from which follows the general relation between s and r:

$$s=k(2-r)+2r-1;$$

for $r=2$ we get

$$s=3 \text{ independent of } k.$$

Only if r is quite different from 2 will it be possible to decide which k gives the best results.[15]

[15] Compare Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948). There $k=\frac{2}{3}$.

VII. ACKNOWLEDGMENT

It is a pleasure to express my gratitude to Professor E. Fermi, who suggested this problem, for his frequent advice and encouragement, and to Professor M. Schein for very stimulating discussions of the experimental problems involved in this work. I would like to express my appreciation to my colleague, Art Rosenfeld, for the reading of the manuscript and for valuable criticism. Thanks are also due to the Hebrew Technical Institute in New York for a grant in aid.

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A NaI (Tl) Scintillation Spectrometer Study of Proton Gamma-Ray Coincidences

ROBERT C. ALLEN,* JOHN E. MAY, AND WALDO RALL
Sloane Physics Laboratory, Yale University,† New Haven, Connecticut
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A proton gamma-ray coincidence study of the $Al^{27}(\alpha,p)Si^{30}$ reaction has been made using a NaI(Tl) scintillation spectrometer. The decay scheme of the excited states of Si^{30} has been established and the energies of the gamma-rays have been measured. The decay of the third excited state was found to be a double cascade process: from the third to the second to the ground state, and from the third to the first to the ground state. The gamma-ray energies in the former process were determined to be 1.28 ± 0.06 Mev and 3.66 ± 0.15 Mev. The energies of both the gamma-rays in the latter transition were found to be between 2.2 and 2.7 Mev. Measurements on the second excited state showed that the decay was direct to the ground state, the energy of the gamma-rays being 3.63 ± 0.15 Mev. The gamma-ray energy from the first excited state was established to be 2.32 ± 0.05 Mev.

I. INTRODUCTION

EXCITED states of nuclei arising from nuclear reactions have been studied primarily by energy measurements of the product particles and by energy measurements of the gamma-rays from the residual nucleus. With the precision now achieved by several research groups, the energies of the levels may be coupled with the gamma-ray energies to suggest a decay scheme.

A coincidence method, in which the gamma-rays associated with a specific level are investigated, can give information as to the precise mode of decay of an excited state. Benson[1] established the existence of coincidences between gamma-rays and protons in the reaction $Al^{27}(\alpha,p)Si^{30}$. Landon[2] extended the technique to include energy measurements of the gamma-rays by absorption methods using a scintillation detector. From his measurements Landon was able to give a probable decay scheme for the excited states of Si^{30}. A NaI(Tl) scintillation spectrometer which provides high efficiency, large solid angle, good resolving time, and fair energy resolution appears well suited to this type of problem. The work described in this paper concerns the application of a NaI(Tl) gamma-ray counter to the energy measurement of the gamma-rays emitted in coincidence with the protons from the reaction $Al^{27}(\alpha,p)Si^{30}$. The reaction was produced with 7.8 Mev cyclotron alpha-particles.

II. Si^{30} ENERGY LEVELS

The low energy levels of Si^{30} as determined from the $Al^{27}(\alpha,p)Si^{30}$ reaction[1,2] lie at approximately 0, 2.4, 3.7, and 5.0 Mev. These agree fairly well with those found from the $Si^{29}(d,p)Si^{30}$ reactions,[3,4] except that Van Patter and his colleagues[5] find that there may be two levels around 3.7 Mev, namely at 3.52 and 3.79 Mev. The assignment of both of these levels to Si^{30}, they point out, is not yet definite, since a study of relative intensities from separated isotope targets did not give conclusive results on these levels. No indication of this

TABLE I. Breakdown of coincidence counts.

Si^{30} excited state	Coincidences	Observed / Accidental	Protons per coincidence	Coincidences per hour
First	3241	16	230	130
Second	8901	18	225	130
Third	20,666	16	145	700

* Part of a dissertation presented to the Faculty of the Graduate School of Yale University, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

† Assisted by the joint program of the ONR and AEC.

[1] B. B. Benson, Phys. Rev. **73**, 7 (1948).

