(d,n) Reactions with 15-Mev Deuterons. II. Neutron Energy Spectra and Yields*

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Energy spectra of neutrons produced by bombarding several thick targets with 15-Mev deuterons were measured at various angles by observing proton recoils in photographic plates and in a triple coincidence-anticoincidence proportional counter telescope. The spectra measured in or near the forward direction strongly indicate a stripping rather than a compound nucleus interaction, while for those measured at large angles the opposite is the case. Analyses of the shapes of the spectra at low energy, and a study of the yields as a function of atomic number indicate that the stripping process is one in which the penetration of a coulomb barrier is important. This, together with an estimate of the relative cross sections for stripping and compound nucleus formation, also seems to favor a stripping process in which the proton is captured by nuclear forces rather than one in which the deuteron is broken up by electric forces.

I. ENERGY SPECTRA

IN connection with the work described in an earlier paper,[1] measurements were made of the energy distributions of neutrons emitted upon bombardment of several thick targets by 15-Mev deuterons in the University of Pittsburgh cyclotron.

A. Experimental Procedure

The low energy portions of the neutron energy spectra were obtained by measuring tracks of recoil protons in 150- and 200-micron Kodak NTB nuclear research emulsions. The plates were wrapped in black paper and placed, without shielding, in positions chosen for their proximity to the target (about 50 cm) and large distances from scattering objects. After developing by standard thick emulsion processing techniques,[2] track counting was carried out as follows: tracks were measured which: (a) started within 20 microns of the top of the emulsion heading in a downward (into the emulsion) direction, (b) started within 15 degrees of the direction from the target in the horizontal plane and between 2 and 12 degrees in the vertical plane.

As a result of using thick emulsions and following specification (a), all but about 4 percent of the tracks ended in the emulsion—almost always because of being scattered back through the top surface—thus eliminating the necessity for the usual large correction factors.[2,3]

For purposes of plotting, track lengths were assorted into groups of $\frac{1}{2}$-Mev width (in some cases at very high energies, wider groups were adopted), the number of tracks in each group being taken as the intensity at the group center. The n-p scattering cross section was taken as inversely proportional to the neutron energy, and plots were made of neutron intensity divided by neutron energy against the neutron energy on a semilog scale. Thus, if the intensity, I, varies as

$$I(E) \sim E \exp(-E/\epsilon), \tag{1}$$

where ϵ is a slowly varying function of E, the plot is a line with slope $-1/\epsilon$. From the similarity between Eq. (1) and a maxwell distribution, ϵ may be called the "effective nuclear temperature." Most measurements were carried out interpreting specification (a) as including tracks which start at the top surface of the emulsion having originated in the paper wrapping. No significant difference was found above $\sim$3 Mev between spectra measured with and without these tracks, and an analysis reveals that little difference is to be expected. In Fig. 1, the dots show the calculated distortion in a typical spectrum (line B) produced by accepting these tracks. The slope of the best straight line (line A) through the portion of the distorted spectrum between 3 and 12 Mev differs from the slope of the actual spectrum by only about 7 percent.[4] This method has the very considerable advantage of greatly increasing the density of countable tracks without increasing the background, thus cutting down the time required by a large factor. Each spectrum represents about 500 tracks. For energies below 3 Mev, the methods described above break down seriously so that separate spectra using standard track measuring techniques[3] were taken in this region. However, because of (a) the possibility of observer bias in selecting tracks owing to the great variation in lengths observable in a single microscope field (for energies above 3 Mev, all tracks travel out of the microscope field in which they start) and (b) the difficulty in judging dip angles of short tracks (they usually are somewhat scattered from their beginnings), these data cannot be considered on a par with those at higher energies. For purposes of interpretation, only the relative shapes for various targets will be considered in this region.

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[1] C. Falk, Phys. Rev. 83, 499 (1951).
[2] M. J. Wilson and W. Vanselow, Phys. Rev. 75, 1144 (1949). Plates were somewhat overdeveloped to reduce grain density differences between low and high energy tracks.
[3] C. F. Powell and F. C. Champion, Proc. Roy. Soc. (London) 183, 64 (1944); H. T. Richards, Phys. Rev. 59, 796 (1941).
[4] This method is valid if ϵ is a monotonically and slowly varying function of the energy.

