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The Scattering of π-Mesons by Deuterons

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A study is made of the expected properties of the cross section for scattering π-mesons by deuterons on the basis of the impulse approximation. The cross section for high energy mesons can then be expressed in a simple manner in terms of the cross sections for meson scattering by free protons and neutrons. This provides a means of deducing the latter when the deuteron and proton cross sections are known.

I. INTRODUCTION

THE interactions of π-mesons with nucleons have been observed to include those which lead to scatterings and those which lead to absorptions. Such interactions can be conveniently classified as of two types: (1) those of mesons with free nucleons and (2) those with nucleons bound in nuclei. The latter types of processes have been observed in several experiments[1-4] and indicate that here both scattering and absorption play an important role. Some implications of these experiments have been discussed previously.[5,6] Processes (1) involve meson interactions with free protons and free neutrons, of which only the former are directly amenable to experimental study.[1,7] Presumably, the free neutron-π cross section must be deduced indirectly from scatterings of type (2).

In this connection there arises, however, the question as to how adequately the free particle scattering cross section can describe the scattering in nuclear matter—i.e., as to the importance of many-body interactions. To obtain information concerning this point as well as the free neutron-π cross section, the scattering of mesons by deuterons would appear to offer the most promise. As a starting point in such an analysis, we shall calculate the properties of the π-deuteron scattering cross section by means of the impulse approximation[8] on the assumption that the meson-nucleon scattering interaction is not modified by the presence of the other nucleon. Granting this assumption, Chew's[8] conditions for the validity of the impulse approximation should be well satisfied, since we are interested in meson velocities much greater than the nucleon velocity in the deuteron and since the expected scattering cross sections are probably less than the nucleon-nucleon cross sections.

The impulse approximation permits us to calculate the ratio of elastic to inelastic scattering as well as the angular distribution and energy spectrum of the scattered mesons in terms of the free nucleon-meson cross sections (subject to certain restrictions to be discussed in Section II).

The energy spectrum of the scattered mesons, particularly at small scattering angles, should provide a means of estimating the role of three particle effects (i.e., of the break down of the assumptions under which the impulse approximation is valid). At larger scattering angles it would appear possible to deduce the π-neutron cross section when these results are combined with the free proton-meson scattering cross section.

A study of meson-deuteron scattering on the basis of weak coupling meson theory has been made by Ferretti and Gallone[9] and by Blair.[10] Blair's results indicate that for scattering angles which are not too small and for meson energies which are not too low, the impulse approximation gives satisfactory results, the exact details depending on the nature of the theory. However, in view of the questionable reliability of meson theories, there seems to be reason for pursuing a phenomenological analysis.

II. THE FORMULATION OF THE SCATTERING PROBLEM IN TERMS OF THE IMPULSE APPROXIMATION

We shall ignore the charge exchange scattering and restrict ourselves to angles sufficiently large that coulomb scattering can be neglected. We shall also not specify whether the scattered meson is positive or negative, since in the impulse approximation any differences will arise through the free nucleon scattering characteristics, which are left arbitrary.

We introduce the scattering matrices,[11] R_P and R_N, referring respectively to scattering on free protons and neutrons. Considering for the moment R_P, we suppose the initial and final meson momenta to be, respectively, $\mathbf{q}_0$ and $\mathbf{q}$, while those for the proton are $\mathbf{p}_0$ and $\mathbf{p}$. We then write R_P in terms of relative momenta as (we use as units $\hbar=c=1$)

$$R_P=[(M+E)^{-1}(M\mathbf{q}-E\mathbf{p})|r_P|(M+E)^{-1}(M\mathbf{q}_0-E\mathbf{p}_0)]\times\delta(\mathbf{q}+\mathbf{p}-\mathbf{q}_0-\mathbf{p}_0), \quad (1)$$

