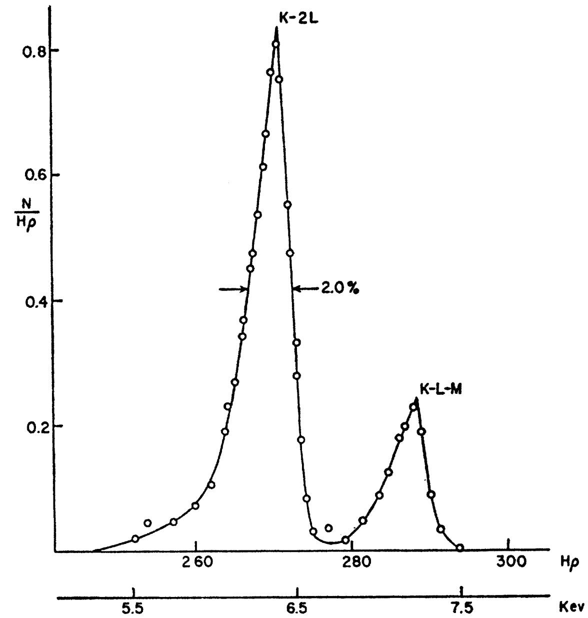

FIG. 1. K Auger electron distribution of Cu^{64}.

K-capture/positron ratio of 2.18 ± 0.20 for Cu^{64}. When this is corrected for the 5 percent branching to the 1.35-Mev excited state of nickel, the K-capture/positron ratio to the ground state is 2.07 ± 0.20, in excellent agreement with the theoretically predicted value of 2.08.[11]

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A Steady-State Transient Technique in Nuclear Induction*

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A NEW nuclear resonance transient phenomenon was initially observed by Bradford[1] and theoretically justified by Uehling,[2] and is now under investigation in our laboratory. Previous investigators[3,4] have described "turn on" transients corresponding to a discontinuous variation of the amplitude of the radiofrequency field. These investigators worked with times between transients that were effectively infinite compared with the time constants, T_1 and T_2, of the nuclear system. The present transient phenomenon is observed during the time between radiofrequency pulses when this time is of the order of the nuclear relaxation time, T_2. The new transient consists of the expected "free" decay following removal of the rf field, plus an attenuated mirror image of this decay, inverted in sign and running backward in time from the onset of the next pulse. This phenomenon occurs as a result of the establishment of a "steady-state" transient in the presence of magnetic field inhomogeneities. A typical photograph of the complete phenomenon is shown in Fig. 1.

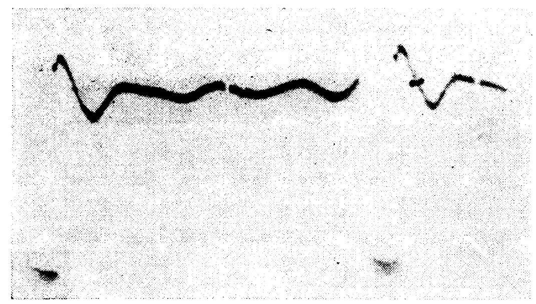

FIG. 1. $0.001M$ $MnSO_4$; prf 400 cps; 3500 μsec sweep.

Uehling has described a solution of the Bloch[5] equations with suitable boundary conditions, specialized for a "delta-function" periodically repeated rf field pulse, normalized so that $\sin\gamma\int H_1dt$ over the pulse is unity, followed by an integration of this solution over the magnetic field inhomogeneities. His result for the transient between pulses in the rotating coordinate system before introduction of field inhomogeneities is

$$M_x' = C[e^{-\tau/T_2}\sin\Delta\tau\cos\Delta t + (1-e^{-\tau/T_2}\cos\Delta\tau)\sin\Delta t]e^{-t/T_2} \quad (1a)$$

$$M_y' = C[(1-e^{-\tau/T_2}\cos\Delta\tau)\cos\Delta t - e^{-\tau/T_2}\sin\Delta\tau\sin\Delta t]e^{-t/T_2} \quad (1b)$$

where

$$C\equiv M_0(1-e^{-\tau/T_2})/[1-e^{-\tau/T_2}(1-e^{-\tau/T_1})\cos\Delta\tau - e^{-2\tau/T_1-\tau/T_2}]$$

and $\Delta\equiv\gamma H_0-\omega$. A reasonable form for the normalized weight factor, $g(\Delta)$, due to magnet inhomogeneities is the uniform distribution $g(\Delta)=1/\Delta_m$, where Δ_m is the "width" of the rectangular