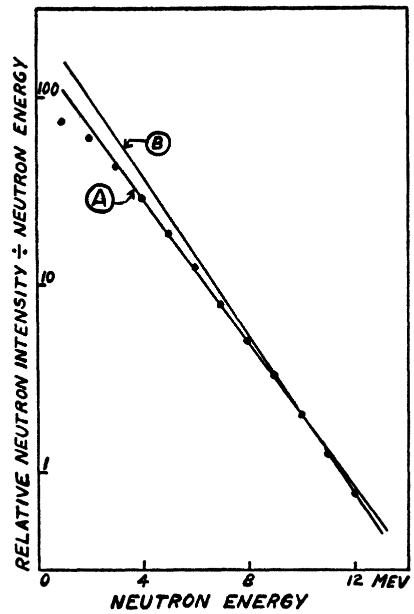

FIG. 1. Calculated distortion in a photographic plate spectrum from accepting tracks which start on the emulsion surface (having started in the paper wrapping). Line B is assumed to be the true spectrum. The dots indicate observed intensities; line A is the best line through the dots between 3 and 12 Mev. The slopes of lines A and B differ about 7 percent.

The high energy part of the spectra were measured with the proportional counter telescope used in an earlier paper.[1] By varying the aluminum absorbers in front of the first and fourth proportional counter it is possible to measure the relative neutron intensity of any energy band above 8.25 Mev. (The minimum value of 8.25 Mev corresponds to the energy necessary for a proton to traverse the system when no absorbers are placed in its path).

The experimental procedure was similar to the one described in Part I[1] for the measurement of angular distributions. The telescope was placed at a fixed angle with respect to the direction of the incident deuteron beam and the relative neutron intensities were measured as a function of energy by varying the thicknesses of the aluminum absorbers. In order to interpret the experimental data it is necessary to analyze what the counter-telescope measures. Corrections have to be made because (a) not all the protons within a given energy interval are counted and (b) the energy intervals are not always the same width.

Neutrons of energy E_n will produce protons of energy $E_p = E_n \cos^2\theta$ at any point within the polyethylene, from the outside layer to the inside layer. The absorbers were designed in such a way that only those protons of energy $E_{\min}$ would be detected which came from the innermost layer of the polyethylene, and only those of $E_{\max}$ which came from the outer layer of polyethylene. Thus, for example, protons of energy $E_{\max}$ and $E_{\min}$ coming from the middle layer of polyethylene would not be detected. As a result $f(E_p)$, the probability of a proton of energy E_p being detected, is zero at $E_{p\,\max}$ and $E_{p\,\min}$ and unity from $(E_{p\,\min}+\Delta_1)$ to $(E_{p\,\max}-\Delta_2)$ where Δ_1 and Δ_2 are the energies lost by protons of energies $E_{p\,\max}$ and $E_{p\,\min}$ in traversing the thickness of the polyethylene foil. The value of $f(E_p)$ in the intervals Δ_1 and Δ_2 was assumed to vary linearly.

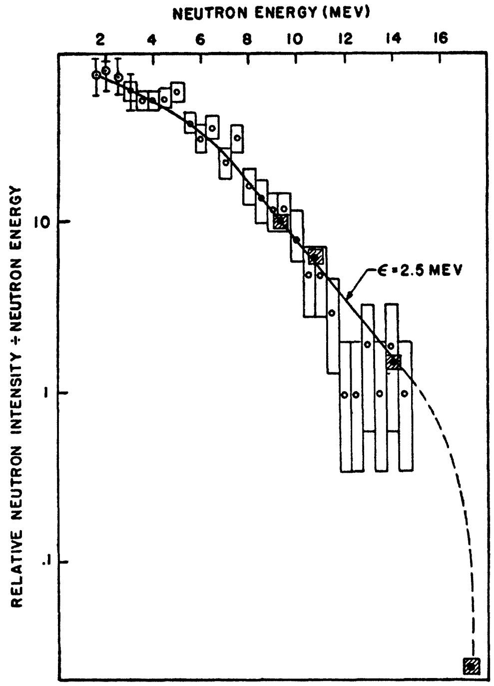

FIG. 2. Neutron energy spectrum at 0° from 15-Mev deuterons on beryllium. Open rectangles and points indicate photographic plate data, and cross-hatched squares are data from counter telescope. The data indicated by the open points (energies of 3 Mev and below) should be considered only indicative of the relative shape of the curve, as it is difficult to estimate the experimental error in this region. The solid line is the best straight line through the "constant" portion.