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1 Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).
2 Camac, Corson, Littauer, Shapiro, Silverman, Wilson, and Woodward, Phys. Rev. **82**, 745 (1951).
3 H. Bradner and S. Rankin, Phys. Rev. **80**, 916 (1950).
4 Bernandini, Booth, Lederman, and Tinlot, Phys. Rev. **82**, 105 (1951).
5 H. A. Bethe and R. R. Wilson, Phys. Rev. **83**, 690 (1951).
6 Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951).
7 Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. **82**, 959 (1951).
8 G. F. Chew, Phys. Rev. **80**, 196 (1950).
9 B. Ferretti and S. Gallone, Phys. Rev. **77**, 153 (1950).
10 J. S. Blair, Ph.D. thesis, University of Illinois (1951). We are indebted to Dr. Blair for sending us a copy of his thesis.
11 C. Møller, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd., Vid. **23**, No. 1 (1945).

where M is the nucleon mass and E is the total energy (rest mass plus kinetic) of the meson in the center-of-mass system. In Eq. (1) it is assumed that the proton velocities are nonrelativistic and remain so when transformed to the center-of-mass system. R_N is given by a similar expression in terms of a function r_N of the relative momenta. We write an arbitrary spin dependence for r_P and r_N as

$$\begin{aligned} r_P &= r_P{}^1 + \boldsymbol{\sigma}^P \cdot \mathbf{r}_P{}^2, \\ r_N &= r_N{}^1 + \boldsymbol{\sigma}^N \cdot \mathbf{r}_N{}^2, \end{aligned} \quad (2)$$

where $\boldsymbol{\sigma}^P$ and $\boldsymbol{\sigma}^N$ are the respective spin operators of the proton and neutron. Then the cross section for scattering by a free proton is

$$d\sigma_P/d\Omega = [(2\pi)^4/v_\pi] \int_{E_f} q^2 dq \delta(E_f - E_0) \times [|r_P{}^1|^2 + |r_P{}^2|^2], \quad (3)$$

with a similar expression for $d\sigma_N/d\Omega$, the cross section for scattering by a free neutron. Here v_π is the initial meson velocity, E_f and E_0 are the final and initial energies, respectively, of the system, and the integration is taken over final energies. For future reference, we define

$$J_0 \equiv \int_{E_f} q^2 dq \delta(E_f - E_0). \quad (4)$$

For the scattering by a deuteron, we assign the neutron respective initial and final momentum variables $\mathbf{n}_0$ and $\mathbf{n}$. Let the deuteron wave function be (we neglect the admixture of D-state)

$$\psi_D = \chi^t \phi_D(\tfrac{1}{2}(\mathbf{p}_0 - \mathbf{n}_0)) \delta(\mathbf{p}_0 + \mathbf{n}_0),$$

and that for the final state of the neutron and proton be

$$\psi_F = \chi^F \phi_F(\tfrac{1}{2}(\mathbf{p} - \mathbf{n})) \delta(\mathbf{p} + \mathbf{n} - \mathbf{K}),$$

where $\mathbf{K}$ is the recoil momentum and the χ's are spin wave functions. We must distinguish three types of final states F: singlet and triplet states for which the neutron and proton are not bound (inelastic scattering), and triplet deuteron states (elastic scattering). We suppose the relative final momentum of the two nucleons to be $\mathbf{k}$ for the inelastic scattering.

The transition amplitude for the scattering is

$$H_{FA} = (\psi_F, (R_P + R_N)\psi_D) \equiv \delta(\mathbf{q} + \mathbf{K} - \mathbf{q}_0) h_{FA}. \quad (5)$$

The differential cross section for the final triplet state inelastic scattering is

$$d\sigma^t = [(2\pi)^4/v_\pi] \int_{E_F} d^3q d^3k \delta(E_F - E_0) \sum_S |h_{FA}{}^t|^2, \quad (6)$$

where $\sum_S$ means the appropriate sum and average over spin substates. E_F and E_0 are the final and initial energies of the system, the integration being taken over the former. A similar expression holds for $d\sigma^s$, the scattering to a final singlet state. For the elastic scattering, we have

$$d\sigma^d = [(2\pi)^4/v_\pi] \int_{F'} d^3q \delta(E_F' - E_0) \sum_S |h_{FA}{}^d|^2, \quad (7)$$

where the integration is taken over the appropriate E_F'.

