parameters, $w_{XX'}{}^M$. If $k=k'$, then, as can be shown from Eqs. (1), $w_{AB}{}^M=w_{BA}{}^M$, so that there are only three distinct parameters. However, Eqs. (1) then reduce to two independent relations, so that an additional relation is still required.

It follows from Eqs. (6) and (8) of reference 1 that the fourier coefficients of the diffuse scattering (in reciprocal space) may be put in the form

$$\alpha_{kk'}{}^M=2(z_{kk'}{}^M-z_{kk'}{}^\infty), \quad (2)$$

where $z_{kk'}{}^M$ was defined as

$$z_{kk'}{}^M=w_{AA'}{}^M+w_{BB'}{}^M. \quad (3)$$

Also (see the following paragraph), we have

$$z_{kk'}{}^\infty=x_A{}^k x_{A'}{}^{k'}+x_B{}^k x_{B'}{}^{k'}, \quad (4)$$

where $z_{kk'}{}^\infty$ is the limiting value of $z_{kk'}{}^M$ as the "magnitude" of M increases indefinitely. Equations (2), (3), and (4), together with the value of $\alpha_{kk'}{}^M$ calculated from diffuse scattering data, provide the additional relation required for the individual determination of the four parameters, $w_{XX'}{}^M$ (three, for $k=k'$).

We now define the conditional probability $p_{XX'}{}^M$ as the probability that the k', $L-M$ site is occupied by X' if the k, L site is occupied by X. Since the probability of the latter event is $x_X{}^k$, it is obvious that

$$w_{XX'}{}^M=x_X{}^k p_{XX'}{}^M. \quad (5)$$

The four probabilities $p_{XX'}{}^M$ may therefore be calculated if the $w_{XX'}{}^M$ are known. It may be seen, either directly, or from Eqs. (1) and (5), that $p_{XA'}{}^M+p_{XB'}{}^M=1$. We also note that $p_{XX'}{}^\infty=x_{X'}{}^{k'}$, which, with Eqs. (3) and (5), justifies Eq. (4).

It has been remarked that the sets $[kk', M]$ and $[k'k, -M]$ are identical, and therefore, $w_{XX'}{}^M=w_{X'X}{}^{-M}$. However, if $k\neq k'$, the set of probabilities, $p_{X'X}{}^{-M}$, that the k, L site be occupied by X if the k', $L-M$ site is occupied by X', is, in general, distinct from the set of probabilities, $p_{XX'}{}^M$, defined in the predeeding paragraph. For a general set, $[kk', M]$, there are, therefore, four additional probabilities, which may be obtained from the relations,

$$w_{XX'}{}^M=x_{X'}{}^{k'}p_{X'X}{}^{-M}. \quad (6)$$

If $k=k'$, Eqs. (5) and (6), and the relation $w_{AB}{}^M=w_{BA}{}^M$, yield $p_{AA}{}^M=p_{AA}{}^{-M}$, $p_{AB}{}^M=p_{AB}{}^{-M}$, etc., so that there are only four distinct probabilities in this case. In the absence of long-range order, all sites will usually be equivalent, and the simplification $k=k'$ will be applicable. However, $p_{AB}{}^M=p_{BA}{}^M$, only if $x_A{}^k=x_B{}^k=\frac{1}{2}$.

The preceding analysis is sufficient to establish the equivalence of the treatment given in reference 1 and the formulation of Cowley,[2] for the special case which he considers (absence of long-range order). Cowley's parameter α_{lmn} is equivalent to $(\alpha_{11}{}^M/\alpha_{11}{}^0)$, or $\alpha_{11}{}^M/4x_Ax_B$, in the present notation (here $k=k'=1$). The writer regrets that in his original article it was implied that Cowley's treatment was not entirely correct.

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High Energy Nuclear Photoeffect in Carbon

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A PREVIOUS letter[1] (referred to as I) described an investigation of the high energy protons emitted by a carbon target under bombardment by bremsstrahlung from an electron synchrotron operated at (195±5) Mev. The method used for investigating the energy distribution of the protons has since been described in more detail.[2] In I an approximate method was used to measure the bremsstrahlung beam intensity. Subsequently, J. W. DeWire and J. C. Keck compared the bremsstrahlung intensities as measured by that method and by counting electron pairs ejected from a gold foil. These measurements showed that the values marked on the cross-section scale in I should be divided by 1.44. The scale values are then correct within the limits of ±40 percent instead of the factor of 3 given in I.