What is measured with the telescope is

$$M = \int_{E_{p\,\min}}^{E_{p\,\max}} I_p(E_p) f(E_p) dE, \tag{2}$$

where $I_p(E_p)$ is the energy spectrum of the protons. Because of the variation in widths of $(E_{p\,\max}-E_{p\,\min})$ and because the Δ's described above are functions of energy, one has to divide the measured intensity M by

$$Q = \int_{E_{p\,\min}}^{E_{p\,\max}} f(E_p) dE. \tag{3}$$

Thus the proton intensity at some average energy value E_p is

$$I_p(E_p) = M/Q. \tag{4}$$

As Q corresponds to the area of a trapezoid, it can be calculated by the use of geometry. The parallel sides of this trapezoid correspond to $A = E_{p\,\max} - E_{p\,\min}$ and $B = (E_{p\,\max}+\Delta_1) - (E_{p\,\min}-\Delta_2)$ and thus the normalizing factor is simply $\frac{1}{2}(A+B)$.

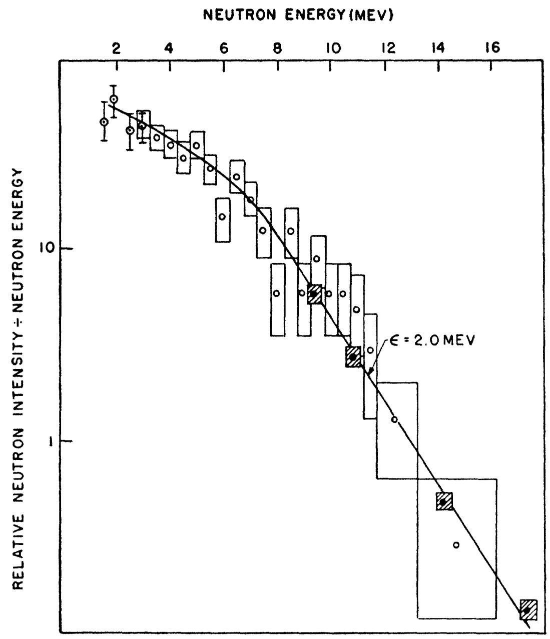

FIG. 3. Neutron energy spectrum at 0° from 15-Mev deuterons on aluminum. See Fig. 2 for explanation. A counter telescope spectrum at 20° agrees within statistical errors.

It should be pointed out that $I_p(E_p)$ is not proportional to the intensity of neutrons of energy $E_n = E_p/\cos^2 15°$. The fact that the (n,p) cross section varies roughly as $1/E$ has to be taken into consideration.[5] Thus,

$$\sigma_{n,p}(E_n)I_n(E_n) = I_p(E_p) = I_n(E_n)/E_n, \tag{5}$$

and the data observed with the counter telescope represents $I(E_n)/E_n$. It was already evident from a histogram plot of the data that the spectrum would be expressed in the form,

$$I(E_n) = E_n e^{-E/\epsilon}, \tag{6}$$

where ϵ is a characteristic constant of the element which produces the spectrum. Consequently, it was possible by successive approximation to obtain a mean for the relative intensity measured over the interval $E_{n\ \max} - E_{n\ \min}$ and the normalized intensities were plotted at these mean energies.

By moving the shield and telescope around to different angles, the spectra were measured at various angles with respect to the direction of the incident deuteron beam. The counter-telescope and photographic plate data, being relative, were arbitrarily adjusted at 8 Mev.

B. Results

Figures 2, 3, and 4 show neutron energy spectra in the forward direction (with respect to the incident deuteron beam) for beryllium, aluminum, and cobalt targets. A spectrum from a copper target was also obtained and found to agree well within statistical errors with Fig. 4. All spectra are characterized by a considerable range in which ϵ [from Eq. (1)] is a very slowly varying function of energy. Average values of ϵ are about 2.5 Mev and are fairly constant or slowly increasing with increasing atomic weights. The experimental accuracy of ϵ is estimated to be ± 10 percent.