Referring to expressions (1) and (5), we can express h_{FA} as

$$\begin{aligned} h_{FA} = \int d^3l \chi^{*F} \Big\{ & \phi_F^*(\mathbf{l} - \tfrac{1}{2}(\mathbf{q} - \mathbf{q}_0)) \\ & \times \left[\frac{M}{M+E} \left(\mathbf{q} + \frac{E}{M}(\mathbf{q} - \mathbf{q}_0) - \frac{E}{M}\mathbf{l} \right) \right. \\ & \times \left. |r_P| \frac{M}{M+E} \left(\mathbf{q}_0 - \frac{E}{M}\mathbf{l} \right) \right] \\ & + \phi_F^*(\mathbf{l} + \tfrac{1}{2}(\mathbf{q} - \mathbf{q}_0)) \\ & \times \left[\frac{M}{M+E} \left(\mathbf{q} + \frac{E}{M}(\mathbf{q} - \mathbf{q}_0) + \frac{E}{M}\mathbf{l} \right) \right. \\ & \times \left. |r_N| \frac{M}{M+E} \left(\mathbf{q}_0 + \frac{E}{M}\mathbf{l} \right) \right] \Big\} \chi^t \phi_D(\mathbf{l}). \end{aligned} \quad (8)$$

To evaluate this integral it is necessary to remove r_P and r_N from the integrand. This can be done rigorously in any one of four limiting cases: (1) ψ_F represents a plane wave; (2) the deuteron binding energy is negligible compared to the recoil energies; (3) the ratio E/M goes to zero; (4) r_P and r_N depend only on the difference of their arguments, as is true in the Born approximation for potential scattering. For large momentum transfers conditions (1) and (2) are valid. For small momentum transfers and elastic scattering we rely largely on condition (3). The validity of condition (4) is doubtful in the present case.

We thus set $l=0$ in r_P and r_N in Eq. (8) and remove them from the integrand. This leads to expressions of the form

$$h_{FA}{}^s = (s|r_P|t) I_1{}^s + (s|r_N|t) I_2{}^s, \quad (9)$$

where the notation $(s|r_P|t)$ means the matrix element of r_P for a transition from a triplet to a singlet state of the neutron-proton system, etc., and

$$\begin{aligned} I_1{}^s &= \int \phi_F{}^{*s}(r) \exp[-i\tfrac{1}{2}(\mathbf{q} - \mathbf{q}_0) \cdot \mathbf{r}] \phi_D(r) d^3r, \\ I_2{}^s &= \int \phi_F{}^{*s}(r) \exp[i\tfrac{1}{2}(\mathbf{q} - \mathbf{q}_0) \cdot \mathbf{r}] \phi_D(r) d^3r. \end{aligned} \quad (10)$$

Here $\phi_D(r)$ and $\phi_F{}^s(r)$ are the coordinate representations of the deuteron and final singlet wave functions of the neutron-proton system. For the elastic and final triplet state inelastic scattering, we have corresponding expressions $h_{FA}{}^d$ and $h_{FA}{}^t$ with the appropriate wave functions $\phi_F(r)$ in the integrals (10).

As the spread in energy of the scattered meson will in most cases be small, we shall be particularly interested in just its angular distribution. In this case we integrate the differential cross section [Eq. (6)] over the energy spectrum of the scattered particles. To do this we again make use of the smallness of E/M and the loose binding of the deuteron and remove the quantities r_N and r_P from the integrand. This should not lead to significant errors, except perhaps for small angle scatterings (for which we can expect only qualitative information in any case).

Referring to the decomposition of r_P and r_N given by Eq. (2), we define

$$\begin{aligned} L_P{}^t &= |r_P{}^1|^2+\tfrac{2}{3}|r_P{}^2|^2, \\ L_P{}^s &= \tfrac{1}{3}|r_P{}^2|^2, \\ M^t &= R_e\{r_P{}^{1*}r_N{}^1+\tfrac{2}{3}\mathbf{r}_P{}^{*2}\cdot\mathbf{r}_N{}^2\}, \\ M^s &= -R_e\{\tfrac{1}{3}\mathbf{r}_P{}^{*2}\cdot\mathbf{r}_N{}^2\}, \end{aligned} \tag{11}$$

with similar quantities $L_N{}^t$ and $L_N{}^s$. "$R_e\{\cdots\}$" means "the real part of . . . ." The quantities, L, represent incoherent contributions to the cross section while the quantities, M, represent the effects of interference of meson waves scattered from the neutron and proton.

