These curves are plotted in Figs. 1 to 4 and numerical values are listed in Tables II and III.

The curves have the peaked shape characteristic of photonuclear reactions. In three of the four cases such cross-section turn-overs cannot be explained as resulting from a cascade competition with other photonuclear reactions but must be due to a peaking of the photonuclear absorption cross section.

The (γ, p) reactions peak at slightly higher photon energies than the (γ, n) reactions and their cross sections near the peak are considerably greater. These large cross sections are remarkably independent of wide variations in photonuclear thresholds and of the characteristics of the residual nuclei. These facts can be explained in terms of a primary interaction of high energy photons with nuclear protons.

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Radiation Reaction in Relativistic Motion of a Particle in a Wave Field

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An approximate solution of the equations of motion of Dirac's classical theory f pointlike particles is obtained for a particle in the field of a plane wave, under the assumption that the radiation reaction terms in these equations can be considered as small. The appearance of runaway terms in this solution is avoided by letting the interaction set in gradually. Considerable simplification is achieved by restriction to the domain of high relativistic energies where the transfer of energy and momentum from the wave to the particle appears to be mainly due to radiation reaction. A quantitative discussion of the conditions of applicability of the formulas obtained is made possible by the assumption that there is correspondence between a photon and a classical wave train of finite length. This assumption leads to the conclusion that the classical formulas can be valid for arbitrarily high energies. An estimate of a lower limit for the duration of the interaction between particle and wave train yields an expression which resembles formulas for lifetimes of unstable particles both in its dependence upon fundamental constants and in its increase with the energy involved in the process.

I. INTRODUCTION

THE transfer of linear momentum from a wave to a particle is usually considered as a typical quantum effect, particularly in the E.R.[1] domain. It is the primary purpose of this paper to show that in classical theory such momentum transfers can be accounted for as radiation reaction effects, and that correspondence can be established between relevant results of classical and quantum theory. The equations of motion of Dirac's classical theory of charged, pointlike particles in an electromagnetic[2] field are used as starting point and transformed in a way which simplifies the treatment of the motion of a particle under the influence of a plane wave (Sec. II). A solution of the transformed equations is worked out explicitly in a first approximation, under the assumption that the radiation reaction terms can be considered as small. Runaway terms in this solution are made to disappear by the device of letting the interaction set in gradually, but no attempt is made to prove the consistency of this procedure (Sec. III). Restriction to the E.R. domain leads to simple formulas which are not likely to depend upon any particular assumptions (Sec. IV).

A quantitative discussion of the conditions of applicability of the first approximation is made possible by the assumption that the particle absorbs the energy of a photon while interacting with a wave train of finite length. This leads to the conclusion that the first approximation is likely to be valid for arbitrarily high energies. At first sight such a conclusion seems surprising, since in the N.R. domain the relative order of magnitude of the radiation reaction terms is given by the expression $\frac{2}{3}e^2\omega/mc^3$ (ω angular frequency) which becomes larger than unity for photon energies $\hbar\omega > 205mc^2$. But radiation reaction makes the particle recede in the direction of incidence of the wave. This effect, though small, is cumulative, and can account for large momentum transfers in the E.R. domain. It leads to such a reduction of the frequency of the wave relative to the particle that, on the average, the ratio of the radiation reaction terms to the main terms in the equations of motion does not exceed the order of magnitude of the fine structure constant, $e^2/\hbar c \cong 1/137$ (Sec. V). Analogous results have been previously obtained in the quantum theory of radiation damping.[3]

The same assumptions lead to an estimate of a lower limit for the duration of the interaction between particle and wave train. The expression obtained for this limit

[1] E.R. for extreme relativistic, N.R. for nonrelativistic.

[2] P. A. M. Dirac, Proc. Roy. Soc. (London) **A167**, 148 (1938).

