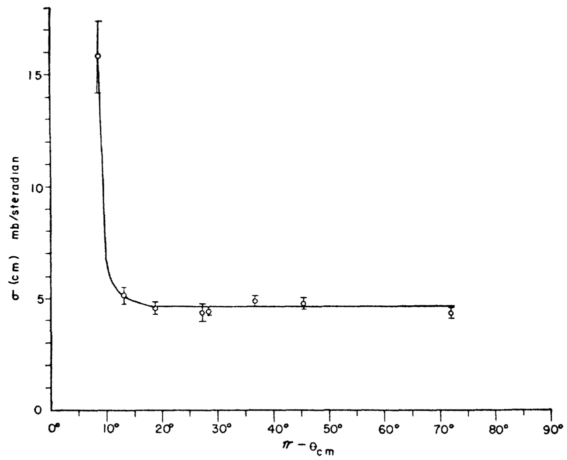

FIG. 1. Differential cross section for proton-proton scattering at 240 Mev.

These results agree with those of Oxley and Schamberger[3] of this laboratory, who obtain an average cross section of 4.97±0.42 mb/sterad from 27.5° to 90° cm. The relative error in both of these experiments is 0.22 mb/sterad, since the errors in the C^{11} cross section and the beta-counter calibration are mutual. The combined value of the two experiments gives 4.81±0.38 mb/sterad for the isotropic part of the cross section.

These measurements are about 30 percent higher than those published by Chamberlain *et al.*[1] The cause of this discrepancy is not clear at the present time.

Details of this experiment will be published at a later date. I wish to thank Professor C. L. Oxley for many suggestions during the course of this work.

* Assisted by the joint program of the ONR and AEC.
[1] Chamberlain, Segre, and Wiegand, Phys. Rev. 83, 923 (1951).
[2] O. A. Towler, Jr., and C. L. Oxley, Phys. Rev. 78, 326 (1950).
[3] C. L. Oxley and R. D. Schamberger, Phys. Rev. (to be published).

Thermodynamic Functions on the Generalized Fermi-Thomas Theory

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(Received July 30, 1951)

RECENTLY, equations of state of the elements[1] based on the generalized Fermi-Thomas theory have been developed. It has been shown[2] that exchange effects may be neglected for high temperatures, and they will not be considered here. In the past, the pressure has been obtained by calculating the rate of flow of momentum through the surface of the atom, which is assumed to be a sphere of radius a. It is also possible to find the pressure from the free energy or the logarithm of the partition function.[3] The purpose of this note is to show that the pressure calculated from the free energy using the generalized Fermi-Thomas model for arbitrary temperatures agrees with that found by mechanical considerations based on the same model, and, in addition, one obtains expressions for the entropy and the specific heat.

Our model leads to the kinetic energy density[4]

$$E_K(r)=(4\pi/h^3)(2m)^{3/2}\beta^{-5/2}I_{3/2}[\eta+\beta eV(r)], \quad (1a)$$

and the electron number density

$$\rho(r)=(4\pi/h^3)(2m)^{3/2}\beta^{-3/2}I_{1/2}[\eta+\beta eV(r)], \quad (1b)$$

where $V(r)$ is the total potential, V_N+V_e, and $\beta=(kT)^{-1}$. Here V_N is the nuclear potential Ze/r, and $-V_e$ satisfies Poisson's equation with the charge density $e\rho(r)=\rho_e$. The boundary conditions are the vanishing of the total potential and the electric field at the surface of the atom. The volume integral of (1a) is the kinetic energy E_{kin} and that of (1b) is Z, the nuclear charge. The potential energy E_{pot} is

$$E_{\text{pot}}=-\tfrac{1}{2}\int_0^a\rho_eV_ed\tau-\int_0^a\rho_eV_Nd\tau=E_{e,e}+E_{e,N}, \quad (2)$$

where $d\tau$ is $4\pi r^2dr$. By mathematical manipulation, one finds the virial theorem

$$E_{\text{kin}}=\tfrac{3}{2}pv-\tfrac{1}{2}E_{\text{pot}}. \quad (3)$$

The difference between our calculation and previous ones is that the pressure is obtained by macroscopic rather than microscopic considerations.

The pressure p and the entropy S are found from the free energy $F=E_{\text{tot}}-TS$, where T is the absolute temperature, by means of the thermodynamic relations,

$$p=-(\partial F/\partial v)_T, \quad (4a)$$

$$S=-(\partial F/\partial T)_v. \quad (4b)$$

The basis of our work is the Gibbs-Helmholtz equation in the form,

$$E_{\text{tot}}=(\partial/\partial\beta)[\beta F], \quad (5)$$

of which we obtain the indefinite integral. This is done by integrating the various terms of E_{tot} by parts and using the relations between the derivatives of $E_K(r)$ and ρ_e obtained by differentiating (1a) with respect to a, β, and r, including the cross derivatives. Employing Green's theorem, one finds

$$F=-\tfrac{2}{3}E_{\text{kin}}-E_{e,e}+ZkT\eta. \quad (6)$$

From the relations (4a) and (4b) it follows that

$$p=\tfrac{2}{3}E_K(a), \quad (7a)$$

$$S=T^{-1}\{(5/3)E_{\text{kin}}+2E_{e,e}+E_{e,N}\}-Zk\eta. \quad (7b)$$

The first of these is the value obtained from mechanical considerations. Consideration of the Fermi factor shows that there is a maximum value of the momentum, $p_m(r)$, given by

$$P_m^2(r)/2m=eV(r)+\eta kT, \quad (8)$$

in the sense that in the limit as T tends to zero the Fermi factor is one for $p<p_m(r)$ and is zero for $p>p_m(r)$. Integrating Eq. (8), which is the same as the last equation of Sec. IV of reference 1, over the phase space at absolute zero yields

$$(5/3)E_{\text{kin}}=-2E_{e,e}-E_{e,e}+ZkT\eta. \quad (9)$$

This shows that $\lim_{T\to 0} S=0$, as it should.

The specific heat at constant volume, C_v, given by the relation,

$$C_v=-\beta(\partial S/\partial\beta)_v, \quad (10)$$

may be expressed in the form,

$$\frac{4}{3}TC_v=\frac{5}{2}pv+\frac{7}{6}E_{e,e}+\frac{1}{6}E_{e,N}-Z\left\{\frac{\partial\eta}{\partial\beta}+e\beta\frac{\partial V_e(0)}{\partial\beta}\right\}+T\left(\frac{\partial p}{\partial T}\right)_v v. \quad (11)$$

This may be shown to agree with that given in reference 1 as T tends to zero. The expression derived in reference 1 is based upon relationships valid at $T=0$, whereas our relations are valid at an arbitrary temperature.

[1] Feynman, Metropolis, and Teller, Phys. Rev. 75, 1561 (1949).
[2] H. Jensen, Z. Physik 111, 373 (1938).
[3] Marshak, Morse, and York, Astrophys. J. 111, 214 (1950).
[4] The notation is the same as that of reference 1, except that our $\rho=-\rho_e$ of it.

The Near Infrared Spectrum of Lightning*

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ON September 18, 1951, a spectrum of lightning in the wavelength range 7100–9100 angstroms was secured on an Eastman spectroscopic plate type 1N. The instrument used was one