Integration over the energy spectra leads to the following differential cross sections for scattering into an angle θ:

$$\begin{aligned} d\sigma^s/d\Omega &= [(2\pi)^4/v_\pi] \\ &\quad\times J_0[(L_P{}^s+L_N{}^s)H_1{}^s(\theta)+2M^sH_2{}^s(\theta)], \\ d\sigma^t/d\Omega &= [(2\pi)^4/v_\pi] \\ &\quad\times J_0[(L_P{}^t+L_N{}^t)H_1{}^t(\theta)+2M^tH_2{}^t(\theta)], \\ d\sigma^d/d\Omega &= [(2\pi)^4/v_\pi]J_0[(L_P{}^t+L_N{}^t)+2M^t]H^d(\theta), \end{aligned} \tag{12}$$

where

$$\begin{aligned} H_1{}^s(\theta) &= (1/J_0)\int q^2dqd^3k\delta(E_F-E_0)|I_1{}^s|^2, \\ H_2{}^s(\theta) &= (1/J_0)\int q^2dqd^3k\delta(E_F-E_0)|I_2{}^{*s}I_1{}^s|. \end{aligned} \tag{13}$$

J_0 is given by Eq. (4) and $I_1{}^s$, $I_2{}^s$ are given by Eqs. (10). The triplet quantities, $H_1{}^t$ and $H_2{}^t$, are obtained by using the triplet state wave functions, $\phi_F{}^t$ in Eqs. (10). From Eq. (7) we have

$$H^d(\theta)=(1/J_0)|I_1{}^d|^2\int_{E_F'} q^2dq\delta(E_F'-E_0), \tag{14}$$

where $I_1{}^d$ is obtained from Eq. (10) by replacing $\phi_F{}^{*s}(r)$ by $\phi_D{}^*(r)$.

For the total cross section, we add the three cross sections (12). Reference to Eqs. (11) shows that at least two-thirds of the final states are triplet, which suggests a simplification in the writing of the cross section. We define

$$\begin{aligned} H_1 &= H^d+H_1{}^t, \\ H_2 &= H^d+H_2{}^t. \end{aligned} \tag{15}$$

Then the sum of the three cross sections (12) can be written as

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \left[\frac{d\sigma_P}{d\Omega}+\frac{d\sigma_N}{d\Omega}\right]H_1+2\cos\omega\left[\frac{d\sigma_P}{d\Omega}\frac{d\sigma_N}{d\Omega}\right]^{\frac{1}{2}}H_2+\frac{(2\pi)^4}{v_\pi}J_0 \\ &\quad\times\{(L_P{}^s+L_N{}^s)(H_1{}^s-H_1)+2M^s(H_2{}^s-H_2)\}. \end{aligned} \tag{16}$$

$d\sigma_P/d\Omega$ and $d\sigma_N/d\Omega$ are defined by Eq. (3). The term proportional to $\cos\omega$ represents an interference effect between waves scattered from the proton and neutron. $\cos\omega$ can, of course, be expressed in terms of the quantities (11). As will be seen below, the last term represents only a small correction to the cross section. We have thus expressed the scattering cross section from deuterium in terms of the cross section from free protons and neutrons.

III. NUMERICAL EVALUATION

We now discuss the dependence of the functions H on the meson energy and scattering angle. We begin with two approximate evaluations, the first of which is the closure approximation.

The closure approximation implies the neglect of the energy of relative motion of the nucleons in the final state on the over-all energy conservation. The meson is assumed to have the energy characteristic of a free particle collision. The completeness relation of the final states then gives [see Eq. (13)]

$$\int d^3k|I_1{}^s|^2=1,$$

$$\int d^3k|I_2{}^{*s}I_1{}^s|=\int\exp[-i(\mathbf{q}-\mathbf{q}_0)\cdot\mathbf{r}]\phi_D{}^2(r)d^3r,$$

etc., so we obtain

$$\begin{aligned} H_1{}^s &= H_1=1, \\ H_2{}^s &= H_2=\int\exp[-i(\mathbf{q}-\mathbf{q}_0)\cdot\mathbf{r}]\phi_D{}^2(r)d^3r. \end{aligned} \tag{17}$$

The last term in Eq. (16) therefore vanishes and we obtain a very simple expression for $d\sigma/d\Omega$ which is expected to be valid for high meson energies (a further discussion is given below).

