then $V_k+V_k^+$ give the usual five operators for the β-interaction. We have previously taken as interaction hamiltonian a linear combination[1]

$$V=G\sum_{k=1}^{5}C_k(V_k+V_k^+),\tag{2}$$

in which the constants C_k must be real in order that V be hermitian.[2,3] Trigg and Feenberg[4] have given the generalization to the (hermitian) linear combinations using complex constants C_k

$$V=G\sum_{k=1}^{5}(C_kV_k+C_k^*V_k^+).\tag{3}$$

Using the conventional transformations of the wave functions in the Dirac equation,[5] it is easily verified that the interactions according to Eqs. (2) and (3) are invariant for (continuous) lorentz transformations, reflection of space, and reversal of time.

We now give two types of symmetries that can be imposed on the interaction.

I. First condition of symmetry.—The interaction energy must be invariant, if we transform all four particles to the anti-particles. (This is done by taking charge-conjugated solutions.[3]) In this way a transformed interaction

$$V'=G\sum_{k=1}^{5}(C_k^*V_k+C_kV_k^+),\tag{4}$$

is obtained from Eq. (3). Biedenharn and Rose have obtained this result by employing Wigner's time-reversal operator, a procedure equivalent to the use of charge conjugation here. In order that Eqs. (3) and (4) have the same meaning, the coefficients C_k must be real, so that Eq. (3) reduces to Eq. (2). This is especially clear if we consider the lower sign expressions (5) occurring in the formulas, taking the polarization into account (see below). These expressions would change their sign, when taking everywhere the "anti-particles."[6]

II. Second condition of symmetry.—We give two alternative formulations *a* and *b* of a second condition of symmetry, proposed earlier. (a) The processes of negatron and positron emission must be symmetrical in such a way that if coulomb interaction is neglected, the expressions for the interaction energy H_{β^-} and H_{β^+} are equal (possibly with the exception of a phase factor $e^{i\vartheta}$) if the following conditions are satisfied: the wave functions of the emitted (positive and negative) electrons and neutrinos must be physically equivalent, and $\psi_f(p)$ and $\psi_i(n)$ for negatron emission must be respectively the same as $\psi_f(n)$ and $\psi_i(p)$ for positron emission (i initial; f final; n neutron; p proton). (b) The interaction energy must be invariant (or differ only by a phase factor $e^{i\vartheta}$), when we take positions as real particles, negatrons as holes, and perform a corresponding change for the neutrinos, but not for the nucleons (see formulas (65), (70), and (75) of reference 1).

The consequences of symmetry principle *II* are as follows:

1. Using (2), we have either combinations of S, A, and P only, or combinations of V and T only.
2. Using (3), one obtains,[4] with $\epsilon_1=\epsilon_4=\epsilon_5=-1$, and $\epsilon_2=\epsilon_3=1$,

$$C_k^*C_l\pm C_kC_l^*=0,\quad\text{if}\quad \epsilon_k\epsilon_l=\mp1\quad(k,l=1,\cdots,5).\tag{5}$$

We have also calculated the transition probabilities for β-emission taking into account the orientation of the nucleus, the polarization of the emitted electron, and the $e-\nu$ angular correlation (for allowed transitions).[7] We then get cross terms that differ in sign for β^+ and β^- emission, with coefficients (5), but for $k, l=1, 2, 3, 4$ only. The pseudoscalar ($k=5$) gives no cross terms with the other invariants for allowed transitions, as the selection rule for the parity of its matrix element is different from that for the other invariants.

If we now compare these results with the consequences (5) of the symmetry principle, we come to the following conclusion: the condition, that the transition probability for allowed transitions (taking the polarization of the electron into account) has no terms that differ in sign for β^+ and β^- emission, is already equivalent to the symmetry principle *II* (except for a possible admixture of the pseudoscalar interaction).

(We may remark here that a common change in the phases of the C_k, *viz.*, $C_k'=e^{i\eta}C_k$, is a purely formal change for which the expressions $|C_k|^2$, $\mathrm{Re}(C_kC_l^*)$, and $\mathrm{Im}(C_kC_l^*)$, which occur as coefficients in the final formulas, are invariant.)

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The Natural Radioactivity of Rubidium

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Uncertainty regarding the decay process[1] of Rb^{87} and its rather unique position in regard to β-theory seemed good reasons for investigating the radiation with a large proportional tube spectrometer.[2] The internal diameter was 5.5 in. and the fully effective length[3] 10 in. The operating pressure was 5 atmospheres of argon +20 cm of methane. The tube was shielded with 2 in. of lead and an array of Geiger tubes could be put in anticoincidence for protection from cosmic radiation. The source, RbCl spread uniformly over the inner surface of a cylinder of aluminum acting as cathode, covered an area of between 800 and 900 cm². Two thicknesses were employed, 1.5 mg/cm² above 40 kev and 0.128 mg/cm² for lower energies.

