For $t=t_f$, $S(t_f, t_i)$ is in the limit $t_i\rightarrow-\infty$, $t_f\rightarrow\infty$ the S-matrix of Dyson, and for $F_H(t_f, t_i)$ one has

$$F_H(t_f, t_i)=S^\dagger(t_f, t_i)F_I(t_f, t_i)S(t_f, t_i). \tag{1}$$

Now $F_H(t_f, t_i)$ is the outgoing operator $F_H{}^{\text{out}}$, and $F_I(t_f, t_i)$, which moves in the Hilbert space like a free field operator, differs from $F_I(t_i, t_i)$ only by a constant phase factor. But at t_i we have

$$F_I(t_i, t_i)=F_H(t_i, t_i)=F_H{}^{\text{in}}.$$

Thus we have shown that, apart from an unimportant constant phase factor, Eq. (1) is just the definition of the S-matrix in the Heisenberg representation

$$F_H{}^{\text{out}}=S^\dagger F_H{}^{\text{in}}S,$$

where F_H stands for any of the field operators.

One can of course define the S-matrix in the Schroedinger representation. In a recent paper, Wu[3] has obtained an S-matrix identical with Dyson's by starting from the Schroedinger representation and using conventional perturbation theory. But he is essentially working in the interaction representation after the general wave amplitude is expanded in terms of the time-dependent energy eigenfunctions of the unperturbed Hamiltonian.

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[2] C. N. Yang and D. Feldman, Phys. Rev. **79**, 972 (1950).
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Principle of Detailed Balance

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According to Hamilton and Peng[1] the principle of detailed balance cannot be expected to hold in quantum theory when perturbation theory is not applicable. In the concrete case which they discuss, the principle holds, however, if averages over the spin variables are taken. Heitler[1] conjectures that this is true, in general. The purpose of this note is to show that the principle of detailed balance holds rigorously in a form slightly different from that implied by Heitler, Hamilton, and Peng, and to give a proof for Heitler's conjecture.

The principle of detailed balance may be stated as follows: *The transition probabilities for a certain process and its inverse are always equal.* This statement needs, however, to be supplemented by a definition of the inverse of a given transition. Hamilton and Peng consider the transition $B\rightarrow A$ to be the inverse of the transition $A\rightarrow B$. Detailed balance would then imply symmetry properties of the S-matrix which cannot be expected to hold in general. On the other hand detailed balance becomes a simple consequence of the invariance of the interaction under time reversal if we adopt the following definition: *The inverse of a given transition is obtained by reversal of the time.*

The operation of time reversal in quantum mechanics has been investigated by Wigner.[2] Generalizing slightly Wigner's results, we note the relation,

$$K\Psi(p, s, t)=U\Psi^*(-p, -s, -t). \tag{1}$$

K denotes the operator of time reversal, p stands for the momenta of all the particles present, and s for their spin variables. The star indicates the complex conjugate. U is a unitary operator which operates on the spin variables only. For one particle with spin $\frac{1}{2}$ we have $U=\sigma_2$, and for one particle with spin 1 we have $U=-1$, if the usual representation of the spin matrices

$$\sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

is assumed. In the most general case we need only form the appropriate product of one-particle operators. From these remarks it can be seen that U is also Hermitian and that

$$(UHU)^*=UH^*U \tag{2}$$

for any operator H which does not change the number of Fermions by an odd number. The interaction Hamiltonian and consequently the S-matrix have this property. The invariance under time reversal of the Hamiltonian in the interaction representation can be stated in the equation,

$$\begin{aligned}(p', s'|H(t)|p, s)&=(p', s'|KH(t)K^{-1}|p, s)\\ &=(-p', -s'|UH(-t)U|-p, -s)^*.\end{aligned} \tag{3}$$

Since H is Hermitian and U unitary, there follows

$$(p', s'|H(t)|p, s)=(-p, -s|UH(-t)U|-p', -s'). \tag{4}$$

In order to exploit the relation (4) for the proof of the principle of detailed balance, we write the S-matrix in the form,

$$S=1+\sum_{n=1}^{\infty}(-i)^n\int dt_n\ldots\int dt_1\theta(t_n, t_{n-1})\ldots\theta(t_2, t_1)H(t_n)\ldots H(t_1), \tag{5}$$

where

$$\theta(t_2, t_1)=\begin{cases}1 & \text{if} \quad t_2>t_1\\ 0 & \text{if} \quad t_2<t_1,\end{cases}$$

and hence

$$\theta(t_2, t_1)=\theta(-t_1, -t_2). \tag{6}$$

From (4) follows

$$\begin{aligned}(p', s'|H(t_n)\ldots H(t_1)|p, s)&\\ =(-p, -s|UH(-t_1)\ldots H(-t_n)U|-p', -s'),&\end{aligned} \tag{7}$$

and hence with (5) and (6)

$$\begin{aligned}(p', s'|S|p, s)&=(-p, -s|USU|-p', -s')\\ &=\pm(-p, -s|S|-p', -s').\end{aligned} \tag{8}$$

The last step follows from the representation for U described above and the fact that S does not change the number of Fermions by an odd number.

From Eq. (8) follows immediately that the transition probabilities for the transition $p, s\rightarrow p', s'$ and $-p', -s'\rightarrow -p, -s$ are equal. This proves the general validity of the principle of detailed balance provided the interaction is invariant under time reversal and an appropriate definition for the inverse of a transition is adopted.

Moreover, upon summation over the spin variables one finds

$$\Sigma_s\Sigma_{s'}|(-p, -s|S|-p', -s')|^2=\Sigma_s\Sigma_{s'}|(p, s|S|p', s')|^2. \tag{9}$$

This follows from the invariance of the transition probability under reflection of the coordinate axes. Combining (8) and (9) we find

$$\Sigma_s\Sigma_{s'}|(p, s|S|p', s')|^2=\Sigma_s\Sigma_{s'}|(p', s'|S|p, s)|^2. \tag{10}$$

This completes the proof of Heitler's conjecture.

I am indebted to Dr. J. M. Jauch for clarifying discussions on the subject of time reversal.

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Perturbation Theory and Configuration Space Methods in Field Theory*

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We have investigated the configuration space treatment of relativistic field theories, both in coordinate and momentum space, with the aim of treating problems of interacting fields.

One of the problems being analyzed is the two-body problem in the neutral scalar and pseudoscalar meson theory. This has been previously investigated by Tamm[1] and by Dancoff,[2] who, how-