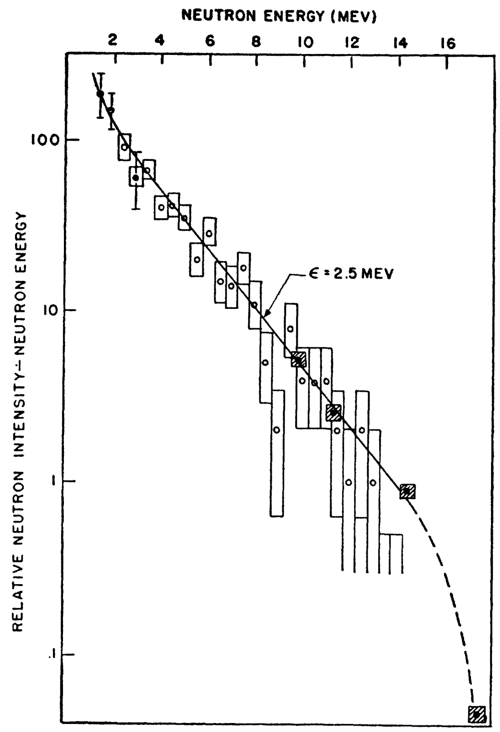

FIG. 4. Neutron energy spectrum at 0° from 15-Mev deuterons on cobalt. See Fig. 2 for explanation. A spectrum from copper and counter telescope spectra at 20° and 45° from cobalt agree within statistical errors.

At energies below about 5 Mev, there is a considerable increase in ϵ for the light elements (beryllium and aluminum) and there seems to be a slight decrease for the heavier ones (cobalt and copper). Although it has been pointed out that there is more danger of experimental errors in this region, it should be noted that considerable faith can be placed in the relative shapes of the various spectra, as they were measured by the same observers using the same methods.

Figures 5 and 6 show spectra measured at 90° (with respect to the incident deuteron beam) for aluminum and copper-cobalt targets (in Fig. 6 the photographic plate data is for copper and the counter telescope data is for cobalt); and a photographic plate spectrum from a copper target measured at 180° was found to agree well with Fig. 6.[6] These are characterized by an almost constant ϵ from the lowest to near the maximum energy, of about 2.0 Mev for aluminum and about 1.7 Mev for copper and cobalt.

[5] R. B. Adair, Revs. Modern Phys. 22, 257 (1950).

[6] Actually, the 180° spectra was found to be steeper by an amount in agreement with the center of mass correction ($\Delta\epsilon \sim 0.15$ Mev); see reference 5.

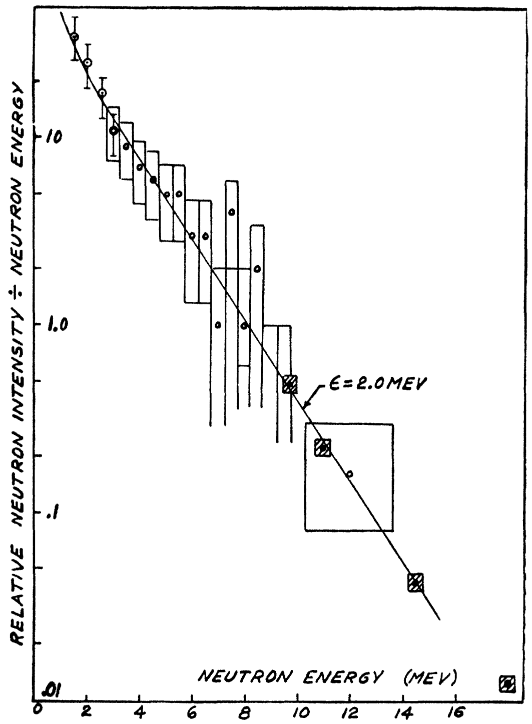

FIG. 5. Neutron energy spectrum at 90° from 15-Mev deuterons on aluminum. See Fig. 2 for explanation.

Counter telescope spectra were obtained for aluminum at 20° and 45°; in each case, the data agrees, within statistical errors, with the data taken at 0°.