(a)
(b)
(c)

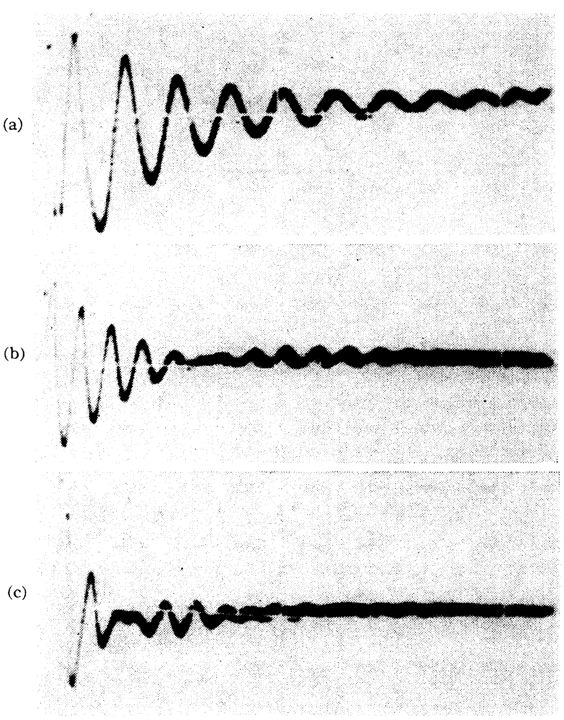

FIG. 2. $0.01M$ $MnSO_4$;prf 100 cps; 2500 μsec sweep. Curve (a): (Δ_m)gauss$\sim$0. Curve (b): (Δ_m)gauss$\sim$0.23. Curve (c): (Δ_m)gauss$\sim$0.37.

field distribution. Integrating this result over the magnetic field inhomogeneities in the "natural" frame $\omega'=\gamma H_0$, in which the limits of Δ are $-\frac{1}{2}\Delta_m$ to $+\frac{1}{2}\Delta_m$, and ignoring the variation of C with Δ, the result is

$$M_x'=0 \tag{2a}$$

$$M_y'=Ce^{-t/T_2}\frac{\sin(\frac{1}{2}\Delta_m t)}{\frac{1}{2}\Delta_m t}-Ce^{-(t+\tau)/T_2}\frac{\sin[\frac{1}{2}\Delta_m(\tau-t)]}{\frac{1}{2}\Delta_m(\tau-t)}. \tag{2b}$$

A controlled leakage of frequency ω from the free running base oscillator in the transmitter is present in the receiving equipment; thus there exists a predominate carrier of amplitude A. In the laboratory frame, the resultant rf voltage to be detected is

$$M_y=M_y'\sin\omega' t+A\sin\omega t,$$

where $\omega'-\omega=\delta$, the nominal angular frequency off resonance. Since $A\gg M_y'$, and $\frac{1}{2}(\omega'+\omega)\sim\omega$, this reduces to

$$M_y=[A+2M_y'\cos(\tfrac{1}{2}\delta t)]\sin\omega t. \tag{3}$$

Thus M_y appears as an amplitude-modulated carrier. M_y' appears as the envelope of the $\cos(\frac{1}{2}\delta t)$ beat, and the first term in Eq. (2b) is just the normal free decay while the second term describes the mirror signal.

Equation (2b) indicates a convenient means for determining the homogeneity of the magnet, i.e., the mean gradient of the field over the sample region. This effect is shown in Fig. 2, in which the inhomogeneity was varied by placing a small screwdriver blade a few cm from the sample. Δ_m can be calculated by noting the time interval between the end of the pulse and the first zero of $\sin(\frac{1}{2}\Delta_m t)$.

These equations can be used for an experimentally simple determination of T_2. For proper experimental conditions, the ratio $M_y'(\tau)$ to $M_y'(0)$ reduces to

$$M_y'(\tau)/M_y'(0)=e^{-2\tau/T_2}. \tag{4}$$

This technique is particularly useful for $T_2>10^{-3}$ sec, a region common to many liquids and usually obscured by magnet inhomogeneities. We are in the process of using this method for the measurement of T_2 for a series of related liquid hydrocarbons.