The β-spectrum observed is shown by the full curve in Fig. 1, constructed from three sets of observations which overlapped each other, two with the thicker source and one with the inner (indicated by triangles). Extra detail near the end point is shown in the inset figure. A Kurie plot is given in Fig. 2 where, as in Fig. 1, the limiting energy E_0 is found to be 275 kev, considerably higher than the generally accepted value[1] of about 130 kev. However the very unusual shape of the spectrum no doubt contributed to the failure of the rather tentative earlier attempt to analyze it correctly. No maximum is observed on the curve, Fig. 1, and the intensity increases rapidly with decreasing energy down to at least

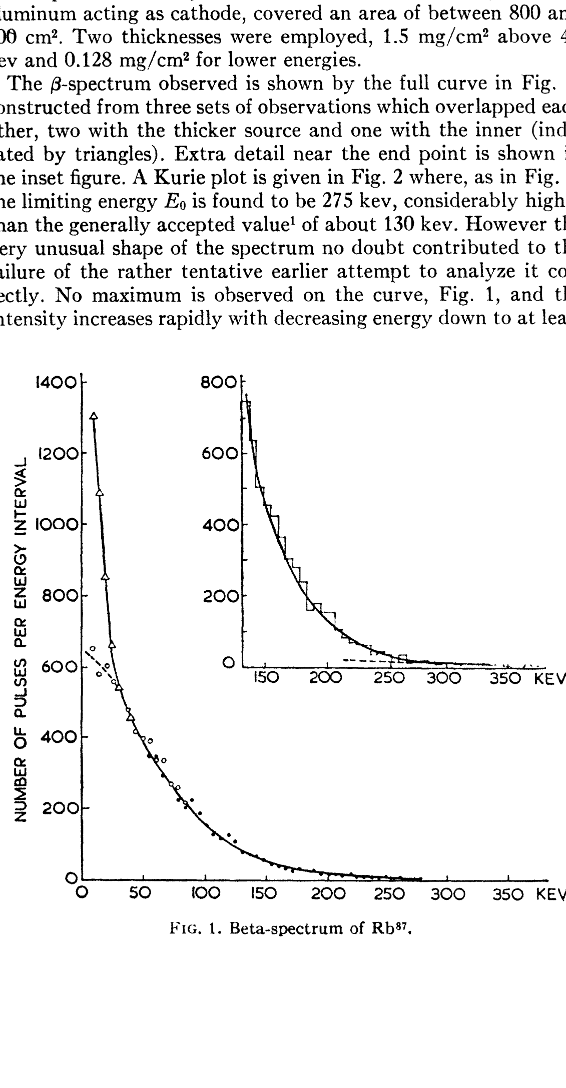

Fig. 1. Beta-spectrum of Rb^{87}.

10 kev, below which point we have not pursued our observations. The average energy is only 44 kev. The known spins of both ground states involve a spin change $\Delta I=3(3/2\rightarrow 9/2)$ and the ft value ($\log ft=17.6$ for $E_0=275$ kev, half-life $\tau=6.15\times 10^{10}$ yr) indicates a definitely third-forbidden transition. The very remarkable departure of the Kurie plot from the straight line qualitatively agrees with expectations for the case $\Delta I=3$, yes. Calculation[4] of the required correction factor C_{3T} has not been attempted, although the adjustment might well prove successful.

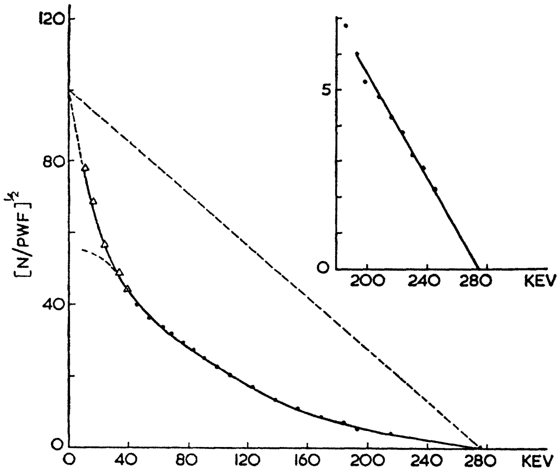

FIG. 2. Fermi plot of Rb^{87} beta-spectrum.

The absence of conversion electron peaks implies simple decay, but to verify this a search for K or L x-rays was carried out with the same equipment. Careful examination of counting rate as the discriminator level was varied through 1.8 kev failed to produce any evidence of L x-rays, while the spectrum observed with aluminum (58 mg/cm^2) covering the source to absorb β-rays showed that the intensity of K x-radiation corresponded to about one quantum per 500 β-particles. Unambiguous allocation to Rb or Sr was not possible. A scintillation spectrometer gave an upper limit of the γ-quantum per 5×10^3 β-rays.[5] Both of these

TABLE I. Decay data for natural β-emitters.