C. Theory

Assuming that the deuterons are captured to form a compound nucleus[7] which then emits neutrons by an "evaporation" process, the energy spectrum can be calculated by methods given by Weisskopf.[8] Such calculations were carried out using different values of the factor "a" defined by Weisskopf's expression,

$$\omega = C\exp(aE)^{\frac{1}{2}}, \tag{5}$$

for the nuclear energy level density, ω, at excitation energy E. Neutron spectra, including "second" neutrons from $(d,2n)$ reactions, were calculated for several deuteron energies up to 15 Mev; these were then weighted in proportion to the differential range of deuterons at that energy[9] and the capture cross section,[8] and added to obtain the calculated thick target spectrum. Figure 7 shows the results of such a calculation for $a=3.6$ Mev^{-1} which corresponds to a nuclear temperature T of 4.7 Mev[8] at 20-Mev excitation (this is the approximate excitation energy for a (d,n) reaction with 15 Mev deuterons). For this particular value of T the calculated curve is characterized by a fairly constant slope ϵ of about 2.5 Mev,[10] which is the same as the experimental values of ϵ for Be, Co, and Cu. The calculated ratio of ϵ to T for this case is about 0.6, and numerical calculations of T and ϵ for other values of "a" reveal that this ratio is fairly independent of the parameter "a."

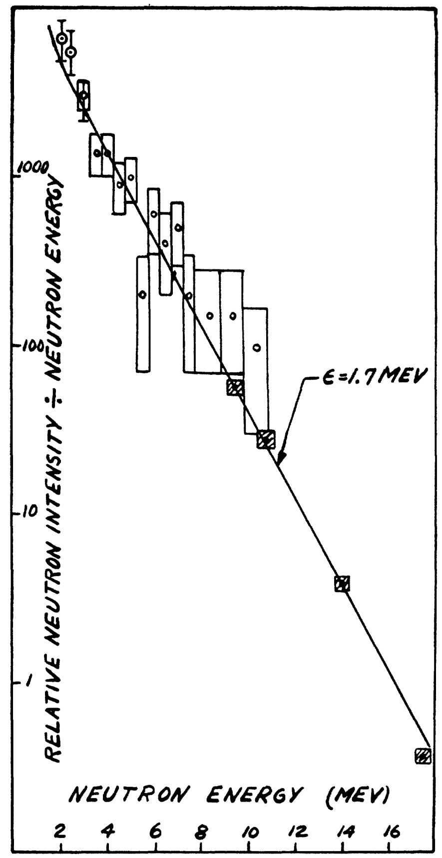

FIG. 6. Neutron energy spectrum at 90° from 15-Mev deuterons on copper-cobalt. See Fig. 2 for explanation. Photographic plate data is for copper and counter telescope data is for cobalt. A photographic plate spectrum from copper at 180° is in good agreement.

The problem of calculating what might be expected from a stripping process is much less straightforward since no completely consistent model for the stripping process has yet been devised. However, in considering neutrons emitted in the forward direction, the following semiclassical treatment might be expected to be at least qualitatively valid:

One might expect the neutron to come off with a momentum equal to its internal momentum in the deuteron, plus half of the deuteron momentum. The

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[7] N. Bohr, Nature **137**, 344, 351 (1936).

[8] V. F. Weisskopf, Lecture Series in Nuclear Physics, LA 29, MDDC 1175 MIT Tech. Report No. 42 (1950, unpublished).

[9] H. Bethe and M. S. Livingston, Revs. Modern Phys. **9**, 265 ff. (1937).

[10] Weisskopf's approximate formula (6)

$$I(E) \propto E\exp(-E/T)$$

is valid only in the low energy region, and this region is distorted by "second" neutrons from $(d,2n)$ reactions. The ratio of ϵ to T is obtained from (5) by numerical methods taking into account these "second neutrons" and the target thickness.

first quantity can be readily calculated from the deuteron wave function,[11] but the second depends on how much momentum the deuteron loses in the coulomb field of the nucleus before the breakup occurs.

For a very light element (i.e., $Z\sim 0$) it does not lose any, so that the spectrum can be readily calculated to give a curve as in Fig. 8. For a fairly heavy element, ($Z\sim 35$), the minimum momentum the deuteron can transfer to the coulomb field before breaking up is none at all (Assumption A), which would give the same curve; the maximum amount would be if the deuteron had to surmount the entire coulomb barrier at the expense of its forward momentum before breakup (Assumption B), which would give a spectrum as the curve labelled "$Z=35$" in Fig. 8. It is interesting to note that the two curves obtained from the opposite extreme assumptions are not very different for energies above ~ 6 Mev, both being roughly representable by (1) with $\epsilon\simeq 3.0$ Mev. It should be noted that this result is independent of the atomic weight of the target nucleus. If the coulomb effect is not taken into consideration, the low energy part of the spectra exhibits a decrease in slope for light elements, as well as for heavy elements. However, in the latter case, the decrease in the slope disappears and is replaced by a slight increase if Assumption B is used. It should be noticed that since the effect of "second neutrons" emitted by the nucleus after the proton has been captured was not taken into account in Fig. 8, only the relative shapes of low energy spectra of different elements can be considered as significant.