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Lagrangian S-Matrix

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SCHWINGER[1] has constructed an elegant and concise formulation of lagrangian field dynamics in which he shows [Eq. (2.133) of reference 1] that the result of applying two independent variations in structure to the lagrangian operator is to change the transformation function by an amount

$$\delta^{(1)}\delta^{(2)}\langle\zeta_1',\sigma_1|\zeta_2'',\sigma_2\rangle=(i/\hbar c)^2\int_{\sigma_2}^{\sigma_1}dx_1\int_{\sigma_2}^{\sigma_1}dx_2\langle\zeta_1',\sigma_1|(\delta^{(1)}\mathfrak{L}[x_1]\delta^{(2)}\mathfrak{L}[x_2])_+|\zeta_2'',\sigma_2\rangle. \tag{1}$$

Here, $|\zeta_2'',\sigma_2\rangle$ is the eigenket of a complete commuting set ζ_2 of operators constructed from the field operators on a spacelike surface σ_2, etc., and the bracket symbol is defined by

$$(A(x_1)B(x_2))_+=\begin{matrix}A(x_1)B(x_2) & \text{if} & (x_1)_0>(x_2)_0,\\ B(x_2)A(x_1) & \text{if} & (x_2)_0>(x_1)_0,\end{matrix} \tag{2}$$

which is an invariant concept, provided the operators involved commute when x_1-x_2 is a spacelike interval, and the positive sense of time is preserved.

This result can be generalized at once to give

$$\delta^{(1)}\cdots\delta^{(n)}\langle\zeta_1',\sigma_1|\zeta_2'',\sigma_2\rangle=(i/\hbar c)^n\int_{\sigma_2}^{\sigma_1}dx_1\cdots\int_{\sigma_2}^{\sigma_1}dx_n\langle\zeta_1',\sigma_1|(\delta^{(1)}\mathfrak{L}[x_1]\cdots\delta^{(n)}\mathfrak{L}[x_n])_+|\zeta_2'',\sigma_2\rangle, \tag{3}$$

which leads to a simple construction of the S-matrix in lagrangian form. Suppose the lagrangian is $\mathfrak{L}+\lambda\mathfrak{L}^i$ where λ is a numerical parameter. Then, if $\mathfrak{L}$ and $\mathfrak{L}^i$ are regarded as fixed, $\langle\zeta_1',\sigma_1|\zeta_2'',\sigma_2\rangle$ is a function of λ; the variation in structure now to be considered comes about by varying λ, and for this case Eq. (3) becomes

$$(d^n/d\lambda^n)\langle\zeta_1',\sigma_1|\zeta_2'',\sigma_2\rangle=(i/\hbar c)^n\int_{\sigma_2}^{\sigma_1}dx_1\cdots\int_{\sigma_2}^{\sigma_1}dx_n\langle\zeta_1',\sigma_1|(\mathfrak{L}^i[x_1]\cdots\mathfrak{L}^i[x_n])_+|\zeta_2'',\sigma_2\rangle. \tag{4}$$

The bras and kets in Eq. (4) refer to a system with the lagrangian $\mathfrak{L}+\lambda\mathfrak{L}^i$. We now invoke Taylor's formula,

$$f(\lambda+1)=\sum_{n=0}^{\infty}\frac{1}{n!}\frac{d^n}{d\lambda^n}f(\lambda), \tag{5}$$

subsequently putting $\lambda=0$. This gives $\langle\zeta_1',\sigma_1|\zeta_2'',\sigma_2\rangle$ for a system with a lagrangian $\mathfrak{L}+\mathfrak{L}^i$ in terms of a series in which the bras and kets refer to a system with a lagrangian $\mathfrak{L}$:

$$\langle\zeta_1',\sigma_1|\zeta_2'',\sigma_2\rangle=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{i}{\hbar c}\right)^n\int_{\sigma_2}^{\sigma_1}dx_1\cdots\int_{\sigma_2}^{\sigma_1}dx_n\langle\zeta_1',\sigma_1|(\mathfrak{L}^i[x_1]\cdots\mathfrak{L}^i[x_n])_+|\zeta_2'',\sigma_2\rangle. \tag{6}$$