The second approximation to the total cross section involves the use of plane waves for the final state, with $H^d(\theta)=0$ [Eq. (14)]. That this is not an unreasonable approximation follows from arguments of Gluckstern and Bethe.[12] Again, the last term in Eq. (16) vanishes

[12] R. L. Gluckstern and H. A. Bethe, Phys. Rev. 81, 761 (1951).

since the plane wave approximation does not distinguish between singlet and triplet states, and we are left with just the terms involving H_1 and H_2. A numerical discussion is given in the next section.

The plane wave approximation fails for those final states in which the neutron and proton have a small energy of relative motion. For such energies only the S-wave is sufficiently distorted to necessitate correction, the waves of higher angular momentum being well represented by plane waves. Thus we can calculate the quantities H by correcting the plane wave approximation for S-waves only.

For the deuteron wave function we use the Chew-Goldberger[13] expression:

$$\phi_D(r)=(N/r)[e^{-\alpha r}-e^{-\beta r}], \tag{18}$$

where N is the appropriate factor of normalization and

$$\alpha=45.5 \text{ Mev}, \quad \beta=7\alpha.$$

For the S-wave phase shifts as a first approximation we can use the asymptotic wave functions.[14]

$$\begin{aligned}\phi_k{}^t&=\exp(-i\delta^t)\sin(kr+\delta^t)/kr,\\ \phi_k{}^s&=\exp(-i\delta^s)\sin(kr+\delta^s)/kr,\end{aligned} \tag{19}$$

where δ^s and δ^t are the appropriate singlet and triplet phase shifts.[15] A correction for the finite range of the singlet $n-p$ potential can be given in terms of the effective ranges according to the arguments of Bethe and Longmire.[16] A correction in the triplet case is obtained by replacing

$$\exp[\pm i(\mathbf{q}-\mathbf{q}_0)\cdot\mathbf{r}]$$

by

$$\{\exp[\pm i(\mathbf{q}-\mathbf{q}_0)\cdot\mathbf{r}]-1\}$$

in Eqs. (10), since the correct $\phi_k{}^t$ must be orthogonal to ϕ_D. The latter expression is small for small r, so we shall not make a further correction for small distances.

The integrals (10) can now be done analytically. With the exception of H^d, the H's must be evaluated numerically. The results are given in the next section.

IV. RESULTS AND DISCUSSION

The values of the quantities $H_1(\theta)$ and $H_2(\theta)$ [Eq. (15)] are plotted in Fig. 1 for several meson energies. The corresponding values of $H_1{}^s(\theta)$ and $H_2{}^s(\theta)$ [Eq. (13)] are plotted in Fig. 2.

We wish first of all to show that to a good approximation one may neglect the second term in Eq. (16). Reference to Figs. 1 and 2 shows that $H_1(\theta)$ and $H_1{}^s(\theta)$ differ at most by about 10 percent. For scattering angles less than 45 degrees, $H_2(\theta)$ and $H_2{}^s(\theta)$ differ at most by 15 percent; at large angles where the difference is larger, the functions themselves are small compared to $H_1(\theta)$ and their difference is less than 10 percent of $H_1(\theta)$. In addition, reference to Eq. (11) shows that the coefficient of $[H_1{}^s(\theta)-H_1(\theta)]$ in Eq. (16) cannot be greater than one-third the coefficient of H_1. It follows, then, that the correction to $d\sigma/d\Omega$ arising from the second term of Eq. (16) cannot be greater than about 4 percent, except for small-angle scatterings—and so may be neglected. We therefore have

$$\frac{d\sigma}{d\Omega}=\left[\frac{d\sigma_P}{d\Omega}+\frac{d\sigma_N}{d\Omega}\right]H_1+2\cos\omega\left[\frac{d\sigma_P}{d\Omega}\frac{d\sigma_N}{d\Omega}\right]^{\frac{1}{2}}H_2, \tag{16'}$$

to a good approximation.