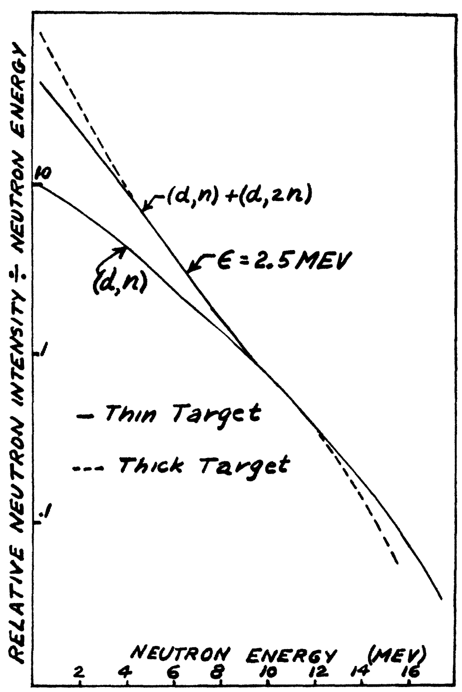

FIG. 7. Theoretical neutron spectrum to be expected from compound nucleus decay. Assumptions: Excitation energy=20 Mev a [for use in Eq. (5)]=3.6 Mev; binding energy of second neutron=10 Mev.

[11] R. Serber, Phys. Rev. **72**, 1008 (1947).

D. Conclusions from Spectra

The measured spectra in the forward directions, Figs. 2, 3, and 4, are in disagreement with the predictions of compound nucleus interaction in three important particulars:

(a) Nuclear temperatures at 20-Mev excitation have never been found to be as high as 4.7 Mev for elements as heavy as copper or cobalt (Weisskopf estimates 3.2 Mev).

(b) There is very good evidence, both experimental[12] and theoretical[13] for the fact that for a given excitation energy, nuclear temperatures decrease with increasing atomic weight. The temperatures in Figs. 2, 3, and 4 are approximately constant or even increasing with increasing atomic weight.

(c) Nothing in compound nucleus theory or in its experimental verifications[12,14–16] suggests the difference in the low energy slopes for light and heavy elements. On the other hand, the measured spectra in the forward direction are in good qualitative and semiquantitative agreement with the predictions of a stripping interaction. The magnitude of ϵ, and its constancy with

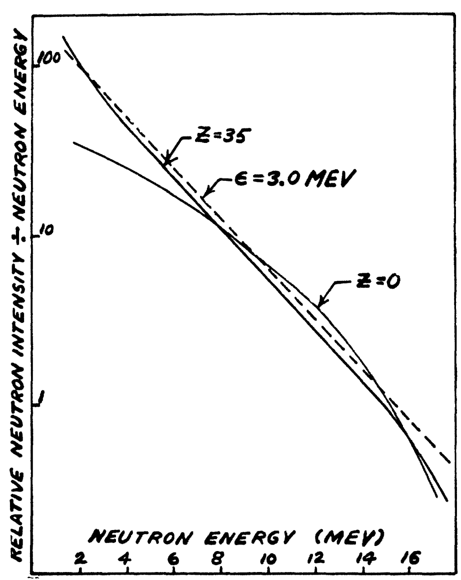

FIG. 8. Theoretical neutron spectrum to be expected from stripping. Dashed line represents Eq. (1) with $\epsilon=-3.0$ Mev. It is included for purposes of orientation.

[12] P. C. Gugelot and M. G. White, Phys. Rev. **76**, 463 (1949). See also reference 3.

[13] H. A. Bethe, Revs. Modern Phys. **9**, 78 (1937).

[14] J. C. Grosskreutz, Phys. Rev. **76**, 482 (1949).

[15] J. L. Fowler and J. M. Slye, Jr., Phys. Rev. **77**, 787 (1950); B. L. Cohen, Phys. Rev. **81**, 184 (1951); H. Wäffler, Helv. Phys. Acta **23**, 239 (1950).

