classical[1] and semiclassical[2] methods would be only slightly altered by the strictly quantum mechanical treatment according to Eq. (3).

IV. FRANCK-CONDON PRINCIPLE

The direct evaluation of the matrix elements for transitions between individual vibrational levels of the initial and final states provides new insight into the Franck-Condon principle. For absorption at $T=0$, one expects the most probable transitions to occur to levels in the excited state with vibrational quantum numbers near $f=41$, for which the classical turning point is at the equilibrium configuration for the ground state. It is clear from Fig. 1 that the probability of transition to levels with f much less than 41 will be small because ψ_i will overlap the exponential tail of ψ_f. On the other hand for f much greater than 41 ψ_i overlaps the periodic portion of ψ_f whose amplitude varies slowly; consequently, the probability of transition given by Eq. (3) will again be small because of effective cancellation arising from the rapid oscillations of ψ_f. In other words, the principal contribution to Eq. (3) results from configurations for the vibrational levels of the final state where the kinetic energy is comparable with the kinetic energy of the vibrational levels of the initial state. This is in accord with the Franck-Condon principle which states that the most probable electronic transitions leave momentum as well as position unchanged.

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Nuclear Interactions of Cosmic Rays in a Silver Chloride Crystal*

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A disk of silver chloride, cut from a large crystal grown by slowly cooling the melt, was operated as an ionization detector at sea level and at Climax, Colorado, elevation 11,200 feet. Calibration was achieved by testing the response of the crystal to single cosmic-ray particles ionizing near minimum. By means of a Geiger counter coincidence system stars produced in the crystal by ionizing particles (protons) were separated from those produced by non-ionizing particles. A pulse-height distribution is plotted for the larger pulses and is in approximate agreement with star data from photographic emulsions for energy releases in the crystal of greater than 80 Mev. Electron showers and slow protons which stop in the crystal are shown to contribute to the rates below this energy. An apparent absorption thickness in air of 114 ± 5 g/cm^2 is obtained for the ionizing star-producing radiation between Climax and sea level. By assuming a geometrical cross section for interaction in the crystal, the intensity of energetic protons at Climax is estimated to be approximately 10 percent of the hard component.

I. INTRODUCTION

ARGUMENTS, such as those given by Rossi,[1] make it reasonable to assume that most nuclear disintegrations (stars) in the atmosphere are produced by the neutrons and protons in the cosmic radiation. The rate of star production has been shown to decrease approximately exponentially with depth in the lower atmosphere, and various observers have given absorption thicknesses for the star producing radiation from 125 to 150 g/cm^2.[2–7] A majority of low energy nuclear events (small stars) observed in photographic emulsions at moderate elevations is produced by non-ionizing radiation, that is, by neutrons. On the other hand, high energy nuclear events (larger stars containing thin prongs) vary somewhat more rapidly with atmospheric depth.[1,6,7] It is probably correct to identify these with penetrating showers. Such stars are more often produced by protons.[6] Direct information on the intensity and altitude variation of the nuclear interactions of protons is not very extensive. It should be noted that star production by pi-mesons may become important under rather thick layers of dense material.

In this paper are presented the results of a counting experiment designed to determine the rate of production of stars in a crystal of silver chloride by ionizing particles. A pulse-height distribution is given for the various events produced in the crystal. By making appropriate correlation assumptions, the distribution is compared with the star data from photographic emulsions.[6,8]

* Assisted by the joint program of the ONR and AEC.
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[1] B. Rossi, Revs. Modern Phys. **20**, 537 (1948).
[2] E. P. George, Nature **162**, 333 (1948).
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[7] See also the results of Bridge, Rossi, and Williams using thin-walled ionization chambers as reported in reference 1 and John Tinlot, Phys. Rev. **73**, 1476 (1948); Phys. Rev. **74**, 1197 (1948).
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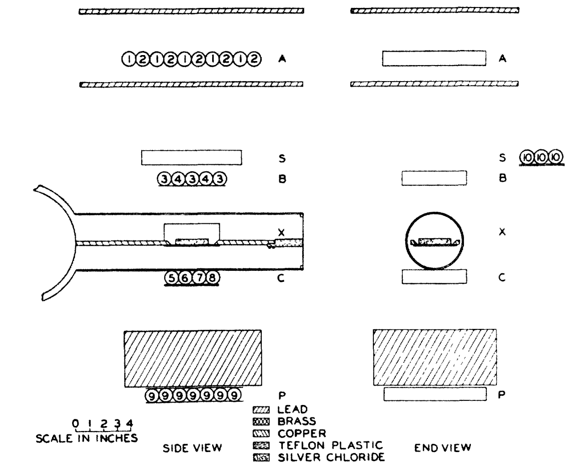

FIG. 1. Counter geometry showing side and end views of the crystal counter vacuum chamber.