[3] For scattering of photons by charged, spinless particles, the case corresponding to the proposed semiclassical treatment, see E. Gora, Z. Physik **120**, 121 (1943). The present paper originated from an attempt to find a classical analog to the results obtained there.

increases with the energy of the incident photon in a way analogous to the relativistic increase of the lifetime of unstable particles, and resembles the formulas for meson lifetimes in its dependence upon fundamental constants. This result suggests that one may have to distinguish between interaction times and lifetimes in interpreting meson processes (Sec. VI).

II. TRANSFORMATION OF THE EQUATIONS OF MOTION

Dirac[2] has shown that the equations

$$\dot{v}_\mu - b\ddot{v}_\mu - b\dot{\mathbf{v}}^2 v_\mu = (e/m)v_\nu F_\mu{}^\nu{}_{\text{in}} \tag{1}$$

represent the correct equations of motion for a charged, pointlike particle in an electromagnetic field. v_μ is the velocity four-vector (v_0, v_1, v_2, v_3), $\dot{\mathbf{v}}^2 = \dot{v}_0^2 - \dot{v}_1^2 - \dot{v}_2^2 - \dot{v}_3^2$, and $b = \frac{2}{3}e^2/mc^3$ is a constant characteristic for the radiation reaction terms. Dots denote differentiation with respect to the proper time of the particle in Eq. (1), but from now on they will be used to denote differentiation with respect to normal time. $F_\mu{}^\nu{}_{\text{in}}$ is the field tensor of the incident field. The units used will in general be such that the velocity of light $c=1$.

In treating the motion of a particle under the influence of a plane wave we use the coordinate system $x=x_1$, $y=x_2$, $z=x_3$, $t=x_0$, and choose the z axis as direction of incidence of the wave which we represent in the usual way by $E_x = B_y$, $E_y = -B_x$, $E_z = B_z = 0$. In conformity with this notation we write

$$\eta = (1-v^2)^{-\frac{1}{2}} = v_0, \quad \pi_x = \eta\dot{x} = v_1, \quad \text{etc.}; \tag{2}$$

$m(\eta, \boldsymbol{\pi})$ is the energy-momentum four-vector of the particle.

Considerable simplification is achieved by introducing the new variable $s=(t-z)$ together with the auxiliary function

$$f = \eta - \pi_z = \eta(1-\dot{z}) = \eta\dot{s}. \tag{3}$$

From these definitions one can easily derive the relations

$$d_t = (1-\dot{z})d_s, \quad \eta d_t = f d_s, \quad dt/ds = \eta/f \tag{4}$$

$(d_t = d/dt,\ d_s = d/ds)$, which will be used repeatedly.

With these definitions subtraction of the last two equations of (1) leads to

$$d_s(f - bff') + bR = 0, \tag{5a}$$

while the first two assume the form

$$d_s(\pi_x - bf\pi_x') + b\pi_x R/f = (e/m)E_x, \tag{5b}$$

$$d_s(\pi_y - bf\pi_y') + b\pi_y R/f = (e/m)E_y. \tag{5c}$$

The accents denote differentiation with respect to s, and

$$R = -\dot{\mathbf{v}}^2 = \eta^2(\dot{\boldsymbol{\pi}}^2 - \dot{\eta}^2) = f^2(\boldsymbol{\pi}'^2 - \eta'^2). \tag{6}$$

Equations (5) contain no other unknown functions but f, π_x, π_y, and their derivatives. The functions π_z and η are linked up with f, π_x, π_y by Eq. (3) and by the four-vector relation $\eta^2 = 1 + \boldsymbol{\pi}^2$. Combining these two relations, we obtain

$$\pi_z = (1 - f^2 + \pi_x^2 + \pi_y^2)/2f, \tag{7a}$$

$$\eta = (1 + f^2 + \pi_x^2 + \pi_y^2)/2f. \tag{7b}$$

Using these expressions in Eq. (6), we find

$$R = f^2(\pi_x'^2 + \pi_y'^2) - 2ff'(\pi_x\pi_x' + \pi_y\pi_y') + f'^2(1 + \pi_x^2 + \pi_y^2). \tag{7c}$$

The system of differential Eqs. (5), supplemented by Eq. (7c), can be solved by the method of successive approximations with b as expansion parameter, provided that this procedure is convergent.