[16] B. L. Cohen, Carnegie Inst. of Tech., Technical Report No. 4 (1950, unpublished).

atomic weight, and the difference between the low-energy slopes are all at least roughly explained.

The data taken at 90° (Figs. 5 and 6), on the contrary, are in good agreement with the predictions of compound nucleus theory. None of the objections (a), (b), (c) listed above applies here, and in fact the observed temperatures ($\epsilon/0.60$) of 2.8 Mev for copper and cobalt, and 3.3 Mev for aluminum are in reasonable agreement with what is known about nuclear temperatures.

We may thus conclude that the evidence from neutron energy spectra, particularly in the case of cobalt and copper, strongly indicates that most of the neutrons in the forward direction are the result of a stripping interaction which takes place after the deuteron has penetrated almost to the surface of the nucleus, while those at large angles arise from a compound nucleus interaction.

II. YIELDS

The relative yield of neutrons from various targets was measured, using the same cyclotron, by Allen *et al.*[17] Their results expressed as a function of atomic number are represented by

$$I(Z)\propto\exp(-Z/18.5). \tag{6}$$

In addition, it has been shown[18] that the angular distribution in reference 12, when corrected for center of mass, are roughly the same for all targets; thus (6) is also approximately valid for neutrons in the forward direction.

Equation (6) seems to indicate that the penetration of a coulomb barrier is an important factor in determining the cross section for the (d,n) reaction at all angles, especially since the yields from alpha-bombardments are also given by (6). As recent calculations by Guth[19] show that the neutron yield from electric disintegration decreases much less rapidly with Z than indicated by Eq. (6), the experimental yield function seems to throw doubt on the possibility that the stripping takes place by electric disintegration, and supports the conclusion that it involves capture of the proton.

An estimate of the relative cross sections for stripping and compound nucleus formation may be obtained from the data of Allen *et al.*[17] and the very probable assumption that angular distributions from compound nucleus reactions are symmetric about 90°.[18,20] This gives the number of neutrons above 3 Mev in energy (the threshold of Allen's detectors) originating from a stripping interaction. Assuming that almost all stripping interactions leave the nucleus with enough excitation to emit a second neutron by compound nucleus decay, the relative cross sections for deuteron capture and stripping are about in the ratio 4:1.

Serber[11] gives the cross section for neutron stripping as

$$\sigma_s=\pi r r_d, \tag{7}$$

where r and r_d are the radii of the target nucleus and deuteron, respectively. At low energies, this must be replaced by

$$\sigma_s=\pi r r_d P, \tag{8}$$

where P is the coulomb barrier penetration factor. The cross section for formation of a compound nucleus is

$$\sigma_c=\pi r^2 P, \tag{9}$$

so that the ratio of the two is

$$\sigma_s/\sigma_c=r/r_d, \tag{10}$$

if the penetration factors in Eqs. (8) and (9) are identical. Using $r=r_0A^{\frac{1}{3}}$ with $r_0=1.4\times10^{-13}$ cm$\simeq r_d$, we have

$$\sigma_c/\sigma_s\simeq A^{\frac{1}{3}}. \tag{11}$$

For the elements considered,[21] $A^{\frac{1}{3}}$ varies from 3 to 6, which is satisfactory agreement with the observed ratio of about 4 considering the various approximations involved.

Summing up all the experimental results, one can reach certain definite conclusions as to the origin of the observed neutrons. The extreme anisotropy of the angular distributions,[1] the large nuclear temperatures obtained from the neutron spectra, and the comparative invariance of T as a function of Z indicate that the majority of the observed neutrons in the forward direction cannot come from a compound nucleus. Whether these neutrons are produced by stripping or electric disintegration of the deuteron is not quite clear; however, the rate at which the yield of neutrons decreases with Z seems to contradict the electric disintegration theory and favor stripping. The latter process is further favored by the shape of the observed spectra. There is no evidence that the neutrons at large angles do not have their origin in compound nucleus decay.

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[17] Allen, Nechaj, Sun, and Jennings, Phys. Rev. **81**, 536 (1951).
[18] B. L. Cohen, Phys. Rev. **81**, 632 (1951).
[19] C. J. Mullin and E. Guth, Phys. Rev. **82**, 141 (1951).
[20] L. Wolfenstein, Phys. Rev. **78**, 322 (1950).

[21] Beryllium is neglected because the center-of-mass correction is too large to allow a good calculation of the experimental ratio.