From observations made at sea level and at an elevation of 11,200 feet (Climax, Colorado), an apparent absorption thickness in air is obtained for the high energy ionizing particles (protons) producing the interactions. Assuming a cross section for interaction in the crystal (157.5 g of silver chloride), the intensity of protons is estimated from the observed rate of star production.

In addition some information about the stars was obtained by observing penetrating particles below with a filter and auxiliary counter arrangement. A few delayed pulses believed to arise from μ-meson decay following a star were observed.

II. EXPERIMENTAL PROCEDURE

The silver chloride crystal has been previously used as a proportional device (a solid ionization chamber) by Voorhies and Street.[9] As described in this paper, the device can be calibrated by the passage of single minimum ionizing particles. It was also shown that this type of calibration can be used in deducing the energy loss in the crystal for more heavily ionizing particles.

Large single, or nearly single, crystals having the desired counting properties could not be obtained commercially. Consequently, crystals were grown by a technique similar to that first employed by Bridgman in 1925.[10] A crucible containing molten silver chloride was very slowly lowered through a temperature gradient into a region where the temperature was below the melting point. Various crucible materials were tried. Both Vycor glass coated with Aquadag, and platinum were found quite satisfactory. A disk 6 cm in diameter and 1 cm in thickness was cut from a boule grown in the platinum crucible. The crystal was heat treated and silver was developed out on the parallel surfaces, forming the electrodes.[11] For the experiment it was mounted in a vacuum chamber which contained a cooling system designed to minimize the amount of heavy material nearby.[12] Since the crystal counter had to be operated at low temperature for long periods of time, it was deemed necessary to have a large cooling reservoir. This was located away from the crystal and not in the solid angle subtended by the counter telescope. A 5-liter commercially available liquid air container was used for this purpose. The edges of the two copper spheres of this container are represented schematically at the left in Fig. 1. The crystal itself was situated near the center of the main vacuum chamber, a thin-walled ($\frac{1}{32}$-inch brass) tube 4 inches in diameter and 16 inches long. Cooling was accomplished by heat conduction through a copper shelf which fitted securely into a socket soldered to the inner sphere. In order to minimize heat leakage the other end of the shelf was supported by a Teflon rod which was held in place with a thin Nichrome support. Installation of the crystal was facilitated by having an opening through which the entire shelf could be inserted. In order to reduce radiation losses, the shelf was brightly polished, the interior of the vacuum chamber chrome plates, and a very thin radiation baffle put around the crystal and its electrical contacts. The entire space within the vacuum chamber, including that between the liquid air container spheres, was continuously pumped.

The counter arrangement was designed to detect nuclear disintegrations which were produced in the crystal by incident ionizing particles. The pulses from the Geiger counter trays A and B (see Fig. 1) and the crystal, X, were placed in triple coincidence $(A+B+X)$, forming a counter telescope which detected ionizing particles incident on the crystal. A discriminator on the crystal output imposed the additional condition that the pulses from the crystal be greater than a predetermined size. Since the energy released in the crystal is proportional to pulse size, the discriminator permits the selection of events wherein the incident ionizing particle is associated with a large energy release in the crystal. Thus protons producing stars are recorded, but also protons simply stopping in the lower part of the crystal give fairly large pulses and are hard to separate from small stars. Also, an electronic shower associated with one or more electrons above would also be recorded. To minimize the number of counts resulting from such showers, a 1-cm lead plate was placed above tray A and another above tray B, and the circuit so arranged that if any of the Geiger tubes marked 1 in tray A (see Fig. 1) were discharged in coincidence with any of the Geiger tubes marked 2, the count was rejected. The Geiger tubes marked 3 and 4 in tray B operated in a similar manner. The assumption is that electronic mul-

[9] H.G. Voorhies and J. C. Street, Phys. Rev. **76**, 1100 (1949). For additional references on the silver chloride crystal counter see: P. J. van Heerden, thesis Utrecht (1945) or Physik **16**, 505 (1950), and R. Hofstadter, Nucleonics **4**, 2 (1949) and **4**, 29 (1949).

[10] P. W. Bridgman, Proc. Am. Acad. Arts Sci. **60**, 305 (1925).