Reference to Fig. 1 shows that $H_1(\theta)=1$ to within about one percent for the energy range investigated (except for 130-Mev mesons, for which H_1 fell to about 0.97 at 180°). Since this is just the value obtained from

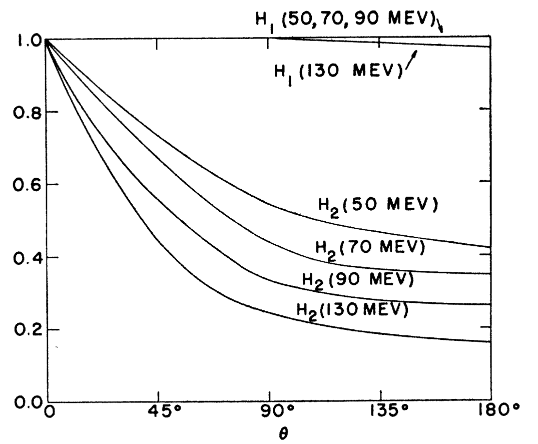

FIG. 1. Values of the functions $H_1(\theta)$ and $H_2(\theta)$ [Eq. (15)] for several incident meson energies. $H_1(\theta)$ was found to be unity at the lower energies, but dropped to about 0.97 at 180° for 130 Mev-mesons.

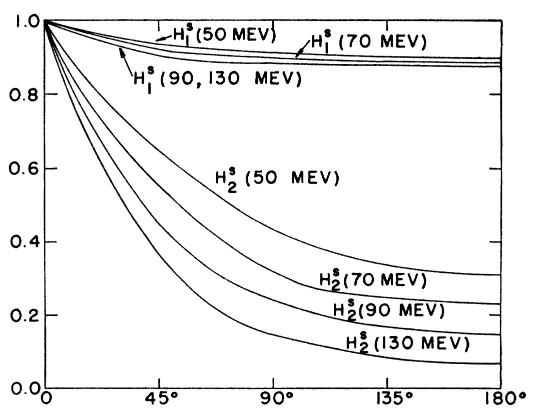

FIG. 2. Values of the functions $H_1{}^s(\theta)$ and $H_2{}^s(\theta)$ [Eq. (13)] for several incident mesons energies. The 90-Mev and 130-Mev curves very nearly coincide.

[13] G. F. Chew and M. L. Goldberger, Phys. Rev. 77, 470 (1950).
[14] See, for instance, Watson and Stuart, Phys. Rev. 82, 738 (1951), where the same integrals are discussed.
[15] The familiar expressions for the phase shifts in terms of the effective ranges and scattering lengths were used (see, for instance, Blatt and Jackson, Phys. Rev. 76, 18 (1949) and H. A. Bethe, Phys. Rev. 76, 38 (1949)).
[16] H. A. Bethe and C. Longmire, Phys. Rev. 77, 647 (1950).

TABLE I. The values of $H_1(\theta)$, as obtained from the plane wave approximation, are given for several incident meson energies. In this approximation, $H_1(\theta)$ was independent of the scattering angle θ.

Incident meson energy in Mev	50	70	90	130
H_1	0.80	0.84	0.86	0.90

the closure approximation [Eq. (17)], it appears that closure gives a surprisingly accurate result. The closure value for H_2 [Eq. (17)] was in very good agreement with $H_2{}^s$ and in fair agreement with the correct value for H_2. Since almost the entire contribution to H_2 comes from the elastic scattering [i.e., $H^d(\theta)$ of Eq. (14)], it was found that excellent agreement could be obtained with closure for H_2 if the phase space factor for elastic scattering were used. This means multiplying the value for H_2 given by Eq. (17) by the factor

$$(1/J_0)\int_{E_F'} q^2 dq\,\delta(E_F'-E_0), \tag{20}$$

[compare Eq. (14)].

Thus, it appears that $d\sigma/d\Omega$ can be given by the closure approximation [with H_2 corrected by the factor (20)] to an accuracy which is probably about as good as the model obtained from the impulse approximation. This is fortunate, as the closure values may be easily calculated from Eq. (17).

