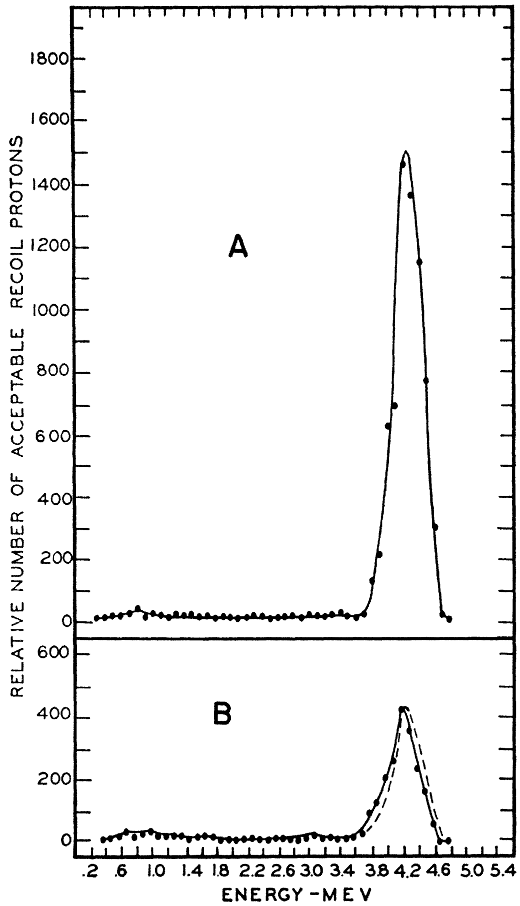

FIG. 4. Curve A is the "background run" or data taken without scatterer. Curve B was observed with the lead scatterer in position.

were no scatterer present. The area under either curve not under the other represents a considerable fraction of the elastically scattered neutrons. Thus, a minimum of twenty percent of the tracks in the principal group of Fig. 2(B) is attributed to elastic scattering, and the remaining percentage is composed of primary neutrons, passing through the scatterer to the photographic plates without suffering scattering of any kind. Since 2500 tracks are contained in the main group of Fig. 2(B), at least 500 are elastically scattered.

When the data below 3.7 Mev were plotted in detail as shown in Fig. 3, it became apparent that *more* acceptable tracks were found without the scatterer in place than when it was present. This fact indicates that most of the background neutrons emanate from a direction from which the plates are shielded with the scatterer in position, probably coming from the target, the target backing, and the glass target chamber. At first thought, the disturbing conclusion was reached that the background, though very small, was sufficient to mask any real inelastic scattering. However, a method of analysis was developed which made possible an estimate of an upper limit for inelastic scattering as compared to elastic scattering. The region of energy from 0.4 Mev to 3.7 Mev was divided into energy intervals of width 0.2 Mev. The value of $\exp(-n_{\mathrm{Bi}}\sigma_{\mathrm{Bi}}x)$ was calculated for each energy interval.[6] From these values, letting I_{0j} be the number of tracks found on the jth interval of the background run, and letting I_j be the background with the scatterer in position, I_j could be calculated. The total number of background tracks on the large interval, 0.4 Mev$\leqslant E_n \leqslant$3.7 Mev, the region of inelasticity, is given by

$$I_0=\sum_1^N I_{0j},$$

and the total number of background tracks in the region of inelasticity with scatterer in position can be calculated from

$$I=\sum_1^N I_j=\sum_1^N I_{0j}\exp[-(n_{\mathrm{Bi}}\sigma_{\mathrm{Bi}})_j x].$$

Thus from Table I, it is seen that the calculated background for bismuth with scatterer in position is 167±8 tracks as compared with 201±10 tracks actually observed at neutron energies of 3.7 Mev or less. The difference, 34±13 tracks, could be explained by scattering from directions not shielded by the scatterer or by elastic scattering of the background neutrons. At any rate, were the inelastic contribution as much as 100 tracks spread over the region below 3.7 Mev, it would have been detected. Were fifty inelastic tracks concentrated in an energy interval of 0.5 Mev or less, they would have been detected. Since there are at least 500 inelastically scattered tracks, the estimates of 100 tracks and 50 tracks set conservative upper limits of 20 percent for an inelastic continuum and 10 percent for an inelastic group in terms of the elastic scattering. Since, on the average, the correction factor for neutron-proton scattering cross section and acceptance probability is a factor of two greater for the region below 3.7 Mev than for the principal group, the upper limits of the percentages are reduced to 10 percent and 5 percent, respectively.

TABLE I. Number of tracks, calculated and observed, in the inelastic region, for Bi.

Observed I_0	Observed I	Calculated I
424±25	201±10	167±8

TABLE II. Numbers of tracks, calculated and observed in the inelastic region, for Pb.

Observed I_0	Observed I	Calculated I
447±16	479±16	155±7

[6] $x=6.1$ cm, the average length of the scatterer.

LEAD

Curves similar to those of bismuth have also been obtained for lead. Figure 4(A) is a plot of the background run for lead, and Fig. 4(B) is one of the neutron distribution with the lead scatterer in position. As described in the section under bismuth, the data of Fig. 4(B) were replotted to give an estimate of the elastic scattering cross section in lead. It turns out that at least sixteen percent or 300 tracks in 1900 can be attributed to elastic scattering.