[11] J. R. Haynes, Rev. Sci. Instr. **19**, 51 (1948).

[12] Silver chloride is an ionic conductor at room temperatures but operates satisfactorily as an ionization detector when cooled approximately to the temperature of liquid nitrogen.

tiplicity in the crystal will usually be associated with electronic multiplicity in trays A or B or both because of the proximity of the lead plates. In addition, a Geiger tube tray, S (Fig. 1), was placed outside the solid angle of the telescope. The 4-fold coincidences $A+B+X+S$ were recorded and interpreted as electronic showers from the side.

Additional information about the event occurring in the crystal was obtained by recording the number of Geiger tubes in tray C, below the crystal, which discharged in coincidence with $(A+B+X)$. This data yielded qualitative information about the number of secondary particles produced in the crystal by the incident primary.

A 10-cm lead absorber was placed beneath tray C and under this, a tray P. The discharge of any Geiger tube in tray P in coincidence with $A+B+X$ was recorded. The events $A+B+X+P$ indicated that at least one penetrating particle was associated with the large energy release in the crystal.

A block diagram of the circuits is shown in Fig. 2. The Geiger counter pulses were shaped so that the resolving times of the various coincidence circuits were about three microseconds. The output pulses from the crystal were fed through a preamplifier and a linear amplifier to a fast oscilloscope whose trace was photographed. Examples of crystal pulses obtained under similar conditions have been shown by Voorhies and Street.[9] Geiger counter and coincidence rates as well as the crystal counting rate were carefully monitored throughout all of the runs. Approximately 600 hours of operation were recorded at Cambridge, Massachusetts, elevation 25 feet above sea level. Approximately 200 hours of operation were recorded at Climax, Colorado, elevation 11,200 feet above sea level. The roofs and walls of the laboratory buildings at both sites were constructed of light materials and their effect on the cosmic ray intensities was neglected.

III. EXPERIMENTAL RESULTS

A. Crystal Response to Single Particles Ionizing Near Minimum

The crystal calibration was checked at regular intervals throughout the experiment by adjusting the amplitude discriminator to pass small pulses resulting from single particles traversing the crystal. Integral counting-rate curves of the type in Fig. 3 were taken. A characteristic region, the knee of the curve, was located and compared with the theoretical predictions of energy loss taking into account the fluctuation problem for fast particles.[13,14] In this manner, the pulse height in microvolts could be correlated with the energy-loss in Mev. In a general way this procedure seems satisfactory, and no doubt provides fairly good relative calibrations of the crystal. However, a difficulty becomes apparent when a flux curve of the type shown in Fig. 3 is examined more carefully in the region beyond the knee toward greater energy pulses. In this region, there are more pulses than can result from fluctuations alone. For example, in Fig. 3, the knee is at about 170 microvolts. If we consider the whole spectrum of sea level mesons, taking fluctuation into account.[15] the most probable pulse size, is about 200 microvolts, in reasonable agreement with the experimental curve. However, not more than 5 percent of the pulses are expected to be greater than 400 microvolts (twice the most probable value). The curve indicates more than 20 percent are greater. In the experiment of Whittemore and Street[15] this trouble did not appear. The present experiment differs, in that protection from small electronic showers was not as good and in that the crystal was run at low saturation. The latter would emphasize the effect of those regions in the crystal which are especially free from trapping centers. Such regions would produce pulses considerably larger than average.

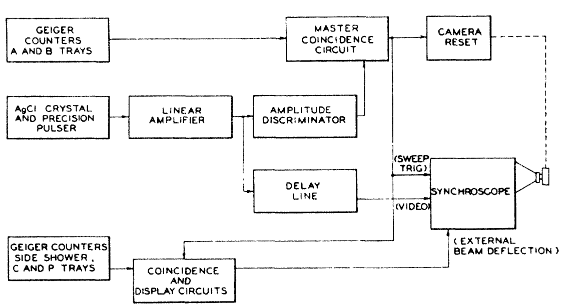

FIG. 2. A simplified block diagram of circuits. The precision pulser was used to calibrate and to check the stability of the linear amplifier and recording circuits.