In contrast to this, the values of H_1 and H_2 obtained from the plane wave approximation require a lengthy numerical calculation and are in general less accurate than the closure values. The quantities H_1, as obtained from the plane wave approximation, were independent of the angle of scattering (to within the estimated computational accuracy of about one percent) and are given in Table I for those meson energies included in Figs. 1 and 2.

The magnitude of the elastic scattering is described by $H^d(\theta)$ [Eqs. (12) and (14)]. Values of $H^d(\theta)$ for several meson energies are given in Fig. 3.

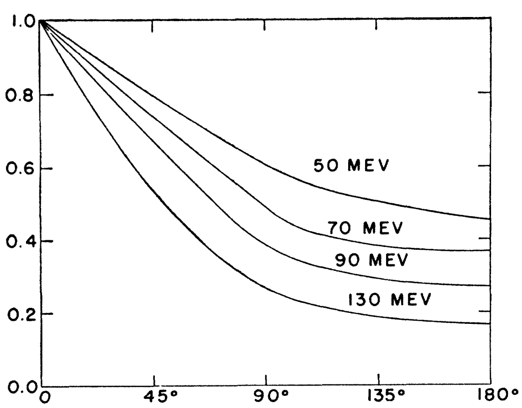

FIG. 3. Plot of the function $H^d(\theta)$ [Eq. (14)] for several incident meson energies.

For the energy spectrum of the scattered mesons, one cannot use an approximation which neglects the final neutron-proton interaction. The mesons scattered elastically have, of course, a fixed energy. Several characteristic energy spectra for the inelastic scattering are given in Fig. 4. These spectra were obtained from $H_1{}^s$ and $H_1{}^t$ by not performing the integrations over dq in Eqs. (13).

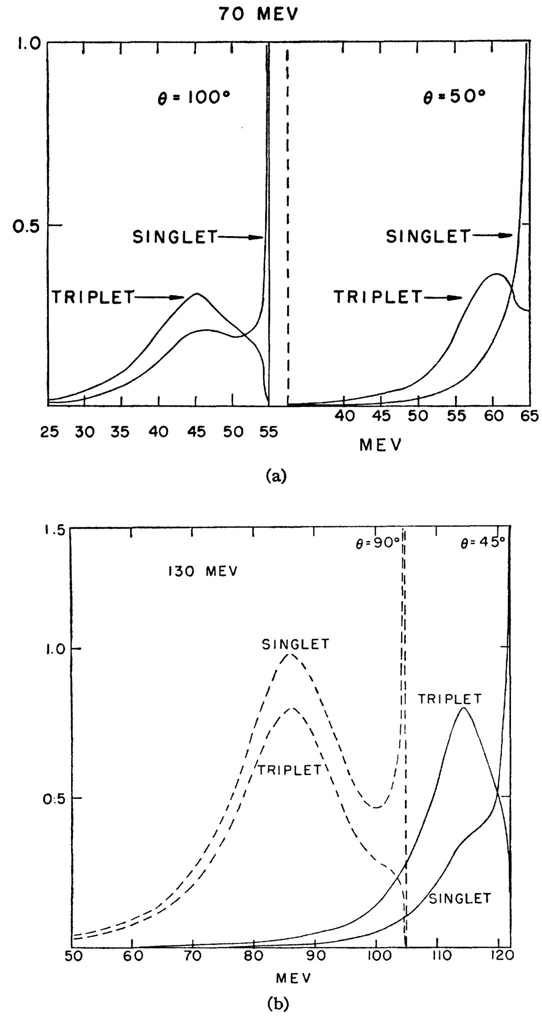

FIG. 4. Meson energy spectrum (in arbitrary units) for inelastic scattering with final singlet and triplet nucleon states. In (a) the spectra are given for an incident energy of 70 Mev and for scattering angles, θ, of 50° and 100°. In 4(b) the incident energy is 130 Mev and the scattering angles are 45° and 90°. The tendency for the curves to peak at the energy corresponding to a free particle collision should be noted.

The use of H_1 only is justified, since the spectra arising from H_2 are nearly the same when the latter is not negligibly small. The energy spectra are of importance, since they should provide a test of the degree of validity of the impulse approximation. It would seem likely that hypothetical three-body interactions should cause considerably larger energy losses for the meson than are predicted on the basis of the present model. As is seen from Fig. 4, the energy loss of the meson on the basis of the present model tends on the average to be slightly less than that for a collision with a free nucleon.