[2] H. H. Landon, Phys. Rev. **83**, 1081 (1951).

[3] R. F. Humphreys and H. T. Motz, Phys. Rev. **80**, 595 (1950).

[4] Van Patter, Sperduto, and Enge, Phys. Rev. **83**, 212(A) (1951).

[5] Private communication from D. M. Van Patter.

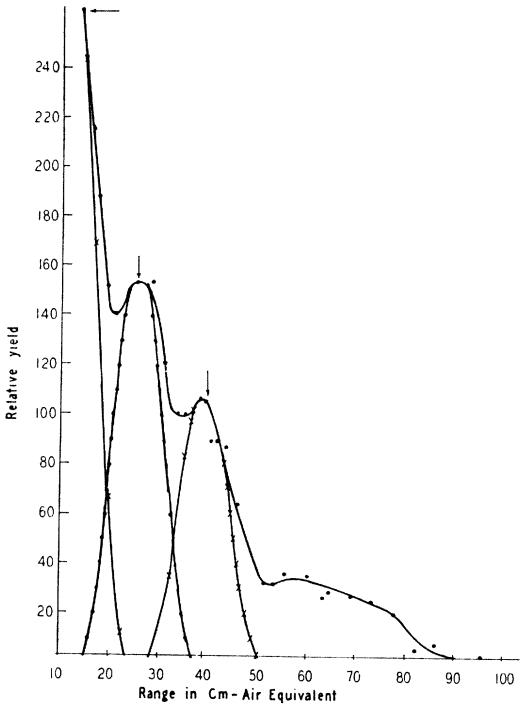

FIG. 1. Proton spectrum—$Al^{27}(\alpha,p)Si^{30}$.

doublet level was found in our work; however, neither the resolution of the proton groups nor the gamma-rays is such that we can rule out such a doublet, especially if the yield from one is low.

The proton spectrum used for the proton-gamma coincidence experiment is shown in Fig. 1, resolution being sacrified to gain solid angle. The protons were detected with a proportional counter and their range measured by absorption in aluminum. The protons corresponding to the first, second, and third excited states were selected by setting the absorption at 39.7, 25.3, and 14.4 cm air equivalent, respectively.

III. EXPERIMENTAL ARRANGEMENT

The arrangement of the bombardment chamber, aluminum target, proton proportional counter, and NaI(Tl) crystal gamma-ray counter is shown in Fig. 2. A block diagram of the circuit is shown in Fig. 3. The

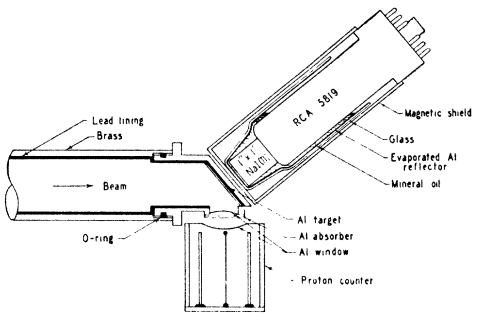

FIG. 2. Diagram of proton and gamma-ray counters.

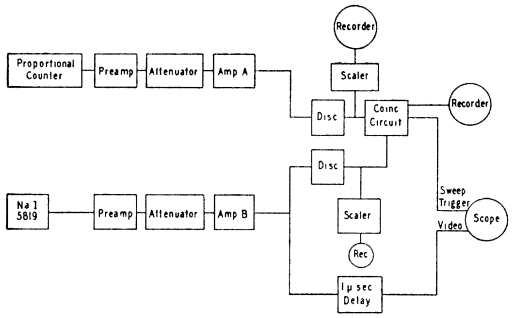

FIG. 3. Block diagram of apparatus.

proton and gamma-ray pulses were amplified, fed to a discriminator and coincidence circuit and to scalers and recorders. The gamma-ray pulses were also fed through a delay line to the y-axis amplifier of a synchroscope. The coincidence output was recorded and also used to trigger the sweep and intensifier gate of the synchroscope. In this way only the gamma-ray pulses in coincidence with a proton were displayed on the face of the cathode-ray tube. The individual pulses were photographed on continuously moving film, using a modified 16-mm movie camera with an f:2.5 lens.