III. SOLUTION OF THE TRANSFORMED EQUATIONS

For the sake of simplicity we assume that the incident wave is circularly polarized, for instance,

$$(e/m)E_x = F\sin\omega s, \quad (e/m)E_y = F\cos\omega s. \tag{8}$$

The appearance of multiples of ω in the radiation reaction terms is thus avoided.

In general, the influence of radiation reaction is likely to be small, and we might expect to obtain a satisfactory approximation by retaining in Eqs. (5) only linear terms in b:

$$f' + bf^2F^2 = 0, \tag{9a}$$

$$\pi_x' - b(\pi_x'' - \pi_x F^2)f = F\sin\omega s, \tag{9b}$$

$$\pi_y' - b(\pi_y'' - \pi_y F^2)f = F\cos\omega s. \tag{9c}$$

The expression $R = f^2F^2$, which has been used here, follows from Eqs. (7c) and (5b, c) with Eqs. (8) if radiation reaction is neglected. The term $bd_s(ff')$ in Eq. (5a) turns out to be of third order in b and is of no influence in this approximation.

With initial conditions of the usual type, for instance, $\boldsymbol{\pi} = \boldsymbol{\pi}' = 0$ for $s=0$ (particle initially at rest), a solution of Eqs. (9) is fully determined; but it contains rapidly increasing, and physically meaningless, radiation reaction terms. It is well known that Dirac's Eqs. (1) lead to such runaway solutions.[4] Probably this difficulty can be avoided in theories where the particle is no longer considered as pointlike, but such theories are necessarily complicated and have not yet been tested.[5]

The appearance of the runaway terms is connected with the use of initial conditions according to which the interaction sets in with full strength at the space-time point of the particle. These terms disappear, at least to any desired order in b, if a suitable starting function is introduced in Eqs. (8) to let the interaction set in gradually. It does not seem likely that such a procedure could be made to fit consistently into the framework of Dirac's classical theory of pointlike particles. Our main

[4] A comprehensive survey of the problems involved has been given by C. J. Eliezer, Revs. Modern Phys. **19**, 147 (1947).

[5] See, for instance, R. E. Peierls and H. McManus, Phys. Rev. **70**, 795 (1946); H. McManus, Proc. Roy. Soc. (London) **A195**, 323 (1949).

reason for using it is that it enables us to derive results which look interesting and new. We do not go beyond the formalism of customary classical theory in using this procedure, but we hope that it might represent a first tentative step towards a formalism of a theory of extended particles and that the results obtained might be confirmed by such a theory.

A function of the type required is $[1-\exp(-n\alpha s)]^n$; it vanishes together with its $(n-1)$st derivatives for $s=0$, and approaches unity for $\alpha s \gtrsim 1$. We multiply the expressions (8) by this function and derive equations corresponding to the linear approximation (9) in b, where now $R=f^2F^2[1-\exp(-n\alpha s)]^{2n}$. The solution of these equations is comparatively simple if it is permissible to omit not only terms of higher than first order in b, but also in $(\alpha n/\omega)$. It does not contain any runaway terms if $n \geqslant 2$. For $\alpha s \gg 1$ this solution reduces to

$$f=A/(1+Bs), \tag{10a}$$

$$\pi_x=(F/\omega)[-\cos\omega s+b\omega f\sin\omega s(1+F^2/\omega^2)], \tag{10b}$$

$$\pi_y=(F/\omega)[\sin\omega s+b\omega f\cos\omega s(1+F^2/\omega^2)], \tag{10c}$$

where

$$A=1/(1-K_ne^2F^2/m\alpha), \quad B=bAF^2. \tag{11}$$

Here we have $K_1=1$, $K_2=25/36$, $K_3=49/90$, $K_4=761/1680$, etc. Using Eqs. (10) in Eq. (7), we obtain

$$\pi_z=(1-f^2+F^2/\omega^2)/2f, \tag{12a}$$

$$\eta=(1+f^2+F^2/\omega^2)/2f, \tag{12b}$$

$$R=f^2F^2. \tag{12c}$$

In these expressions all the linear terms in b cancel out.

