

field distribution. Integrating this result over the magnetic field inhomogeneities in the "natural" frame $\omega' = \gamma H_0$, in which the limits of Δ are $-\frac{1}{2}\Delta_m$ to $+\frac{1}{2}\Delta_m$, and ignoring the variation of C with Δ , the result is

$$M_x' = 0 \quad (2a)$$

$$M_y' = C e^{-t/T_2} \frac{\sin(\frac{1}{2}\Delta_m t)}{\frac{1}{2}\Delta_m t} - C e^{-(t+\tau)/T_2} \frac{\sin[\frac{1}{2}\Delta_m(\tau-t)]}{\frac{1}{2}\Delta_m(\tau-t)}. \quad (2b)$$

A controlled leakage of frequency ω from the free running base oscillator in the transmitter is present in the receiving equipment; thus there exists a predominate carrier of amplitude A . In the laboratory frame, the resultant rf voltage to be detected is

$$M_y = M_y' \sin\omega't + A \sin\omega t,$$

where $\omega' - \omega = \delta$, the nominal angular frequency off resonance. Since $A \gg M_y'$, and $\frac{1}{2}(\omega' + \omega) \sim \omega$, this reduces to

$$M_y = [A + 2M_y' \cos(\frac{1}{2}\delta t)] \sin\omega t. \quad (3)$$

Thus M_y appears as an amplitude-modulated carrier. M_y' appears as the envelope of the $\cos(\frac{1}{2}\delta t)$ beat, and the first term in Eq. (2b) is just the normal free decay while the second term describes the mirror signal.

Equation (2b) indicates a convenient means for determining the homogeneity of the magnet, i.e., the mean gradient of the field over the sample region. This effect is shown in Fig. 2, in which the inhomogeneity was varied by placing a small screwdriver blade a few cm from the sample. Δ_m can be calculated by noting the time interval between the end of the pulse and the first zero of $\sin(\frac{1}{2}\Delta_m t)$.

These equations can be used for an experimentally simple determination of T_2 . For proper experimental conditions, the ratio $M_y'(\tau)$ to $M_y'(0)$ reduces to

$$M_y'(\tau)/M_y'(0) = e^{-2\tau/T_2}. \quad (4)$$

This technique is particularly useful for $T_2 > 10^{-3}$ sec, a region common to many liquids and usually obscured by magnet inhomogeneities. We are in the process of using this method for the measurement of T_2 for a series of related liquid hydrocarbons.

* Supported in part by the AEC.

¹ R. S. Bradford, thesis, University of Washington, 1951.

² E. A. Uehling, private communication.

³ H. C. Torrey, Phys. Rev. **76**, 1059 (1949).

⁴ E. L. Hahn, Phys. Rev. **76**, 461 (1949); Phys. Rev. **77**, 297 (1950); Phys. Rev. **76**, 145 (1949); Phys. Rev. **80**, 580 (1950).

⁵ F. Bloch, Phys. Rev. **70**, 460 (1946).

Lagrangian S-Matrix

W. K. BURTON*

H. H. Wills Physical Laboratory, Royal Fort, Bristol, England

(Received August 8, 1951)

SCHWINGER¹ has constructed an elegant and concise formulation of lagrangian field dynamics in which he shows [Eq. (2.133) of reference 1] that the result of applying two independent variations in structure to the lagrangian operator is to change the transformation function by an amount

$$\delta^{(1)}\delta^{(2)}\langle \zeta_1', \sigma_1 | \zeta_2'', \sigma_2 \rangle = (i/\hbar c)^2 \int_{\sigma_2}^{\sigma_1} dx_1 \int_{\sigma_2}^{\sigma_1} dx_2 \times dx_2 \langle \zeta_1', \sigma_1 | (\delta^{(1)}\mathcal{L}[x_1] \delta^{(2)}\mathcal{L}[x_2])_+ | \zeta_2'', \sigma_2 \rangle. \quad (1)$$

Here, $|\zeta_2'', \sigma_2\rangle$ is the eigenket of a complete commuting set ζ_2 of operators constructed from the field operators on a spacelike surface σ_2 , etc., and the bracket symbol is defined by

$$(A(x_1)B(x_2))_+ = \begin{cases} A(x_1)B(x_2) & \text{if } (x_1)_0 > (x_2)_0, \\ B(x_2)A(x_1) & \text{if } (x_2)_0 > (x_1)_0, \end{cases} \quad (2)$$

which is an invariant concept, provided the operators involved commute when $x_1 - x_2$ is a spacelike interval, and the positive sense of time is preserved.