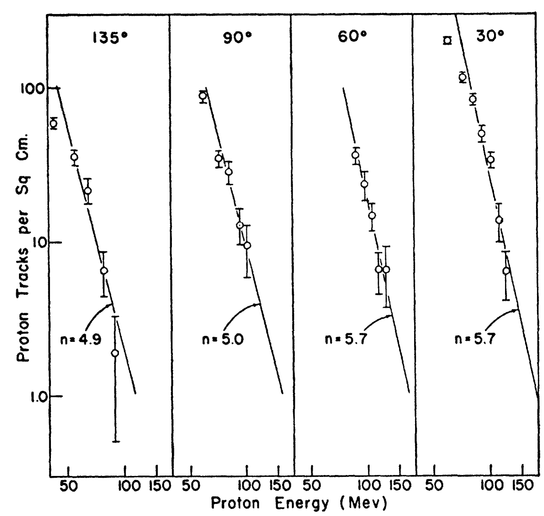

FIG. 1. Integral proton spectra. The normalized number (N) of protons with an energy exceeding E, plotted against E. Lines of the form $N=N_0E^{-n}$ are not fitted mathematically to the spectra but serve to illustrate the rate of fall-off of N with increasing E. Errors are statistical probable errors. Angles are laboratory angles.

It is of interest to compute the total nuclear cross section for the production of high energy protons. (For consistency in the figures we use the same scales as in I.) Figure 1 shows the relative integral proton spectra at 135°, 90°, 60°, and 30° to the bremsstrahlung beam, corrected for the loss of proton tracks in the earlier photographic plates of the stacks due to the fact that Ilford C2 plates record only protons with residual energies less than about 50 Mev. No statistically significant numbers of protons were observed at energies 10 Mev higher than the highest energy points plotted. The experiment does not yield information about the proton spectra at lower energies down to a few Mev, although there were indications that the integral spectra at lower energies could be represented by an n much less than 5. (The experimental conditions ruled out the confusion of protons with other high energy particles from the target, except for the possible presence of deuterons or tritons. There appears to be no reason, however, why deuterons or tritons could account for more than a small fraction of the observed tracks. Moreover, near the upper energy limit of the recorded proton spectrum, the range-energy relations discriminate against deuterons or tritons.) Figure 2 shows the differential cross section per steradian ($d\sigma/d\Omega$) plotted as a function of the angle of emission (θ) for protons with energies (E) greater than 70 Mev and greater than 90 Mev. These cross sections are computed directly from the corresponding points in Fig. 1. Total nuclear cross sections (σ) are obtained by graphical evaluation of $\sigma=(1/1.44)\int_0^\pi 2\pi \sin\theta(d\sigma/d\Omega)d\theta$. It is assumed that $d\sigma/d\Omega$ does not diverge strongly for $\theta<30°$ or $\theta>135°$. For $E>70$ Mev, we obtain $\sigma=(2.3\pm1.6)\times10^{-28}$ cm² per unit Q, and for $E>90$ Mev, $\sigma=(9\pm6)\times10^{-29}$ cm² per unit Q. The errors given for σ are estimated maximum possible errors.

As implied in I, the strong forward bias in the angular distribution of the high energy protons requires, because of momentum considerations, that only a small subgroup of nucleons be involved in the primary interaction with a high energy photon. Levinger[3] has pointed out that the results are in qualitative agreement with a two-nucleon ("quasi-deuteron") model for the interaction. From the point of view of momentum, the results could also be consistent with a sub-group as large as an α-particle. (Tamor[4] has used an α-particle model in discussing the emission of high energy protons by nuclei which have captured negative π-mesons.) The