Source	E_0(Mev)	Half-life $\tau(10^{10}$ yr)	$\log ft$
Rb^{87}	0.275	6.15	17.6
K^{40}	1.36	0.11	18.05
Lu^{176}	0.215 or 0.4	2.4	18.02 or 18.91
Re^{187}	0.043	400	17.73
Nd^{150}	0.011	5	13.73

low intensity radiations could be due to bombardment of the source by β-particles, and we conclude that the process $Rb^{87}\rightarrow Sr^{87}+\beta^-$ is a simple ground to ground-state transition, contradicting some previous findings. The conversion peaks described by Ollano[6] would appear to be statistical fluctuations, and previously observed $\beta-e$ coincidences[7] perhaps had their origin in reflection.

With the thin source we found that 0.10075 g of RbCl gave 1477 counts per min. Applying corrections for reflection[8] at the support (+7.5 percent) and self-absorption (−4.5 percent), the half-life is

$$\tau=(6.15\pm 0.3)\times 10^{10}\text{ yr},$$

in good agreement with the latest value measured directly[7] and that deduced from the uranium-lead method.[9]

Our new value for E_0 increases $\log ft$ to 17.6 and shows, when tabulated with recent data[10] for the other natural sources ($Z<80$), a notable grouping of $\log ft$ values near 18 (see Table I). Nd^{150} seems exceptional, but preliminary work[11] shows Libby's data[12] are probably wrong and E_0 may greatly exceed 11 kev. Further examination is necessary but Nd^{150} may prove consistent in ft, and a considerable gap will exist between ft for these elements and that for the nearest neighbor, Be^{10} at $\log ft=13.65$.

We should like to thank Professor P. I. Dee for his interest and encouragement in this work. A fuller account will be published elsewhere.

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Acoustic Streaming and the Conservation of Angular Momentum*

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In this note we derive a quantity related to the rate at which angular momentum is transferred from an acoustic wave to a lossy medium; as Eckart[1] has recently pointed out, this transfer of angular momentum generates time-independent vorticity. In the interest of simplicity inviscid diabatic flow will be considered, although it is necessary to take into account viscous forces if the streaming velocity is to be calculated from the transfer of angular momentum. The artifice of attributing acoustic losses (including those due to shear viscosity) to the diabatic character of the flow simplifies the argument without altering its validity.

Euler's equation of motion may be written[1]

$$(\partial\rho\mathbf{u}/\partial t)+\rho(\mathbf{u}\cdot\nabla)\mathbf{u}+u\nabla\cdot\rho\mathbf{u}=-\nabla p. \quad (1)$$

Equation (1) may also be written in terms of the flux of momentum density dyadic $\rho\mathbf{uu}$

$$(\partial\rho\mathbf{u}/\partial t)+\nabla\cdot\rho\mathbf{uu}=-\nabla p. \quad (2)$$

We now assume, with Eckart, that the solution to Eq. (2) is the sum of a first- and a second-order term, $\mathbf{u}_1$ and $\mathbf{u}_2$, respectively, where $\mathbf{u}_1$ is irrotational and has a simple harmonic time dependence. Upon taking the curl of Eq. (2), there results

$$\nabla\times\rho_0\partial\mathbf{u}_2/\partial t=-(\partial/\partial t)(\mathbf{u}_1\times\nabla\rho_1)-\nabla\times\nabla\cdot\rho_0\mathbf{u}_1\mathbf{u}_1, \quad (3)$$

where we denote the equilibrium density by ρ_0, and in which ρ_1 represents the density associated with the solution $\mathbf{u}_1$ to the first-order equations of motion. The time average of the right-hand side of Eq. (3) is equal to

$$-\nabla\times\nabla\cdot\langle\rho_0\mathbf{u}_1\mathbf{u}_1\rangle_{\text{Av}}, \quad (4)$$

which is related to the average rate at which angular momentum is communicated from the wave to the medium. In the steady state, conservation of angular momentum requires that (4) be balanced by stresses arising from the streaming of the fluid. It is pertinent to note that (4) is the curl of the divergence of the average value of Brillouin's[2] radiation stress tensor. Should the fluid be homogeneous, ρ_0 may be taken outside the differential operators. For a homogeneous medium with $\nabla\times\mathbf{u}_1=0$, we have

$$\nabla\times\nabla\cdot\langle\rho_0\mathbf{u}_1\mathbf{u}_1\rangle_{\text{Av}}=\rho_0[\nabla(\nabla\cdot\mathbf{u}_1)\times\mathbf{u}_1]_{\text{Av}}. \quad (5)$$