Two types of calibration were made periodically throughout each run. Energy calibrations were made by taking time exposures[6] of the pulse distribution from the 1.28-Mev Na^{22} gamma-ray. A typical exposure is shown in Fig. 4. The top two bands correspond to the photoelectric and Compton processes of the 1.28-Mev gamma-ray, while the lower two are associated with Na^{22} positron annihilation radiation.

Square pulses of one microsecond duration and of

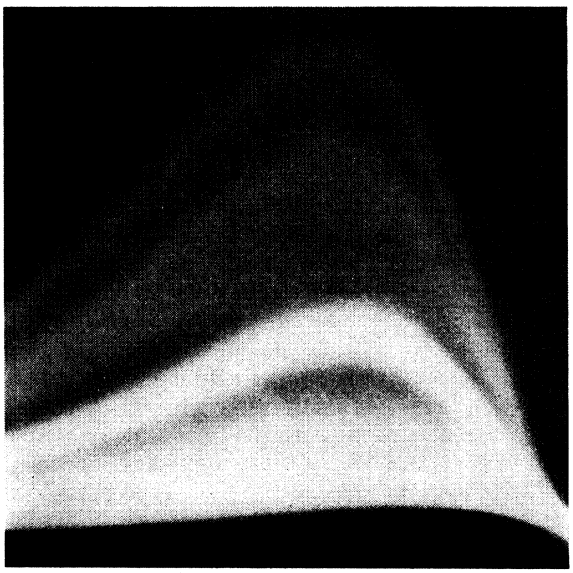

FIG. 4. Photographic distribution of Na^{22}.

[6] R. Hofstadter and J. A. McIntyre, Phys. Rev. **80**, 631 (1950).

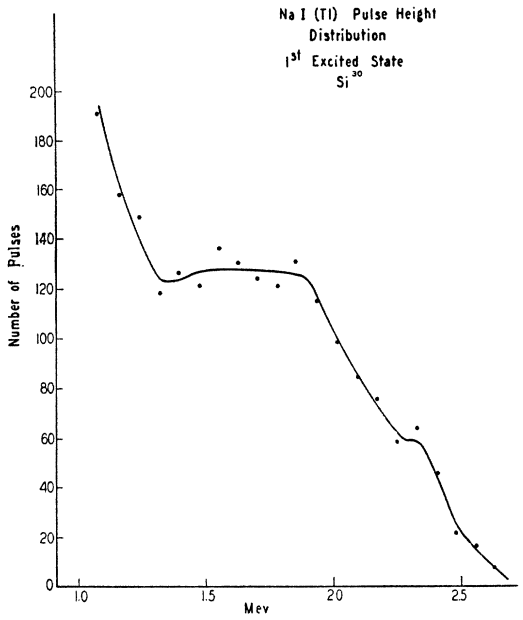

FIG. 5. Pulse-height distribution. First excited state—Si^{30}.

variable pulse height were introduced into the amplifier system and photographed to provide a frequent check on the stability of the electronic equipment.

IV. PULSE-HEIGHT DISTRIBUTIONS

Table I lists for each excited state the total number of coincidences, the ratio of observed to accidental coincidences, the number of protons required per coincidence, and the number of coincidences per hour. The low coincidence counting rates were required in order to maintain a high observed to accidental coincidence ratio.

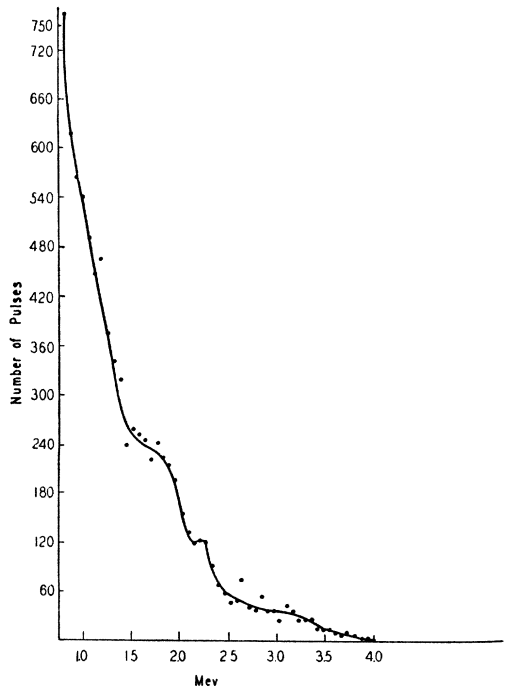

FIG. 6. Pulse-height distribution. Second excited state—Si^{30}.