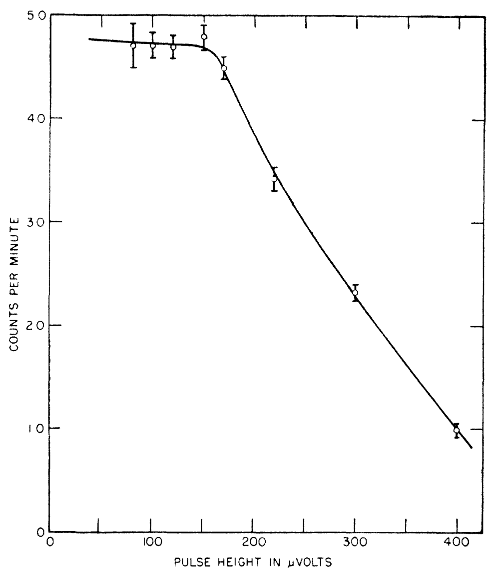

FIG. 3. A typical flux curve for cosmic rays at sea level, events ABX for low discriminator settings. Plotted are counts per minute against discriminator settings in microvolts.

[13] K. Symon, thesis, Harvard University (1948).

[14] L. Landau, J. Phys. (U.S.S.R.) **VIII**, No. 4, 201 (1944).

[15] W. L. Whittemore and J. C. Street, Phys. Rev. **76**, 1786 (1949).

TABLE I. Comparison of expected and observed values of flux. The expected rates are calculated from the values of vertical intensity given in reference 1, and the geometry of the apparatus. The observed rates are taken from flux curves of the type shown in Fig. 3.

	Flux at Cambridge F_{sl} (min^{-1})	Flux at Climax F_{cl} (min^{-1})	F_{cl}/F_{sl}
Calculated from the geometry and known intensities:			
Hard	4.85	9.95	2.05
Soft	1.46	9.65	6.60
Total	6.30	19.6	3.10
Observed experimental rates:	4.8±0.2	11.0±0.5	2.29

The expected flux of particles passing through the counter telescope formed by tray A, tray B, and the crystal can be calculated from the known vertical intensities and angular distributions of the hard and soft components of cosmic rays.[1] These flux rates both at sea level and at Climax are presented in the top rows of Table I. The experimental rates, obtained by extrapolating the integral flux curve to zero pulse height are also given (see Fig. 3). It should be noted that the observed altitude dependence agrees more closely with that calculated for the hard component. At sea level, the two 1-cm lead plates in the counter telescope should eliminate more than half of the soft component by absorption. The shower protecting anticoincidence circuit also strongly reduces the rate of this component. Taking these effects into account, the expected rate should be about 5 per minute. Since the experimental value of 4.8±0.2 agrees favorably with this estimated

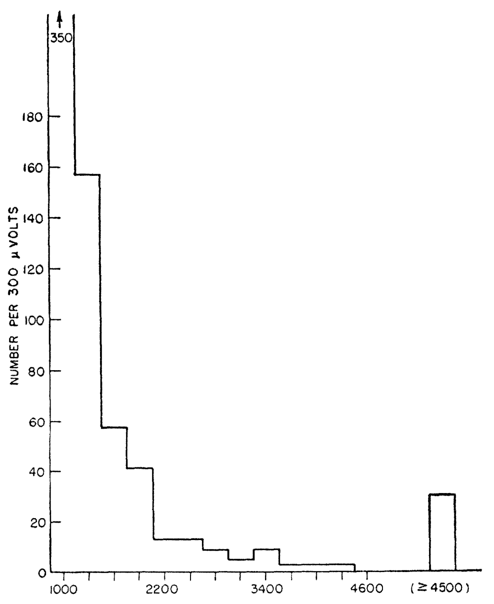

FIG. 4. A typical pulse-height group at Climax for all pulses above 850 microvolts, corresponding to about 43 Mev of energy loss in the crystal.

rate, it may be assumed that the crystal was detecting nearly all of the particles passing through it.[16]

B. Large Pulses and Their Interpretation

After calibration, the discriminator was raised to a level corresponding to roughly 50 Mev of released energy (seven times that lost by a single particle ionizing at minimum). All pulses $(A+B+X)$ for a given discriminator setting were photographed and measured. The lower limit of the discriminator setting was large enough to eliminate pulses arising from mesons passing through the crystal or stopping as a consequence of normal ionization. Similarly, most proton pulses of normal ionization were eliminated.

Over long periods of time the sensitivity of the crystal was found to vary. After about ten days of continuous operation at Climax, a calibration run would show a decrease of about ten percent in pulse height for a given energy loss. Such an effect was attributed to a polarization slowly induced in the crystal by the applied field and continuous exposure to cosmic rays.