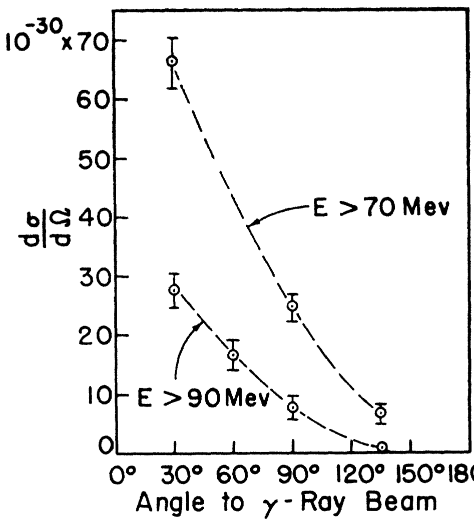

FIG. 2. Differential nuclear cross section of carbon for the production of high energy protons plotted against the angle of proton emission. The differential cross section is expressed as cm² per steradian per unit Q. Errors are probable errors.

cross section is too large to admit Compton recoil protons as a main effect, while, in addition, the high energy protons at large angles would not be produced by the Compton effect.

A synchrotron energy near 200 Mev is advantageous for studying the high energy photoeffect. It is about as high an energy as can be used while avoiding possible complications due to appreciable photomeson production.[5]

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Radiative Corrections to the Intensities for Hydrogenlike Atoms

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THE radiative corrections to the transition probabilities for the transitions of excited hydrogenlike atoms with single photon emission are calculated, in nonrelativistic approximation.

The hamiltonian, which has been studied by Pauli and Fierz,[1] is used, and it is

$$H=(\mathbf{p}^2/2m)+V(\mathbf{q}-i\lambda\Sigma_K(k^3V)^{-\frac{1}{2}}\mathbf{e}_K(a_K-a_K^+))+\Sigma_K a_K^+a_K\hbar ck, \quad (1)$$

where $\mathbf{p}$ and $\mathbf{q}$ are the momentum and coordinate of the electron, $V(\mathbf{q})$ is the electrostatic potential energy, a_K and a_K^+ are destruction and creation operators for a light quantum with wave number K, $\mathbf{e}_K$ is a unit vector in the direction of polarization, m is the observed mass of the electron, and

$$\lambda=[(2\pi\hbar/mc)(e^2/mc^2)]^{\frac{1}{2}}.$$

We expand the second term of Eq. (1) and obtain[2]

$$\begin{aligned}V(\mathbf{q})&-i\lambda\Sigma_K(k^3V)^{-\frac{1}{2}}(a_K-a_K^+)\,\mathrm{grad}_K V(\mathbf{q})\\&-\tfrac{1}{2}\lambda^2\{\Sigma_K(k^3V)^{-\frac{1}{2}}(a_K-a_K^+)\,\mathrm{grad}_K\}^2V(\mathbf{q})\\&+i\tfrac{1}{6}\lambda^3\{\Sigma_K(k^3V)^{-\frac{1}{2}}(a_K-a_K^+)\,\mathrm{grad}_K\}^3V(\mathbf{q})+\cdots,\end{aligned} \quad (2)$$

where $\mathrm{grad}_K=\mathbf{e}_K\cdot\mathrm{grad}$.

The fourth term of expansion (2) gives the main contribution to the transition which we now consider. Other terms contribute to cancel the infrared divergence. We rearrange the operators a_K and a_K^+ in the fourth terms of Eq. (2), using the commutation relations $[a_K, a_L^+]=\delta_{KL}$, and choose the following single-photon emission terms:

$$\begin{aligned}\tfrac{1}{6}i\lambda^3 &\sum_{KLM}\frac{1}{V^{\frac{3}{2}}k^3l^3m^3}(\mathrm{grad}_K\,\mathrm{grad}_L\,\mathrm{grad}_M)\\&\times V(\mathbf{q})(a_K^+\delta_{LM}+a_L^+\delta_{MK}+a_M^+\delta_{KL})\\&=\tfrac{1}{2}i\sum_L\frac{1}{Vl^3}(\mathrm{grad}_L)^2\sum_K\frac{1}{(k^3V)^{\frac{1}{2}}}\mathrm{grad}_K V(\mathbf{q})a_K^+\\&=i\frac{2}{3(2\pi)^2}\lambda^3B\Delta\sum_K\left(\frac{1}{k^3V}\right)^{\frac{1}{2}}\mathrm{grad}_K V(\mathbf{q})a_K^+,\end{aligned} \quad (3)$$