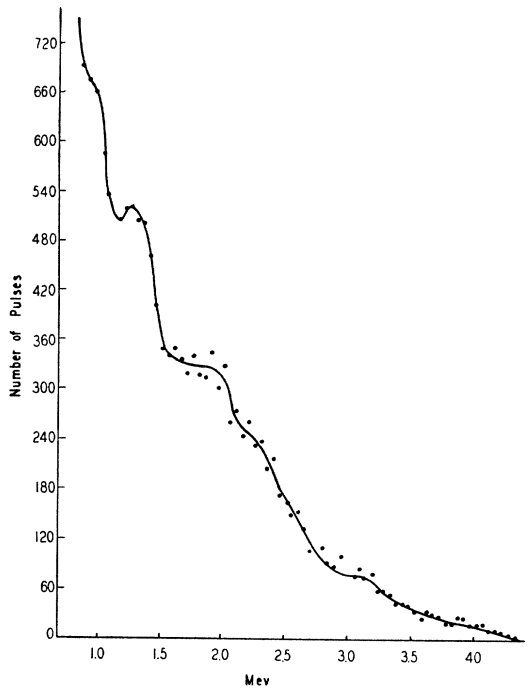

FIG. 7. Pulse-height distribution. Third excited state—Si^{30}.

For the analysis of the pulse-height data, the film was projected on a ruled screen and the height of each pulse was recorded. The resultant pulse-height distributions are shown in Figs. 5–7. Figure 5 gives the pulse height distribution of the gamma-rays from the first excited state of Si^{30}. The photoelectric and Compton peaks are apparent and establish an energy of 2.32 ± 0.05 Mev in excellent agreement with the value of the energy level.

For the second excited state the number of protons per coincidence was the same as that for the first excited state. This is definite evidence that the transition is direct to ground. Figure 6, however, shows the presence of two gamma-rays. The peak near 2.3 Mev and the broad distribution from 1.5 Mev to 2.0 Mev are attributed to photoelectric and Compton effects associated with gamma-rays from the first excited

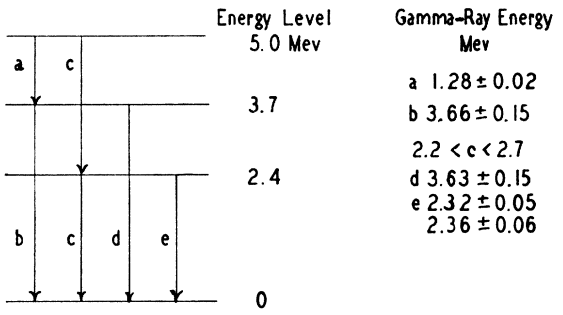

FIG. 8. Decay scheme of Si^{30}.

state. It is believed that this was due to inadequate separation of the proton groups. That is, some protons corresponding to the first excited state were counted when the total absorption in the proton path was 25.3 cm air equivalent. This distribution gives an energy of 2.36±0.06 Mev for the gamma-ray from the first excited state in agreement with the value cited above. The distribution above 2.5 Mev is much less distinct, but definitely indicates a higher energy gamma-ray. The bulge at 3.1 Mev is attributed to pair production with the escape of one of the annihilation quanta. The energy of this gamma-ray is 3.63±0.15 Mev.