Apart from the factor A in f which should not affect the order of magnitude, Eqs. (10) represent just the forced solution of Eqs. (9). Thus, the use of a starting function makes it possible to leave aside the free solution of Eqs. (9) which contains the runaway terms.

We have still to consider whether the procedure is consistent apart from the more fundamental difficulties involved. It seems reasonable to expect that the influence of the starting function should not be considerable, or that $A\sim 1$. According to Eq. (11), this will be the case if $K_ne^2F^2/m<\alpha$. Further, since we have omitted terms of order $(\alpha n/\omega)^2$ in Eq. (10), $(\alpha n/\omega)$ should be small. We obtain thus lower and upper limits for α:

$$K_ne^2F^2/m<\alpha \ll \omega/n. \tag{13}$$

Such a condition can obviously only be fulfilled if $e^2F^2/m \ll \omega$. A similar condition (see Eq. (21) below) is obtained for the validity of the linear approximation in b.

IV. ENERGY AND MOMENTUM RELATIONS

To interpret our formulas, we need a few energy and momentum relations. With the help of Eqs. (2), (3), and (10) we calculate the energy and the momentum which the particle absorbs:

$$\begin{aligned} E_{\text{abs}}=P_{\text{abs}}&=m\int^{t_1}(\dot{x}F\sin\omega s+\dot{y}F\cos\omega s)dt \\ &=mbF^2(1+F^2/\omega^2)s_1; \end{aligned} \tag{14}$$

here t_1 is the value of t which corresponds to the length s_1 of the incident wave train. To determine the energy and the momentum which the particle emits, we use Eqs. (10a), (12a, c), and (14):

$$E_{\text{em}}=m\int^{t_1}bRdt=\tfrac{1}{2}(E_{\text{abs}}+mf_1Bs_1), \tag{15a}$$

$$P_{\text{em}}=m\int^{t_1}bR\dot{z}dt=\tfrac{1}{2}(E_{\text{abs}}-mf_1Bs_1). \tag{15b}$$

The energy and the momentum which the particle retains should be given by the difference between the absorbed and the reemitted energies and momenta. One can easily verify that

$$m(\eta_1-\eta_0)=E_{\text{abs}}-E_{\text{em}}, \tag{16a}$$

$$m(\pi_{z1}-\pi_{z0})=P_{\text{abs}}-P_{\text{em}}; \tag{16b}$$

but, since the formulas used here are valid only for $s\gg 1/\alpha$, η_0 and π_{z0} represent the expressions obtained from Eqs. (12a, b) with $s=0$, and not the correct values for a particle initially at rest. This inconsistency is of little importance in the E.R. domain, where a satisfactory approximation is obtained by substituting 1, 0 for η_0, π_{z0} in our formulas; but, in general, such a simplification is not permissible.

To verify this statement, we consider first that our formulas describe a particle moving both in the direction of incidence of the wave and around this direction. In the N.R. case, where $\eta\cong 1$, $F\ll\omega$, $A\cong 1$, and $f\cong 1$, the kinetic energy of the particle, $m(\eta-1)\cong\frac{1}{2}m(\pi_x^2+\pi_y^2)\cong\frac{1}{2}mF^2/\omega^2$, is mainly due to an approximately circular motion in the (x, y) plane under the direct influence of the incident wave and Eqs. (12a, b) reduce to $\pi_z=\frac{1}{2}(F^2/\omega^2)+bF^2s$, $\eta=1+\frac{1}{2}(F^2/\omega^2)$. The final value of the radiation reaction term in π_z is not likely to exceed the order of magnitude of the direct interaction term, F^2/ω^2, since it seems plausible to consider the classical coherence length $1/b\omega^2$ as an upper limit for s_1 and with this expression $bF^2s_1=F^2/\omega^2$. A term of this order is of negligible influence upon the kinetic energy. Thus, we have $(\eta_0-1)<(\eta_1-1)$, and $\pi_{z0}\sim\pi_{z1}$. From these relations, it is evident that formulas like Eq. (16) do not apply to the N.R. case.