TABLE II. Complete $(A+B+X)$ coincidence data combined and listed in integral form with pulse sizes expressed in Mev dissipated in the crystal.

	Climax			Sea level		
Pulse size in Mev dissipated	No. of counts of equal or greater size	Hours	Rate hr^{-1}	No. of counts of equal or greater size	Hours	Rate hr^{-1}
400	21	169.1	0.124	...	...	...
300	45	202.1	0.221	8	560	0.014
200	102	236.2	0.432	10	560	0.018
100	314	236.2	1.33	30	560	0.0536
80	504	236.2	2.13	53	560	0.0947
60	1016	236.2	4.30	138	560	0.246
40	652	67.2	9.71	403	560	0.720

The crystal was depolarized by warming to room temperature with the collecting field off and then recooling. As would be expected, the polarization was observed to set in more slowly at sea level than at Climax.

The data were separated into groups of the type shown in Fig. 4 where the calibration of the crystal was constant to within ten percent for each group but was different from group to group. Using the appropriate calibration figure for each group, the scale of pulse height in microvolts was then replaced with one of energy loss in Mev and the groups combined. The complete data for sea level and for Climax are tabulated in integral form in Table II. The Climax data are plotted on a smoothed differential basis in Fig. 5. Curve A gives the number of counts in the crystal per 20 Mev per hour for energy release greater than 40 Mev. Curve B is the tail of the curve on an expanded scale. The statistical precision of these plots can be readily deduced

[16] The actual fraction of the soft component detected by the apparatus can be estimated from the increase in rates with elevation. A simple calculation from the data of Table I gives $\frac{1}{6}$ for this fraction.

from the number of counts listed in Table II and to avoid confusion is not shown in Fig. 5. The average rate between 100 and 200 Mev is uncertain to within about 7 percent. We shall show later that only the data above 80 Mev may be interpreted as due solely to star production in the crystal.

The counter telescope defines an angle from the vertical of roughly 20°. From the angular distribution of fast star-producing particles, as given by the Bristol group,[8] we estimate that our telescope selects a fraction 0.48 of these particles. Our crystal weighs 157.5 g. Therefore, to convert the rates above 80 Mev (Table II) to the number per gram per hour produced by particles incident at all angles, it is necessary to apply a factor of $1/(0.48\times157.5)=0.013$ to the tabulated values.

We have tried to make a comparison of our observations with the very different sort of data taken from photographic emulsions.[8] The intensities for the Bristol

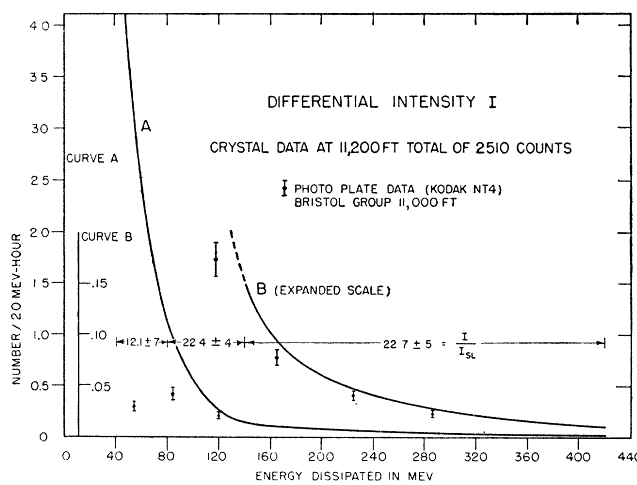

FIG. 5. Crystal data at Climax for $(A+B+X)$ coincidence events plotted on a differential basis. The ratios I/I_{sl} are the factors by which the rates in the indicated regions have to be reduced to fit the sea level data.

"p-stars," have been converted to counts per hour in the crystal, taking account of the solid angle subtended by the counter telescope and the angular distribution of star-producing particles. In addition, by using Table IV in reference 8 and range-energy curves for silver chloride, a correction was made for the average energy carried out of the crystal by escaping particles. The resulting conversion from number of prongs to Mev dissipated in the crystal is given in Table III. The final transformed intensities are plotted with statistical errors in Fig. 5. The two points below 100 Mev are plotted in the scale of curve A, the other four on the expanded scale of curve B. Above 80 Mev the agreement is fair. For smaller pulses, the crystal rate is considerably larger. Even for the larger pulses the emulsion points are a little low but this is not significant, considering the roughness of the conversion procedure. For example, the composition of the emulsion is different

TABLE III. Calculated conversion relations—emulsion to crystal data.