For the third excited state the proton per coincidence value indicates a cascade process. In the distribution shown in Fig. 7, the peak near 1.3 Mev and the bulge at 1.0 Mev are attributed to the photoelectric and Compton effects of a 1.28±0.6 Mev gamma-ray. Above 2.7 Mev the curve is very similar to that obtained for the second excited state. Attributing the peak near 3.1 Mev again to pair production with the escape of one quantum, this high energy gamma-ray is 3.66±0.15 Mev. The distribution from 1.5–2.7 Mev is attributed to two gamma-rays with photoelectric peaks between 2.2 Mev and 2.7 Mev. The absence of pulses corresponding to an energy of 5 Mev indicates that there is no direct transition to the ground state. Thus the third excited state decays by a double cascade.

The decay scheme of the excited states of Si^{30} is shown in Fig. 8. The agreement with the work of Landon[2] is quite good, the exception being that Landon found positive evidence only for the cascade from the third excited state to the second to ground. The alternate mode of decay, however, is estimated from the present data to be not more than 15 percent of the total.

The authors feel that improvement of the spectrometer resolution and improvement of the stability of the electronic equipment over the long periods required for the accumulation of data, will facilitate the extension of this technique to more complicated decay schemes. The authors wish to express their appreciation to Professor E. C. Pollard for his guidance throughout this program.

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Elastic Scattering of Electrons*

HERMAN FESHBACH
Department of Physics and Laboratory for Nuclear Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts

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This paper considers the effect of the finite size of the nucleus on the electrostatic scattering of electrons whose energy greatly exceeds their rest mass. It is shown that the phase shift for a given total electron angular momentum j, is independent of their orbital angular momentum l; that the phase shift depends on the parameter R describing the extension of the nuclear charge only through the combination pR where p is the electron momentum. If the additional assumption $pR \ll 1$ is made, it may be shown that the phase shift η_0 for $j=\frac{1}{2}$ electron is independent of the model describing the distribution of nuclear charge. For models for which the potential is finite at the origin, η_0 depends upon the model only through the volume integral of the potential over the nucleus.

1. INTRODUCTION

THE effect of the finite size of the atomic nucleus on the scattering of high energy (~20 Mev) electrons has been recently demonstrated experimentally[1] and has been the subject of several theoretical discussions.[2–5] In all of these, it is assumed that the nucleus can be represented by a charge density, $\rho(r)$, where r is the distance from the center of the nucleus, an approximation which is valid only for electron wavelengths λ considerably larger than interparticle distances in the nucleus. Taking this to be about $\frac{1}{2}(e^2/mc^2)$, we find that these theories are valid for electron energies E considerably smaller than 274 mc^2. Under this assumption, the most accurate results have been obtained by numerical analysis; the most complete calculation being that of Acheson in which the scattering for a variety of elements ($13 \leq z \leq 79$) and energies (15 Mev $\leq E \leq 30$ Mev) has been considered. Rose and Parzen also consider the more general problem of formulating suitable methods for these calculations, deriving, among other results, variational principles for the phase shifts.

In the present paper some general properties of the phase shifts required to describe the scattering, will be derived. These will help to reduce the labor of the numerical calculations as well as provide a basis for interpreting the experimental results.

* Assisted by the joint program of the ONR and AEC.

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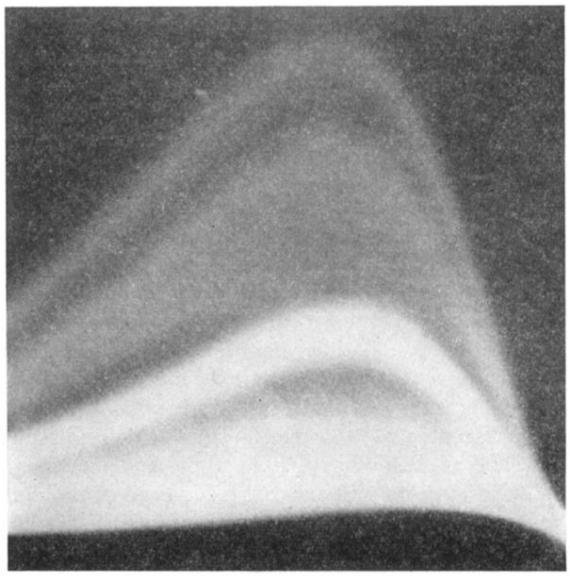

FIG. 4. Photographic distribution of Na^{22}.