In the E.R. domain, we find $\eta_1\gg 1$; and, according to Eqs. (3), (10), and (12), provided that $F\lesssim\omega$ (see Sec. VI), we have $f_1\cong 1/bF^2s_1\cong\frac{1}{2}/2\eta_1$, and $\eta_1\cong\pi_{z1}\cong\frac{1}{2}bF^2s_1$. These relations show that in the E.R. domain the influence of radiation reaction predominates. From Eqs. (12a, b) it follows further that $\pi_{z0}\sim 1\ll\pi_{z1}$, $(\eta_0-1)\sim 1$

$\ll \eta_1$. Obviously, it is permissible to substitute 1, 0 for η_0, π_{z0} in the approximation where terms of relative order $1/\eta_1$ are omitted. The contribution of the particle's motion in the (x, y) plane to its total energy, $\sim(m/2\eta_1)(F^2/\omega^2)$, is also negligible in this case. The approximate validity of our formulas in the E.R. domain is thus demonstrated. To derive formulas valid for all energies, one would have to take into account explicitly both the starting process and a final stage where the particle reemits the energy of its motion in the (x, y) plane. Since this would lead to considerable complications, we prefer to restrict our considerations to the E.R. case.

V. COMPARISON WITH QUANTUM THEORY

In comparing our classical formulas with quantum theory, we have to use the corresponding formulas for scattering of photons by spinless particles which can be derived from expressions given by Pauli.[6] In the E.R. case, where the ratio of the photon energy to the rest energy of the particle $\gamma = \hbar\omega/mc^2 \gg 1$, the total scattering cross section is

$$\Phi \cong \pi r_0^2/\gamma \quad (r_0 = e^2/mc^2). \tag{17}$$

Terms of relative order of magnitude $1/\gamma$ have been omitted. The average energy of the secondary photons and their average momentum in the direction of incidence of the primary photon follow from

$$(E_{\mathrm{em}})_{\mathrm{Av}} = m\left(\int \gamma' d\Phi\right)\Big/\Phi,$$

$$(P_{\mathrm{em}})_{\mathrm{Av}} = m\left(\int \gamma' \cos\vartheta d\Phi\right)\Big/\Phi,$$

where $\gamma' = \gamma/[1+\gamma(1-\cos\vartheta)]$ refers to the secondary photon which is scattered in the direction ϑ. Evaluating these integrals and using Eq. (17), we obtain the E.R. relations:

$$(E_{\mathrm{em}})_{\mathrm{Av}} \cong (P_{\mathrm{em}})_{\mathrm{Av}} \cong \tfrac{1}{2} m\gamma. \tag{18}$$

Since m is the energy initially absorbed by the particle, we see that $(E_{\mathrm{em}})_{\mathrm{Av}}/E_{\mathrm{abs}} \cong \frac{1}{2}$. The same result follows from the classical formulas (14), (15), since $mf_1Bs_1 \sim m \ll E_{\mathrm{abs}}$ in the E.R. domain. Thus both classical and quantum theory lead to the result that in the E.R. case about one-half of the energy which scattering spinless particles initially absorb is retained by them and the other half is emitted in form of secondary radiation.

The classical expressions (15) become identical with the quantum-theoretical expressions (18) if it is assumed that the particle absorbs the energy of a photon, $E_{\mathrm{abs}} = \hbar\omega$, or

$$E_{\mathrm{abs}}/m = bF^2(1+F^2/\omega^2)s_1 = \gamma. \tag{19}$$

We propose to use this relation to supplement our classical formulas. It appears that such a procedure may contribute to a better understanding of why customary quantum-theoretical methods can lead to correct results for high energy photon-electron processes. In fact, the expression for the interaction is still taken over from classical theory there, and it is by no means obvious that classical concepts can be valid for arbitrarily high energies.