Size of "p-star" in number of heavy prongs	2	4	6	8	10	12	14
Estimated average energy dissipated in crystal in Mev	25	55	80	125	175	225	290

from that of the crystal. In addition it seems likely that some events produced in the chamber walls, or other adjacent portions of the apparatus were detected. Although these two effects can readily explain the slight difference above 80 Mev, they cannot explain the large difference between the emulsion and the crystal data for small pulses. It should be stressed that in making this comparison we have not adjusted any arbitrary factor, but have transformed the emulsion data on an absolute basis. The fact that the absolute intensities do agree reasonably well and the fact that the shape of the crystal and emulsion curves are the same above 80 Mev provide evidence for the assumption that in this energy range our arrangement is detecting mainly stars of the type observed in photographic emulsions.

Occasionally a star produced in the crystal will not be counted because a particle is projected upward into the anticoincidence arrangement of trays A and B (see Sec. II). However, such events should be rare because a small solid angle is subtended by alternate tubes in these trays and because the particles are absorbed by the material between these counters and the crystal. Air shower particles accompanying a nuclear particle which produced a star in the crystal might also trip this anticoincidence circuit. That both these occurrences are infrequent is born out by the relatively small frequency (1.3 percent) with which a count in the side shower tray, S, was registered in coincidence with a star in the crystal.

A comparison of the rates at sea level and at Climax is made in Table IV. The errors stated are the standard deviations calculated from the number of counts involved. It is estimated that the effect of the uncertainty in the calibration of the crystal for the various runs is considerably less than the statistical variation due to the small number of counts. Above 80 Mev, the increase with elevation is consistent with the assumption that mainly the effects of the nuclear component of the cosmic radiation were detected. Below 80 Mev, the

TABLE IV. Counting rates at sea level and at elevation 11,200 feet.

Range of energy loss in Mev	40–60	60–80	80–140	140 and greater
(I_{sl}) counts hour^{-1} at sea level	0.474	0.152	0.062	0.032
(I)counts hour^{-1} at elevation 11,200 feet	5.41	2.17	1.38	0.75
I/I_{sl}	11.5±0.9	14.3±1.7	22.4±4.0	22.7±5.0

smaller increase with elevation can be interpreted as the result of a partial contribution from the electronic component. For all counts above 80 Mev, the increase in rate with altitude is $I/I_{sl}=22.5\pm3.0$. Assuming exponential absorption of the star producing radiation in the 355 g/cm² of air between Climax and Cambridge the absorption thickness corresponding to the above factor is:

$$L_{\text{air}}=114\pm5\ \text{g/cm}^2.$$

C. Auxiliary Data, Including Observations with the *C* and *P* Counter Systems

At least one tube in the *C* tray beneath the crystal was triggered for 50 percent of the counts above 40 Mev. Two or more counters were triggered for 10 percent of the counts, indicating that in these cases at least two particles were able to get out of the crystal and penetrate the $\frac{3}{32}$-inch brass walls. When the shower protecting anticoincidence circuit associated with trays *A* and *B* was disconnected the counting rate data taken at Climax for energy releases of 40 Mev or greater increased about four times. Nearly every count which normally would have triggered this anticoincidence circuit showed a multiple count in tray *C*. Double counts in the *C* tray, then, were a fairly sensitive indication of electron showers as well as of multiple particles from stars.

At Climax a particle was detected in the *P* tray below the 10 cm of lead under the crystal for about 10 percent of the counts with 50 to 180 Mev energy-loss. For pulses larger than 180 Mev the *P* tray was triggered 27 percent of the time. One could argue that such *P* counts result from a μ-meson which has produced a large shower in the crystal or in the lead above, through knock-on or other processes. However, the expected rate of such events can be estimated and shown to be quite small. (See Appendix.) In addition, some *P* counts were registered at sea level at a rate (26 counts in 406 hours) very roughly 1/14 that at Climax. A short run was made with circuits which registered how many counters in the *P* tray fired. Among 200 events above 40 Mev only 21 showed a count below the 10 cm of lead. In only one of these 21 cases did more than one counter in the *P*-tray fire. It appears that showers under the lead are much less frequent than single particles. A simple interpretation of these *P* events is that a high energy proton produces an interaction in the crystal and then continues on with sufficient energy to penetrate the 113 g/cm² of lead (an event similar to that shown in Plate VI, reference 8).