V. CONCLUSIONS

The use of Eq. (16′) should provide a direct means of deducing the values of $d\sigma_N/d\Omega$ (that is, the cross section for scattering mesons on free neutrons) from experiments on the scattering by deuterons and protons for momentum transfers large enough that H_2 is small. Comparison of the scattering from deuterium at large and small angles should provide information about the relative phase of waves scattered from neutrons and protons.[17] This phase is described by the factor $\cos\omega$ in Eq. (16′), which may, of course, be a function of the scattering angle.

[17] H. A. Bethe and R. R. Wilson, reference 5, have shown that this is of importance in describing the scattering of mesons in complex nuclei.

For these purposes the readily applied closure approximation is satisfactory.

We are indebted to Mr. Richard I. Mitchell, Mr. Glen Culler, Mr. Burns Macdonald, and other members of the computing group at the Radiation Laboratory for performing most of the numerical calculations described in this paper.

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Note added in proof:—Since many meson scattering experiments are being done by attenuation methods, it is desirable to know the cross section for charge exchange scattering. This can be calculated simply from the theory above, using the closure approximation. If we let $d\sigma_p^{\text{exch}}/d\Omega$ be the exchange cross section for π^- on protons, we obtain for the exchange cross section for π^- on deuterons:

$$\frac{d\sigma_{\pi^--D}^{\text{exch}}}{d\Omega}=\frac{d\sigma_p^{\text{exch}}}{d\Omega}[1-f\mathrm{H}_2],$$

where H_2 is given by Eq. (17) and f is a fraction depending upon the amount of spin flip.

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Term Values in the $3d^5 4s$ Configuration of Fe III

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The theoretical formulas for $d^5 s$ are compared with the experimental data of Fe III. The mean deviation between theory and experiment is found to be ± 852 cm^{-1}.

We compared the values of the parameters B and G_2 which are required to give correctly the observed separations between certain pairs of terms. Adding to the theory a correction term proportional to $L(L+1)$ leads to consistent values of all the radial integral parameters. The mean deviation between the theory corrected in this manner and experiment is reduced to 105 cm^{-1}.

I. INTRODUCTION

In conjunction with the results of two previous papers,[1,2] this study completes the analysis of the term values of the $3d^5 4s$ and $3d^6$ configurations in Fe III and Mn II.

A second purpose of this study is to show that the errors between theory and experiment are simply related to the orbital quantum number. Since 23* of the 32 possible terms of the $3d^5 4s$ configuration of Fe III are known, there is a large amount of experimental data to verify this conclusion. In addition, the term values are not appreciably affected by interactions with nearby configurations, so that the relationship of the errors to the orbital quantum number represents a polarization effect. If this effect is assumed similar in the same configurations of different atoms, one can predict more accurately the positions of terms for the experimentalist. We have tentatively made such an application of the results of this paper in a previous paper and have also used the results to verify the experimental assignments of terms to their configurations.[2]

[1] R. E. Trees, Phys. Rev. **82**, 683 (1951).
[2] R. E. Trees, Phys. Rev. **83**, 756 (1951).
* *Note added in proof:*—There are 26 known term values; the 3 highest levels were inadvertently overlooked. The values for these levels are included in Table I. The failure to include them in the least squares calculation has no effect on the conclusions.

II. TERM VALUES

The experimental term values are taken from Edlen and Swing.[3] The theoretical formulas are the same as those used for $d^5 s$ in our previous calculations[2] for Mn II, and the parameters were evaluated by least squares. The results of the calculation are given in column one of Table I; the mean deviation between theory and experiment is 852 cm^{-1}.

The theoretical formulas for all terms observed in $3d^5 4s$ of Fe III are rational with the exception of the two terms based on the ^2D parent. Partly as a matter of convenience, these two terms were not included in the calculation. However, our subsequent results (Table II) will indicate that the $(^2D)^1D$ term observed at

[3] B. Edlen and P. Swings, Astrophys. J. **95**, 532 (1942).