This result can be generalized at once to give

$$\delta^{(1)} \dots \delta^{(n)} \langle \zeta_1', \sigma_1 | \zeta_2'', \sigma_2 \rangle = (i/\hbar c)^n \int_{\sigma_2}^{\sigma_1} dx_1 \dots \int_{\sigma_2}^{\sigma_1} dx_n \langle \zeta_1', \sigma_1 | (\delta^{(1)}\mathcal{L}[x_1] \dots \delta^{(n)}\mathcal{L}[x_n])_+ | \zeta_2'', \sigma_2 \rangle, \quad (3)$$

which leads to a simple construction of the S-matrix in lagrangian form. Suppose the lagrangian is $\mathcal{L} + \lambda \mathcal{L}^i$ where λ is a numerical parameter. Then, if \mathcal{L} and \mathcal{L}^i are regarded as fixed, $\langle \zeta_1', \sigma_1 | \zeta_2'', \sigma_2 \rangle$ is a function of λ ; the variation in structure now to be considered comes about by varying λ , and for this case Eq. (3) becomes

$$(d^n/d\lambda^n) \langle \zeta_1', \sigma_1 | \zeta_2'', \sigma_2 \rangle = (i/\hbar c)^n \int_{\sigma_2}^{\sigma_1} dx_1 \dots \int_{\sigma_2}^{\sigma_1} dx_n \langle \zeta_1', \sigma_1 | (\mathcal{L}^i[x_1] \dots \mathcal{L}^i[x_n])_+ | \zeta_2'', \sigma_2 \rangle. \quad (4)$$

The bras and kets in Eq. (4) refer to a system with the lagrangian $\mathcal{L} + \lambda \mathcal{L}^i$. We now invoke Taylor's formula,

$$f(\lambda+1) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n}{d\lambda^n} f(\lambda), \quad (5)$$

subsequently putting $\lambda=0$. This gives $\langle \zeta_1', \sigma_1 | \zeta_2'', \sigma_2 \rangle$ for a system with a lagrangian $\mathcal{L} + \mathcal{L}^i$ in terms of a series in which the bras and kets refer to a system with a lagrangian \mathcal{L} :

$$\langle \zeta_1', \sigma_1 | \zeta_2'', \sigma_2 \rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i}{\hbar c} \right)^n \int_{\sigma_2}^{\sigma_1} dx_1 \dots \int_{\sigma_2}^{\sigma_1} dx_n \langle \zeta_1', \sigma_1 | (\mathcal{L}^i[x_1] \dots \mathcal{L}^i[x_n])_+ | \zeta_2'', \sigma_2 \rangle. \quad (6)$$

Hence, the S-matrix is given by

$$S = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i}{\hbar c} \right)^n \int_{-\infty}^{\infty} dx_1 \dots \int_{-\infty}^{\infty} dx_n (\mathcal{L}^i[x_1] \dots \mathcal{L}^i[x_n])_+. \quad (7)$$

¹ J. Schwinger, Phys. Rev. **82**, 914 (1951).

* Seconded from I.C.I., Ltd., Butterwick Research Laboratories, The Frythe, Welwyn, Herts, England. Now at Department of Natural Philosophy University of Glasgow, Scotland.

Erratum: Spectrometer and Coincidence Studies on Np²³⁸

[Phys. Rev. **79**, 410 (1950)]

M. S. FREEDMAN, A. H. JAFFEY, AND F. WAGNER, JR.
Argonne National Laboratory, Chicago, Illinois

IN Table I the conversion coefficients of the 0.1028-Mev gamma-ray relative to the 0.258-Mev beta-ray were incorrectly reported to be 4.7 percent (M) and 7.5 percent (L). The correct figures are 2.5 percent (M) and 3.9 percent (L).