Let us consider the conditions of applicability of our classical formulas. It is evident from Eqs. (9) or (10), and it can be confirmed by working out higher approximations, that radiation reaction terms which do not contain s explicitly will, in general, be small if

$$b\omega f \ll 1, \tag{20}$$

$$bf(F^2/\omega) \ll 1. \tag{21}$$

Since for particles initially at rest $f \leqslant 1$, these conditions are certainly fulfilled if they are fulfilled for $f=1$.

If $f \cong 1$, it follows from Eq. (20) that $\hbar\omega \ll \hbar/b \cong 205\ mc^2$. Obviously, for high energies and $f \cong 1$, the use of our classical formulas is not permissible. But if high relativistic energies are transferred to the particle during the interaction, it will rapidly recede from the incident wave, and the function f can be expected to decrease considerably during the interaction.

To determine the average E.R. value of f, we make use of Eq. (19). If the particle acquires the energy $\eta_1 m \cong \frac{1}{2}\gamma m$ during the interaction, it follows from Eq. (3) that

$$f_1 \cong 1/2\eta_1 \cong 1/\gamma. \tag{22}$$

The average value of f can be obtained from the relations

$$(f)_{\mathrm{Av}} = \left(\int^{t_1} f dt\right)\Big/\left(\int^{t_1} dt\right) = \left(\int^{s_1} \eta ds\right)\Big/\left[\int^{s_1} (\eta/f) ds\right],$$

where Eq. (4) has been used to carry out the transformation from t to s; t_1 is the time during which the interaction takes place.

With f and η as given by Eqs. (10a) and (12b), these integrals are easily worked out. Using the E.R. relation $f_1 \cong A/Bs_1 \ll 1$, and omitting terms of relative order $1/Bs_1$, we obtain

$$\int^{s_1} \eta ds \cong \tfrac{1}{4}(1+F^2/\omega^2)Bs_1^2/A, \tag{23}$$

$$t_1 = \int^{s_1} (\eta/f) ds \cong \tfrac{1}{6}(1+F^2/\omega^2)B^2 s_1^3/A^2. \tag{24}$$

Using these expressions and Eq. (22), we obtain the E.R. formula:

$$(f)_{\mathrm{Av}} \cong 3A/2Bs_1 \cong 3f_1/2 \cong 3/2\gamma. \tag{25}$$

[6] W. Pauli, Revs. Modern Phys. **13**, 230 (1941).

With this expression for f, condition (20) assumes the form

$$b\omega(3mc^2/2\hbar\omega)=e^2/\hbar c\ll 1. \tag{20'}$$

Since $e^2/\hbar c\cong 1/137$, it appears that, except during an initial stage of the process when f is relatively large, condition (20) can be fulfilled for arbitrarily high frequencies of the incident photon. It is possible that the influence of this initial stage upon the total process is small and that classical results still apply, in a correspondencelike way, even for $\hbar\omega>205\ mc^2$. As mentioned already in Sec. I, this somewhat surprising result may prove of some help in understanding the quantum-theoretical result that, for the process considered, radiation reaction corrections do not exceed the relative order of magnitude $e^2/\hbar c$.

VI. ESTIMATES OF LOWER LIMITS FOR s_1 AND t_1

We have still to consider condition (21). Since condition (20) leads to the plausible condition (20′), it would seem desirable that condition (21) should not lead to an additional restriction of the domain of applicability of our formulas. It can be seen immediately that condition (21) will be fulfilled simultaneously with condition (20) if

$$F\lesssim\omega,\quad\text{or}\quad E\lesssim m\omega/e\cong(e/137r_0^2)\gamma. \tag{26}$$

For $\gamma\sim 1$, this upper limit for E is of the order of magnitude of the critical field intensity in positron theory,[7] $e/137r_0^2$; and for $\gamma>137$ it is larger than e/r_0^2, the field intensity on the surface of the classical electron model. Certainly, neither classical nor quantum theory will be expected to apply if the field intensities exceed these limits, and so it seems plausible to require that condition (26) be fulfilled.