The effect of additional lead absorbers placed above the crystal chamber was investigated qualitatively at Climax. A lead absorber 1.3 cm thick placed directly over the chamber produced less than ten percent change in the rate. For a short run of 24 hours at Climax, 12.7 cm of lead was inserted in the counter telescope between trays *A* and *B*. Forty-two counts from 50 to 90 Mev were recorded at a rate 25 percent of that without the lead. Only 16 counts above 90 Mev were recorded, indicating a rate roughly 40 percent of the rate without the lead. This leads to a lead absorption thickness of 170 ± 60 g/cm². The stronger absorption of the counts 50 to 90 Mev is in agreement with the assumption that in this range the rates are partly due to electron showers.

It should be mentioned that a few oscilloscope traces were recorded in which a second crystal pulse followed the main crystal pulse, delayed by a few microseconds. These amplified and shaped pulses were three microseconds long so we could readily resolve delays greater than four microseconds. There were three such cases occurring in 1300 counts at Climax. For these same 1300 counts at least ten events were recorded superimposed on the crystal pulse itself, with delays of from one to four microseconds. The pulse height and delay of these few counts were all consistent with a μ^+-meson β^+-decay process with the parent π^+-meson originating in connection with a star produced in the crystal.

D. Interpretation of the Smaller Pulses

On the assumption that, for the smaller pulses, our apparatus detects the same fraction of incident electrons at Climax as at sea level, we can calculate the rate, S, resulting from the soft component and the rate, N, resulting from the nuclear component. The electron rate is taken to increase between sea level and Climax by a factor of 6.6, the nuclear component by 22. Then, from the data in Table IV, we find for pulses of 40 to 60 Mev at Climax, $S+N=5.41$ per hour per 20 Mev interval, and at sea level, $S/6.6+N/22=0.474$ per hour. Solving these two equations we get $N=3.3$ per hour. A similar computation for pulses of 60 to 80 Mev at Climax and at sea level yield $N=1.7$. This is still well above the rate (0.4 per hour) given by the emulsion data for stars in this energy range (see Fig. 5). A part of the discrepancy can be accounted for in terms of single protons of moderate energies incident from above and stopping in or just below the crystal. Using range-energy curves and taking account of angular effects, a calculation shows that protons incident on the top lead plate in the momentum range $5\text{–}6\times10^8$ ev/c contribute pulses of 40 to 60 Mev in the crystal. Using the data of Miller *et al.*[17] a calculation applied to the present experiment indicates a rate of 1.3 per hour for such events. After subtracting this proton contribution from the above determined rate of 3.3 per hour for N, the remainder is still high in comparison with emulsion data (2.0 per hour compared with 0.3 per hour). It is not clear, without further investigation, whether the discrepancy arises from errors in estimating the electron and single-proton contributions, or whether other events, which contribute in this region of pulse size, have been overlooked.

[17] Miller, Henderson, Potter, Todd, and Wotring, Phys. Rev. **79**, 459 (1950).

E. Estimate of Proton Intensity

Assuming a cross section for the production of nuclear interactions in the crystal, the vertical intensity of the star-producing protons can be estimated. It is necessary to use the angle distribution of incident protons given by Brown *et al.* (reference 8, Fig. 4, page 867). From this we deduce that the vertical intensity is 1.16 times the average intensity for rays in the 0-to-20 degree angle covered by our counter telescope. The solid angle subtended is 0.47 sterad. The number of events recorded in the crystal per hour, a quantity which we call R, would be given by

$$R=(0.47\times 3600/1.16)A\sigma TI_v.$$

Here I_v is the vertical intensity ($\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}\ \mathrm{sterad}^{-1}$), σ is the cross section for collision (of the type observed) measured in cm^2/g, T is the thickness of the crystal in $\mathrm{g/cm}^2$, and A is the area of the crystal (28.5 cm^2). To obtain σT, we calculated it separately for the silver and the chloride in the crystal, using geometrical cross sections, and added them. At Climax the value of R from our data for all pulses 100 Mev or greater is 1.3 per hour. In this range there is a reasonable agreement with emulsion data. For smaller pulses we may either extrapolate down to 40-Mev dissipation along a curve indicated from emulsion data (see Fig. 5), obtaining $R_1(\mathrm{total})=2.7$ per hour, or use our rates corrected as described in Sec. III-D above obtaining $R_2(\mathrm{total})=5.3$ per hour. For lack of a better criterion we arbitrarily choose the average of these, $R=4$ per hour. Solving for I_v, we obtain $1.6\times 10^{-3}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}\ \mathrm{sterad}^{-1}$. This represents 10 percent of the hard component at 11,200 feet elevation (atmospheric depth 675 $\mathrm{g/cm}^2$). It is to be pointed out that the result is uncertain in the choice of R, as indicated above, and as to how well σT can be represented by geometrical cross sections.

