tures from less than 1°K to 90°K. Benzie and Cooke[4] suggest, on the basis of specific-heat data, that a noncubic contribution to the crystalline electric field splits the ^6S iron state into three equally spaced doublets.

It might be expected that iron potassium alum, having a similar crystalline structure to that of iron ammonium alum, would show absorption spectra similar to those of the ammonium salt. In other salts the chief effect of replacing the ammonium radical by a potassium atom has been a relatively small change in the zero-field splitting but no marked difference in the character of the crystalline field, and hence, the paramagnetic spectra.[5] Figure 2 shows the spectra observed for iron potassium alum diluted with aluminum potassium alum at the same temperatures and for the same crystal orientation used for the ammonium salt. No structure is observed at room temperature. The positions of broad peaks at 77°K and 4°K are indicated by arrows in the figure. The 77°K and 4°K spectra differ essentially only in the appearance of two additional peaks in the 77°K spectrum. Although these spectra are markedly different from the corresponding spectra for the ammonium salt and are incompatible with the cubic-field assumption, it appears that the splitting undergoes no severe changes in the region from 77°K to 4°K.

Unpublished work by Mr. P. E. Baker of this laboratory shows that for a frequency of 9375 Mc/sec aluminum ammonium alum has a large dielectric absorption peak at 175°K, whereas aluminum potassium alum shows no appreciable dielectric absorption in the region of this temperature. This effect may be relevant to the differences found in the paramagnetic spectra for the ammonium and potassium varieties of iron alum.

* This work has been supported by the ONR, by the Rutgers University Research Council, and by RCA.

[1] Weidner, Weiss, Whitmer, and Blosser, Phys. Rev. **76**, 1727 (1949).
[2] Ubbink, Poulis, and Gorter, Physica **17**, 213 (1951).
[3] J. H. Van Vleck, Am. J. Phys. **18**, 495 (1950).
[4] R. J. Benzie and A. H. Cooke, Proc. Phys. Soc. (London) **A63**, 213 (1950).
[5] Whitmer, Weidner, Hsiang, and Weiss, Phys. Rev. **74**, 1478 (1948).

Behavior in Pb of the Nucleonic Component in the Stratosphere*

Maurice M. Shapiro
Nucleonics Division, Naval Research Laboratory, Washington, D. C.

AND

Andrew F. Gabrysh
Department of Physics, Catholic University of America, Washington, D. C.
(Received August 13, 1951)

PREVIOUS communications[1,2] described an experiment (denoted by I) on the behavior of the star-producing radiation in the stratosphere as it passes from air into Pb. It was pointed out[2,3] that the transition effects must be strongly influenced by geometry, since these effects are due to secondary processes generated inside the absorber. Results which support this conclusion and also confirm other findings in experiment I, have been obtained from two "Skyhook" balloon flight exposures.[4]

Figure 1 shows the Pb block inside of which 6 Eastman NTB3 (ultrasensitive) emulsions 15×5 cm and 200μ thick were inserted vertically in a central slot. As in experiment I, this arrangement permitted a measurement of the star frequency at any desired depth in the block. The absorber differed from that in I in having (a) no covering slab of Pb above the emulsions, and (b) more restricted lateral dimensions, particularly in the base which was narrower, instead of wider, than the upper portion.

The apparatus floated for 8 hours at a pressure altitude of 1 to 4 cm Hg, the time-average pressure "at ceiling" being 2.3 cm Hg. Data used in this study were collected only from the upper (wider) portion of the block, in order to avoid—or at least to reduce—complications introduced by rays incident at large zenith angles. As an index of star size we adopted the number of "black" and "gray" tracks (grain density >1.4 times minimum). σ-stars were excluded.

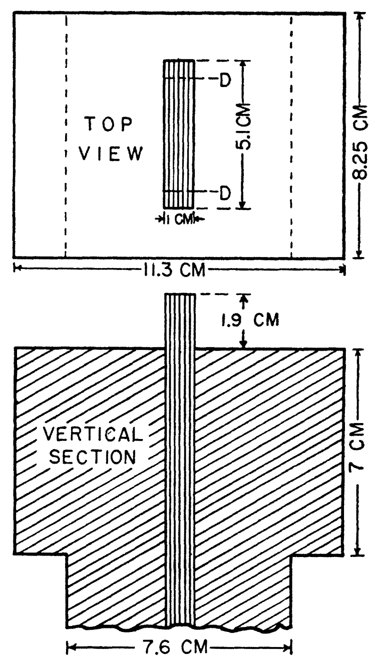

Fig. 1. Pb absorber, showing top view and vertical section cutting through the emulsions. In scanning the photoplates, star data were gathered only from the region DD in any horizontal plane, and from the upper portion of the Pb block. Zero-point data were collected from the region directly above the Pb. Total depth of the block was 12.5 cm.