where B is the Bethe term, $\int_{k_0}^{K}dl/l=\log(mc^2/\hbar ck_0)$. We replace the $\Delta V(\mathbf{q})$ by the charge density of nucleus $\rho(\mathbf{q})$; then Eq. (3) becomes as follows:

$$i(2/3\pi)e\lambda^3B\Sigma_K(k^3V)^{-\frac{1}{2}}\,\mathrm{grad}_K\rho(\mathbf{q})a_K^+. \quad (4)$$

The transition probabilities per unit time for the transitions of hydrogenlike atoms from the excited state ψ_A to the lower state ψ_B, caused by the interaction term (4), are

$$\gamma=\frac{2}{9}\frac{1}{\pi^4}\lambda^6B^2\frac{e^2}{\hbar^2c}\frac{1}{k}\int|\{\mathrm{grad}_K\rho(\mathbf{q})\}_{AB}|^2d\Omega \quad (5)$$

where $\{\mathrm{grad}_K\rho(\mathbf{q})\}_{AB}=\int\bar{\psi}_A(\mathbf{q})(\mathrm{grad}_K\rho(\mathbf{q}))\psi_B(\mathbf{q})d\mathbf{q}$. The quantity γ is proportional to the inverse of k. The smaller the energy difference between two levels of the atom, the greater is the transition probability between the levels with one-photon emission due to the radiative correction.

The value of γ depends on the size and shape of the charge distribution $\rho(\mathbf{q})$. If the charge has uniform spherical distribution of radius R, then we have

$$\rho=\rho(r)=\rho_0=3e/4\pi R^3 \quad \text{for} \quad r<R, \quad \rho=0 \quad \text{for} \quad r>R,$$
$$\mathrm{grad}_K\rho=\rho_0\delta(r-R)\cos\theta,$$
$$\begin{aligned}\{\mathrm{grad}_K\rho\}_{AB}&=\int\rho_0\bar{\psi}_A(R,\theta,\varphi)\cos\theta\psi_B(R,\theta,\varphi)R^2d\Omega\\&=(3e/4\pi)\int R^{-1}\bar{\psi}_A(R,\theta,\varphi)\cos\theta\psi_B(R,\theta,\varphi)d\Omega,\end{aligned}$$

and γ is given by

$$\gamma=\frac{1}{2\pi^5}\frac{e^4}{\hbar^2c^2}\lambda^6B^2\frac{c}{k}\left|\int\frac{1}{R}\bar{\psi}_A(R,\theta,\varphi)\cos\theta\psi_B(R,\theta,\varphi)d\Omega\right|^2. \quad (6)$$

Breit and Teller[3] calculated the probability for the $2s\rightarrow1s$ transition of hydrogen with the emission of two photons and obtained 1/7 sec for the mean life of the $2s$ state. The $2s_{\frac{1}{2}}$ state of hydrogen is raised by the radiative reaction and is 0.03 cm^{-1} higher than the $2p_{\frac{1}{2}}$ state.[4] For the probability of the $2s_{\frac{1}{2}}\rightarrow2p_{\frac{1}{2}}$ transition caused by the radiative reaction, Eq. (6) gives the value 2×10^4 sec^{-1}. Here we assume $B=10$, and R is much smaller than the Bohr radius.

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[4] W. E. Lamb and R. C. Retherford, Phys. Rev. **72**, 241 (1947).

Mixed Invariants in Beta-Decay and Symmetries Imposed on the Interaction Hamiltonian

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THE interaction hamiltonian, which is used in the theory of beta-radioactivity, can be a linear combination of the scalar (1), vector (2), tensor (3), axial vector (4), and pseudoscalar (5) interactions. If we put

$$V_k=(\psi^*\,{}^k\Omega_L\varphi)Q\,{}^k\Omega_N \quad (k=1,\cdots,5), \quad (1)$$