From Table II, it is seen that 479 ± 16 tracks were observed at 3.7 Mev or less, the region of inelasticity, whereas 155 ± 7 tracks would be expected from background alone. Then 324 ± 17 tracks in the region of 3.7 Mev or less must be attributed to inelastic scattering of the 4.3-Mev primary neutrons in lead. This is to be compared with at least 300 tracks elastically scattered. The average correction for neutron-proton scattering cross section and acceptance probability further reduces the inelastic total by a factor of two, so that the inelastic scattering cross section for 4.3-Mev neutrons in lead is

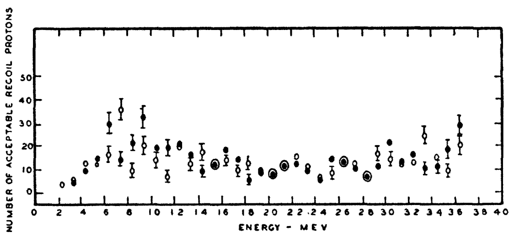

FIG. 5. Distribution of neutron energies less than, or equal to, 3.7 Mev collected with and without the lead scatterer in position. The data are plotted in energy intervals of 0.10 Mev. The open circles refer to the "background run" with no scatterer, and the closed circles to data obtained with scatterer in position.

less than one-half as great as the elastic cross section. The data for lead obtained both with and without the scatterer in position are shown plotted in intervals of 0.10 Mev in Fig. 5.

Thus far, inelastic scattering in lead has been established purely from a consideration of the gross effects of Table II. A more detailed approach is necessary to obtain an actual curve of the inelastic contribution. Dividing the region 0.4 Mev$\leqslant E_n \leqslant$3.7 Mev into intervals of width 0.2 Mev, the actual background with scatterer in position was calculated for each interval according to the equation

$$I_j = I_{0j}\exp[-(n_{\mathrm{Pb}}\sigma_{\mathrm{Pb}})_j x].$$

I_{0j} is the number of tracks observed in the jth energy interval without scatterer, and I_j is taken to be the true background with scatterer in position. This value, I_j, was then subtracted from the number of tracks observed in the energy interval with scatterer in position.

The resulting curve is plotted in Fig. 6. When the observed curve is corrected for variation with energy of the neutron-proton scattering cross section and ac-

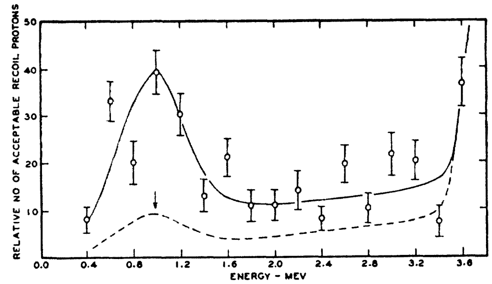

FIG. 6. Distribution in energy of 4.3-Mev neutrons inelastically scattered in naturally occurring lead. The broken line is the distribution after correction for variation with energy of n-p scattering cross section and acceptance probability.

ceptance probability, the broken line of Fig. 6 is obtained. This curve is interpreted to represent a neutron group at an approximate neutron energy of 1.0 Mev, corresponding to an energy level in naturally occurring lead at $\sim$3.3 Mev. The evidence for existence of the level in lead at 3.3 Mev finds some confirmation in natural radioactivity, since, by the coincidence method, it has been shown[7] that each beta-ray of the transition $\mathrm{ThC}'' \xrightarrow{\beta} {}^*\mathrm{Pb}^{208}$ is followed, on the average, by 3.2 Mev of gamma-ray energy. About fifty percent of the naturally occurring lead nuclei are Pb^{208}. The remaining percentage is distributed among Pb^{204}, Pb^{206}, and Pb^{207}. A level at 3.45 Mev has been reported[8] excited in the reaction $\mathrm{Pb}^{207}(d,p)\mathrm{Pb}^{208}$. The odd fact that the well-known level at 2.62 Mev in Pb^{208} is excited in radioactive decay and not in the above-mentioned primary disintegration has been previously emphasized.[9] This point is strengthened by the neutron scattering experiments.

The question of carbon contamination arises when the group at 1.0 Mev is considered in detail. The experimental procedure outlined earlier in the paper should have taken into account the carbon effects. Moreover, the observed peak at $\sim$1.0 Mev occurs at too large an energy for the bombarding energy of 1.15 Mev to assign it to carbon. The effect of elastic scattering is to shift the peak to a lower value rather than to a slightly higher one. Carbon peaks observed at the same bombarding energy and with the same geometry occur at about 0.8 Mev or less.[10] In Fig. 6, contributions to the inelastic peak are appreciable at 1.3 Mev.

CONCLUSIONS

Whatever the interpretation of the fine structure of the results may be, it can be concluded from the data of this paper that in the inelastic scattering process, bismuth behaves entirely differently from lead, since much more inelastic scattering of fast neutrons of energy 4.3 Mev has been found to occur in lead.

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