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APPENDIX

We seek an upper limit to the probability that a meson of an energy belonging to the observed sea level cosmic-ray spectrum will produce an electron cascade large enough to be recorded as a P event. To be specific, we estimate the chance of production of an electron cascade of more than 7 ionizing particles formed in 2 cm of Pb. From Heitler,[18] we deduce that such showers require initiating electrons or photons of energies greater than 500 Mev or so. We assume as a rough approximation that such electrons or photons are produced by mesons in the first centimeter of lead, since the second centimeter is needed for effective shower growth. The processes involved are as follows:

(a) Electrons emerging as knock-ons from the direct collision of incident mesons.

(b) Photons appearing as the bremsstrahlung of the incident mesons in the fields of heavy lead nuclei.

(c) Direct pair production in the fields of lead nuclei. We consider these processes in succession.

In each case we assume that the incident mesons have momenta belonging to the spectrum[19]

$$S(P)=AP^{-3.3}.$$

The specific exponent chosen is not critical, in view of the other more serious approximations made.

(a) Electronic knock-ons: The basic cross section used is that given by Janossy[20] for collisions of spin-$\frac{1}{2}$ mesons with electrons. To produce a 500-Mev electron by collision, momentum and energy conservation require an incident meson of at least 2000 Mev. To simplify calculation, however, it was assumed that any amount of momentum could be transferred, up to the incident momentum p. The collision cross section was thus integrated over electron momenta from 500 Mev/c to p. The result was weighted by the meson spectrum $S(p)$, above, and integrated over meson momenta from 2000 Mev/c to infinity. This procedure reduces the calculation to a series of elementary integrations, and will give an upper bound to the frequency of such knock-on processes. By this method one obtains $f_k=1.2\times 10^{-4}$ as the probability that a cosmic-ray meson will produce in 1 cm of lead a knock-on of greater than 500-Mev energy.

(b) Bremsstrahlung: We use the formulas of Heitler,[21] adapted to incident particles with 200-electron masses. The extreme relativistic approximation was applied to find the probability of incident cosmic-ray mesons forming photons of energies greater than 500 Mev in one cm of lead. This probability is calculated in the same manner as that for knock-on electrons, and is found to be $f_k=4.3\times 10^{-4}$, for an unscreened nucleus, and $=3.3\times 10^{-5}$, assuming complete Thomas-Fermi screening. The correct value is probably closer to the second of these.

(c) Direct pair production: In this case we use the formulas of Hayakawa and Tomonaga.[22] For ready calculation from their equations it is convenient to carry the integration over meson momenta only up to 6×10^4 Mev/c. For a spectrum $S(p)\alpha p^{-3}$, only one meson in 10^4 has a momentum above this. The probability that a pair of 500-Mev total energy is formed is then $f_p=6.4\times 10^{-7}$ when $S(p)=Ap^{-3.3}$.

This is smaller than the probability for bremsstrahlung near an unscreened nucleus by about $\alpha=1/137$, as is to be expected, since direct pair production is a higher order process.

In conclusion, we note that an upper limit to the probability of production of such large cascades by cosmic-ray mesons is about 5×10^{-4}, even assuming unscreened nuclei in the bremsstrahlung process. Actually the best estimate for the probability is about 2×10^{-4}. Thus at sea level, where about 300 mesons per hour traverse the equipment, some 6×10^{-2} seven-particle showers would be expected in a similar time. As noted in Sec. III-C the anticoincidence arrangement eliminates at least three-quarters of these showers and some of them would spread by scattering enough for several of the particles to miss the crystal. Therefore, the expected rate of such events at sea level is about 10^{-2} per hour and at Climax approximately twice this. That these rates are indeed negligible is shown by Table II. They are roughly only 1 percent of the rate at sea level and 0.2 percent at Climax, for dissipated energies greater than 40 Mev.

[18] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, pp. 237–239.

[19] L. Janossy, *Cosmic Rays* (Oxford-Clarendon Press, England, 1948), p. 171.

[20] See reference 19, p. 130.

[21] See reference 18, pp. 168–170.

[22] S. Hayakawa and S. Tomonaga, Prog. Theor. Phys. 4, 287 (1949).