This requirement leads to interesting conclusions concerning the admissible values of s_1 and t_1. An estimate of a lower limit for s_1 is obtained by substituting ω for F in the expression $s_1=\gamma/bF^2(1+F^2/\omega^2)$ which follows from Eq. (19):

$$s_1\gtrsim\gamma/2b\omega^2=(3\hbar c/8\pi e^2)\lambda, \tag{27}$$

where λ is the wavelength of the incident radiation. Since s_1 represents an extension of the incident wave-train along its direction of propagation and $(3\hbar c/8\pi e^2)\cong 16.3$, this looks like a reasonable condition.

The corresponding order of magnitude of a lower limit for t_1 is obtained by combining the E.R. expression (24) with (19) and substituting ω for F. This leads to the estimate

$$t_1\gtrsim\tau_0\gamma,\quad \tau_0=\hbar^2/16me^2c. \tag{28}$$

Since τ_0 is a constant, this lower limit for t_1 increases with the energy of the incident photon in a way analogous to the relativistic increase of the lifetime of unstable particles.

In our classical theory t_1 represents the duration of the interaction between wave and particle. In deriving the condition (28) for t_1, no unusual assumptions have been made, apart from the use of a starting function which may be nothing more but a mathematical device and which does not directly affect our results. It seems thus likely that such lower limits for interaction times really exist. Analogous results for normal Compton scattering would probably lead to the prediction of a lower limit for the delay in the appearance of the secondary particles. Times of the order given by Eq. (28), $\tau_0\cong 10^{-20}$ sec, are, however, far below the present limit of experimental accuracy, around 10^{-8} sec, in attempts to observe such a delay.[8]

Results of this kind might be of some interest in meson theory. Perhaps it is not just coincidence that the expression for τ_0 in condition (28) resembles the formulas for meson lifetimes in its dependence upon fundamental constants. In earlier theories of meson decay where the existence of mesons of different masses has not been taken into account, meson lifetimes of the order of magnitude $\hbar^2/\mu f^2c$ have been obtained; μ is the meson mass, and f is a constant of the dimension of a charge, $\sim(10^{-8}-10^{-7})e$, which determines the interaction between mesons and light particles in the same way as e determines the coupling between the electromagnetic field and charged particles. There is direct analogy between these meson theories and the quantum theory of electromagnetic radiation in all their aspects, including the classical foundations.[9] If our semiclassical results corresponded to physical reality, one might expect analogous results for interaction processes between mesons and light particles, and that would imply lower limits for interaction times which might be of the same order of magnitude as the meson lifetimes. In recent versions of meson theory which distinguish between mesons of different masses, the situation is more involved, but still essentially the same. It seems conceivable that an interaction process between some type of meson and a high energy light particle might last long enough to be observable. At present, such an observation would probably be interpreted as indicating formation and decay of a heavier meson. A possibility of reducing the number of independent elementary particles by taking finite interaction times into account is thus suggested.

It was in particular this possibility which seemed to make the semiclassical treatment of radiation reaction worth pursuing. A quantum-theoretical investigation of relevant problems would seem desirable, but apparently no attempt in this direction has as yet been made.

[7] See, for instance, W. Heisenberg and H. Euler, Z. Physik **98**, 714 (1936).

[8] R. Hofstadter and J. A. McIntyre, Phys. Rev. **78**, 24 (1950); W. G. Cross and N. F. Ramsey, Phys. Rev. **80**, 929 (1950).

[9] See, for instance, H. Yukawa, Proc. Math. Phys. Soc. (Japan) **17**, 48 (1935); H. A. Bethe and L. W. Nordheim, Phys. Rev. **57**, 998 (1940).