Hence, the S-matrix is given by

$$S=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{i}{\hbar c}\right)^n\int_{-\infty}^{\infty}dx_1\cdots\int_{-\infty}^{\infty}dx_n(\mathfrak{L}^i[x_1]\cdots\mathfrak{L}^i[x_n])_+. \tag{7}$$

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Erratum: Spectrometer and Coincidence Studies on Np^{238}

[Phys. Rev. **79**, 410 (1950)]
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IN Table I the conversion coefficients of the 0.1028-Mev gamma-ray relative to the 0.258-Mev beta-ray were incorrectly reported to be 4.7 percent (M) and 7.5 percent (L). The correct figures are 2.5 percent (M) and 3.9 percent (L).

Half-Lives of Cu^{66}, Cu^{64}, Fe^{59}, and Ce^{144}

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IN the course of our work, the half-lives of Cu^{66}, Cu^{64}, Fe^{59}, and Ce^{144} have been determined with considerable precision. The half-lives of the copper isotopes were determined on samples of electrolytic copper foil irradiated with water-moderated radium beryllium neutrons. The Fe^{59} sample was from a cyclotron irradiation unit, free of Fe^{55}, obtained from the Isotopes Division of the Atomic Energy Commission. The Ce^{144} sample was obtained by the radiochemical isolation of cerium from fission products after a decay of over one year so that the sample would be free of Ce^{141}. The counting has been done with a flow proportional counter (nucleometer) using an external sample counter. All counts were checked against a standard to insure a constant counting efficiency.

We have obtained a half-life of 5.12 ± 0.05 min for Cu^{66}, in good agreement with the values of 5.18 ± 0.10 min by Cameron and Katz,[1] and 5.05 min by L. Meitner,[2] but differing from the value of 4.34 ± 0.03 min by Silver.[3] An irradiation was made of a Cd-shielded Cu foil which showed that most of the activation was due

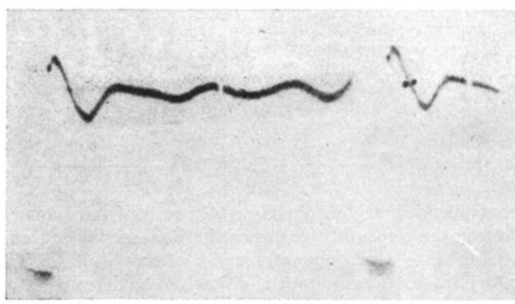

FIG. 1. $0.001M$ $MnSO_4$; prf 400 cps; 3500 μsec sweep.

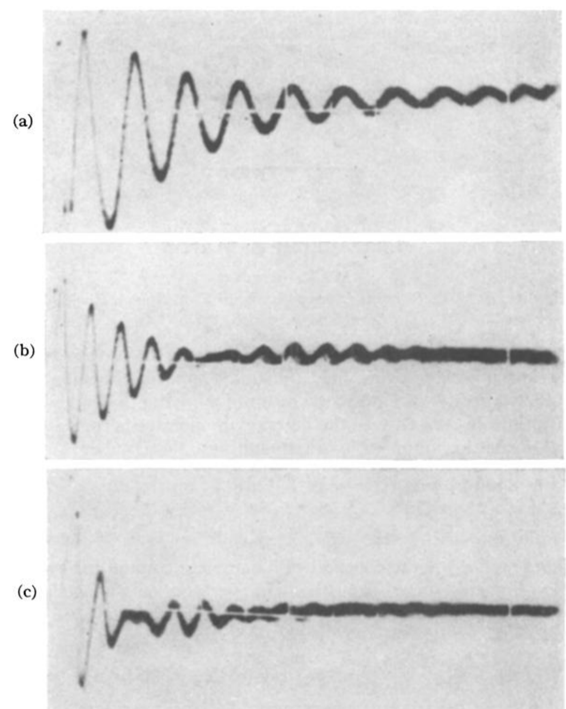

FIG. 2. $0.01M$ $MnSO_4$;prf 100 cps; 2500 μsec sweep. Curve (a): (Δ_m)gauss ~ 0. Curve (b): (Δ_m)gauss ~ 0.23. Curve (c): (Δ_m)gauss ~ 0.37.