Half-Lives of Cu⁶⁶, Cu⁶⁴, Fe⁵⁹, and Ce¹⁴⁴

ROBERT P. SCHUMAN AND ANITA CAMILLI

Knolls Atomic Power Laboratory, Schenectady, New York

(Received August 7, 1951)

IN the course of our work, the half-lives of Cu⁶⁶, Cu⁶⁴, Fe⁵⁹, and Ce¹⁴⁴ have been determined with considerable precision. The half-lives of the copper isotopes were determined on samples of electrolytic copper foil irradiated with water-moderated radium beryllium neutrons. The Fe⁵⁹ sample was from a cyclotron irradiation unit, free of Fe⁵⁵, obtained from the Isotopes Division of the Atomic Energy Commission. The Ce¹⁴⁴ sample was obtained by the radiochemical isolation of cerium from fission products after a decay of over one year so that the sample would be free of Ce¹⁴¹. The counting has been done with a flow proportional counter (nucleometer) using an external sample counter. All counts were checked against a standard to insure a constant counting efficiency.

We have obtained a half-life of 5.12 ± 0.05 min for Cu⁶⁶, in good agreement with the values of 5.18 ± 0.10 min by Cameron and Katz,¹ and 5.05 min by L. Meitner,² but differing from the value of 4.34 ± 0.03 min by Silver.³ An irradiation was made of a Cd-shielded Cu foil which showed that most of the activation was due

to thermal neutrons; the Cd-shielded Cu foil also showed a 5-min half-life. Our value for the Cu⁶⁴ half-life is 12.74 ± 0.07 hr, in agreement with the present literature.⁴

Our value for the half-life of Fe⁶⁹ is 45.1 ± 0.2 days. The decay of the sample has been followed for over eight half-lives, and does not show any tendency to tail out into a longer half-life. The Ce¹⁴⁴ sample has been followed for two half-lives, and shows a half-life of 282 ± 3 days. Our half-life for Fe⁶⁹ is slightly lower than the published values of 45.5 days⁵ and 46.3 days.⁴ The half-life of Ce¹⁴⁴ is in general agreement with the literature values of 275 days, 290 days, and 310 days.⁴

We have also followed samples of Fe⁵⁵ and Pm¹⁴⁷ for a period of one year. The Fe⁵⁵ sample was obtained from the Isotopes Division of the Atomic Energy Commission, and the Pm¹⁴⁷ sample was radiochemically isolated from fission products. Both samples were shown to be radiochemically pure by absorption curves. The preliminary value of the Fe⁵⁵ half-life is 3.0 ± 0.15 years, in good agreement with the better value of 2.94 yr.⁶ The Pm¹⁴⁷ half-life is 2.6 ± 0.2 years, compared with literature values of 2.26 years⁷ and 3.7 years.⁴

¹ A. G. W. Cameron and L. Katz, *Phys. Rev.* **80**, 904 (1950).

² L. Meitner, *Arkiv Mat. Astron. Fysik* **33A**, No. 3 (1946).

³ L. M. Silver, *Phys. Rev.* **76**, 589 (1949).

⁴ *Nuclear Data*, National Bureau of Standards Circular 499 (1950).

⁵ J. Govaerts, *Bull. Soc. Roy. Sci. Liege* **12**, 555 (1943).

⁶ G. L. Brownell and C. J. Meletskos, *Phys. Rev.* **80**, 1102 (1950).

⁷ Inghram, Hayden, and Hess, *Phys. Rev.* **79**, 271 (1950).

Microwave Paramagnetic Spectra of Iron Alum Salts*

C. A. WHITMER AND R. T. WEIDNER

Department of Physics, Rutgers University, New Brunswick, New Jersey
(Received August 10, 1951)

THE paramagnetic resonance absorption at 9375 Mc/sec by diluted single crystals or iron ammonium alum and iron potassium alum has been observed for temperatures of 4°, 77°, and 293°K. The spectra show that the splitting of the ground state of the iron ion cannot be accounted for by assuming a crystalline electric field of purely cubic symmetry and that the iron and potassium salts have markedly different spectra.