Figure 2, based on 934 stars, gives the relative frequency as a function of depth in the Pb for stars with ≥ 3 prongs and also for small (3- to 6-prong) stars. A prominent feature is the peak at ≈ 27 g/cm^2, a depth which agrees closely with that observed in two other experiments.[1,5] This agreement is not unexpected, despite the diverse geometries employed, since the main differences between them occurred at depths >40 g/cm^2. At shallow depths, the several Pb absorbers were more nearly alike. Another interesting, though minor, feature is the indication of a slight initial decrease in frequency of the small stars, which corroborates a similar observation in I. A possible explanation of this effect was suggested in reference 2.

The major difference between the present results and those in I is the absence of the broad peak at ≈ 60–70 g/cm^2 which in the previous results suggested that a saturation of the star-producing

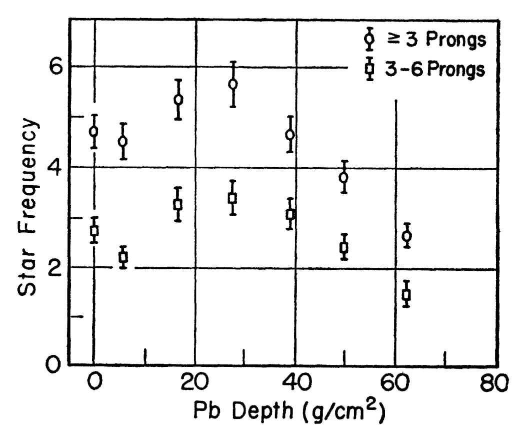

Fig. 2. Transition effect curves obtained using the absorber in Fig. 1, for (a) stars regardless of size (circles), and (b) small stars alone (squares). Star frequency is expressed in arbitrary units. Probable errors, based on counting-statistics alone, are given.

secondaries sets in at about this depth (see Fig. 3 of reference 1). This difference probably stems from the lack of a massive Pb base, such as that in I, in which a substantial development of the nucleonic cascade could take place. If this interpretation is correct, it supports the view that a considerable fraction of the secondary star-producing radiation generated in the Pb has an upward component.[1,6]

In the second stratosphere exposure, the mean pressure altitude above 70,000 ft for 7 hours was 2.5 cm Hg. A particularly simple geometry was involved: a massive horizontal Pb slab, 30×30 ×2.15 cm, was sent aloft for a counter experiment.[7] This provided an opportunity to measure the rates of star production in a central location above and below the Pb slab. We used NT4 emulsions, 200μ thick, made by the Kodak Company in England.

The following ratios of star frequencies below/above the 2.15 cm Pb were observed: for stars with ≥ 3 prongs, 1.74 ± 0.16; for small (≤ 6-fold) stars, 1.86 ± 0.22; for large (>6-fold) stars, 1.55 ± 0.22. Probable errors are given throughout. Our results showed no significant difference between vertical and horizontal plates above the lead. These values of the ratios are larger than those obtained in our absorbers having smaller lateral dimensions. They agree better with ratios observed by Blau *et al.*,[8] using Ilford *C.2* emulsions in a rather shallow absorber.

We are grateful to Mr. Bertram Stiller for his able help with the flights and the emulsion processing, and to Mr. Francis Booth for scanning some of the plates. It is a pleasure to acknowledge the support and advice of Dr. F. N. D. Kurie. One of us (A. F. G.) is indebted to Professor F. L. Talbott for guidance and encouragement.

* Phys. Rev. **83**, 874 (1951).
[1] Shapiro, Stiller, Birnbaum, and O'Dell, Phys. Rev. **83**, 455 (1951).
[2] Maurice M. Shapiro, Phys. Rev. **83**, 456 (1951).
[3] Dallaporta, Merlin, and Puppi, Nuovo cimento **7**, 99 (1950).
[4] Chronologically earlier than those in reference 1. These flights, arranged with the help of the ONR and the General Mills Laboratories, took place on June 21, 1949 and October 15, 1949 at geomagnetic latitude 56°N.
[5] In I (reference 1), the evidence for this peak, though fairly good, was not conclusive. The authors nevertheless felt justified in referring to it as a definite effect because (a) the confirmatory evidence presented here was known to them, and (b) a similar maximum had been observed at ≈ 23 g/cm² by Blau, Nafe, and Bramson, Phys. Rev. **78**, 320(A) (1950).
[6] J. J. Lord and M. Schein, Phys. Rev. **75**, 1956 (1949).
[7] By Dr. G. J. Perlow, whose kind cooperation is gratefully acknowledged.
[8] Marietta Blau (private communication).