Figure 1 shows the absorption spectra for iron ammonium alum. The single crystals, grown from a mixed solution of iron ammonium alum and aluminum ammonium alum, contained one iron atom for every 38 aluminum atoms. The measurements were made with the dc magnetic field perpendicular to the (100) crystalline face because the spectra are most highly dispersed for this orientation. Earlier measurements¹ showed that the room-temperature spectra were in accord with the theoretical patterns deduced on the basis of the splitting of the ⁶S state of the iron ion into a double degenerate and a quadruply degenerate level by a crystalline electric field of cubic symmetry. The theoretical spectrum fitting the observed absorption at 293°K is shown by vertical lines in Fig. 1. The main central peak was observed, however, to be relatively more intense than the theory predicted. A zero-field splitting of 0.032 cm^{-1} was deduced for the diluted samples. The five distinct peaks in the spectrum at 77°K are incompatible with the theoretical cubic-field spectrum for the choice of any single value of the zero-field splitting; the increased breadth of the spectrum over that for room temperature suggests, however, that the splitting increases in going from 293°K to 77°K.

The spectrum observed for iron ammonium alum at 4°K is in agreement with the measurements of Ubbink, Poullis, and Gorter,² who attribute the very weak resonances occurring at fields one-half the values for the main side peaks to ordinarily "forbidden" transitions in which the magnetic quantum number changes by ± 2 . The main side peaks show an asymmetry with respect to the main resonance; and the latter is relatively more intense and narrow than the main resonance observed at 293°K. If the asymmetry in the side peaks is disregarded, the cubic-field assumption

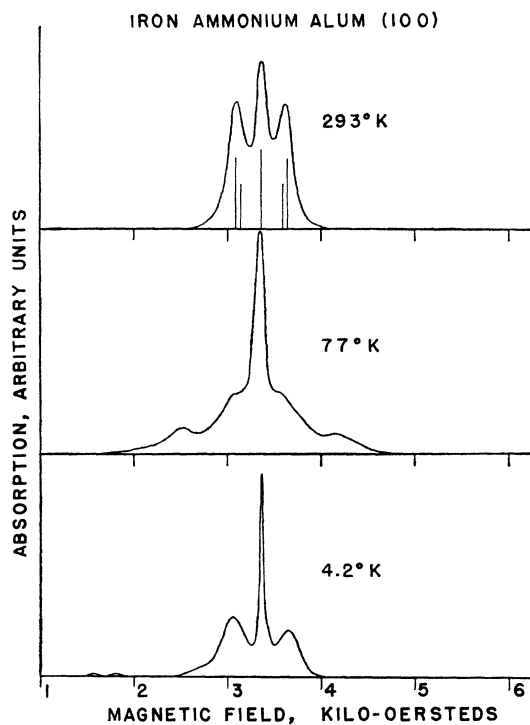


FIG. 1. Absorption spectra for diluted iron ammonium alum in the (100) orientation. The theoretical cubic-field spectrum is shown in the 293°K curve.

leads to the value 0.036 cm^{-1} for the zero-field splitting. This value is in clear disagreement with the value 0.14 cm^{-1} , which specific heat measurements³ yield for the splitting for tempera-

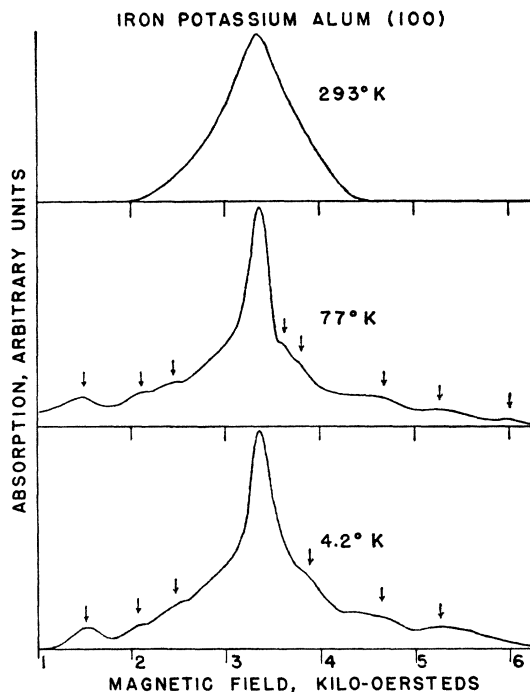


FIG. 2. Absorption spectra for diluted iron potassium alum in the (100) orientation. The arrows indicate the positions of absorption peaks.