Statistical Significance of Small Samples of Cosmic-Ray Counts

VICTOR H. REGENER
Department of Physics, University of New Mexico, Albuquerque, New Mexico
(Received August 13, 1951)

It is occasionally desirable in cosmic-ray research that conclusions of a tentative nature be drawn from a small total number of counts obtained in a counter tube experiment. When the number n of measured counts is large, it is customary to indicate the statistical uncertainty by quoting precision limits located at $n \pm \sqrt{n}$. This procedure is usually described as giving a probability of 0.1587, or roughly 16 percent, that the unknown average number of counts a for the duration of the experiment lies below the lower precision limit $n-\sqrt{n}$, and also a probability of 16 percent that a lies above $n+\sqrt{n}$. When n is small, this use of the standard deviation of a gaussian distribution should, of course, be replaced by a similar prescription based on the less easily manipulated poisson distribution, but the quoted statement of probability also contains the illegitimate assumption of a constant *a priori* probability for the existence of the average count a on both sides of the value n. This difficulty is here briefly examined for the poisson distribution, and appropriate precision limits for low values of n are given.

If the poisson probability $W_n = a^n e^{-a}(n!)^{-1}$ for the occurrence of just n counts could be interpreted as being a normalized probability density distribution $W_n(a)$ for the average count a, an integration over $W_n(a)$ between appropriate limits of a would appear to give the probability that the average count lies between these limits. Inherent in such an interpretation is the illegitimate assumption that all average counts a contained between arbitrary limits are *a priori* equally probable or, in other words, that physical reality contains a population of counting rates which is known to be uniformly dense. Fisher[1] made it clear that ignorance of an *a priori* distribution does not justify the assumption that it is constant any more than that it is of any other form.[2]

A legitimate procedure for the assignment of precision limits has been suggested by Fisher.[3] Consider, for example, a total measured count $n=3$. It is possible to determine a value a_1 for a hypothetical average count which lies so far below 3 that the total probability for the actual count to fall anywhere below 3 assumes a specified value p. Above 3, another value a_2 can be determined so that the total probability for the count to fall anywhere above 3 assumes again the value p. If p is made 0.8413, the result is 1.37 for a_1, and 5.92 for a_2. The fiducial limits a_1 and a_2 should replace the customary precision limits for small values of n.

TABLE I. Poisson fiducial limits and normal limits for $p=0.8413$, $x_p=1.00$.

Lower limit $d_0=0$				Upper limit $d_0=1.00$		
Normal	d	Poisson fiducial	n	Poisson fiducial	d	Normal
...	...	0	0	1.84	1.84	0
0	0.17	0.17	1	3.30	1.30	2.00
0.59	0.12	0.71	2	4.64	1.23	3.41
1.27	0.10	1.37	3	5.92	1.19	4.73
2.00	0.09	2.09	4	7.16	1.16	6.00
3.55	0.07	3.62	6	9.58	1.13	8.45
6.84	0.05	6.89	10	14.26	1.10	13.16
15.53	0.04	15.57	20	25.54	1.07	24.47
42.93	0.02	42.95	50	58.11	1.04	57.07

TABLE II. Poisson fiducial limits and normal limits for $p=0.9500$, $x_p=1.645$.

Lower limit $d_0=0.57$				Upper limit $d_0=1.57$		
Normal	d	Poisson fiducial	n	Poisson fiducial	d	Normal
...	...	0	0	3.00	3.00	0
...	...	0.05	1	4.74	2.09	2.65
...	...	0.35	2	6.30	1.98	4.32
0.15	0.67	0.82	3	7.75	1.90	5.85
0.71	0.66	1.37	4	9.15	1.86	7.29
1.97	0.64	2.61	6	11.84	1.81	10.03
4.80	0.63	5.43	10	16.96	1.76	15.20
12.64	0.62	13.26	20	29.06	1.70	27.36
38.37	0.59	38.96	50	63.28	1.65	61.63

These fiducial limits are computed by evaluating A in the expression

$$P = e^{-A} \sum_{m=0}^{N} A^m (m!)^{-1},$$

with $P=p$, $N=n-1$, $A=a_1$ in case of the lower limit a_1, and with $P=1-p$, $N=n$, $A=a_2$ in case of the upper limit a_2. The identity

$$1-P = (N!)^{-1} \int_0^A A^N e^{-A} dA = F$$

allows[4] the use of published tables for the incomplete factorial function F. Tables I and II give poisson fiducial limits for certain values of n from zero to 50, as computed from interpolations of Pearson's table[5] with $p=0.8413$ (Table I) and $p=0.9500$ (Table II). For comparison, the tables also contain conventional precision limits based on the gaussian distribution, here called normal limits, where the probability defined by the statement at the beginning of this note now corresponds to $1-p$. The poisson fiducial limits differ from the normal limits by an additive amount d