TABLE I. Isotope splittings in erbium, in wave-number units. $\Delta\nu_{\text{mean}}$ are the mean values of measurements using four different spacer sizes. The upper numbers are $\nu_{168}-\nu_{170}$ and the lower numbers are $\nu_{166}-\nu_{168}$. R gives the ratio of the splittings.

λ, A	$\Delta\nu_{\text{mean}}$	R	λ, A	$\Delta\nu_{\text{mean}}$	R
4729	0.046_6 0.046_6	1.00	4496	0.044_2 0.046_3	0.96
4722	-0.050_6 -0.051_3	0.99	4426	-0.047_2 -0.050_0	0.94
4673	-0.049_5 -0.046_9	1.06	4424	-0.043_8 -0.040_4	1.08
4606	-0.045_6 -0.045_6	1.00	4409	-0.045_1 -0.046_2	0.98
4552	0.045_8 0.047_6	0.96	4331	-0.053_0 -0.052_0	1.02
4531	0.047_5 0.044_6	1.07			

s-electrons. For instance, the usual change of a *p*-electron to an *s*-electron would give a positive shift. No analysis is available for erbium, but the isotope structure may give useful clues. The hollow cathode source usually enhances the ionized spectra, and since the shifts are of the magnitude to be expected from the second spectrum, it is presumed that the lines showing the shifts are caused by Er II. The positive isotope shifts probably arise from the electronic transitions $4f^{12}6p$ to $4f^{12}6s$ and $4f^{11}6s6p$ to $4f^{11}6s^2$, and the negative shifts probably arise from the two-electron transition $4f^{11}6s6p$ to $4f^{11}5d^2$. The configurations $4f^{12}6s$ and $4f^{11}6s6p$ should give similar splittings; the configuration $4f^{11}6s^2$ would give splittings twice as great if it were not that the mutual screening of the 6*s*-electrons tends to reduce the effect.

The work is being continued, lines are being measured in other regions of the spectrum, and the complete results will be reported later.

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Threshold Values of Internal Conversion Coefficients for the *K*-Shell

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RECENT computations of the internal conversion coefficients for the *K*-shell have been reported for various Z and for k (γ-ray energy) greater than 0.3–0.5 electron masses.[1] The extrapolation of these results to lower energies is uncertain for two reasons: first, the mathematical formulation of the problem is of such complexity that no simple extrapolation rule can be used; second, the numbers are computed for unscreened wave functions.

In an attempt to resolve the mathematical difficulties, computations have been made on the threshold values of the conversion coefficients. These computations were performed by taking the limiting values of the formulas of reference 1, as p, the electron momentum, approaches zero positively. Under these conditions many simplifications arise, and it is possible to compute the results on a desk machine.

TABLE I. Threshold values of internal conversion coefficients for the *K*-shell.

Z	α_1	α_2	α_3	α_4	α_5
10	7.329(3)	8.184(5)	4.281(7)	1.340(9)	2.867(10)
20	4.510(2)	1.251(4)	1.596(5)	1.232(6)	6.459(6)
30	8.720(1)	1.030(3)	6.060(3)	1.922(4)	4.338(4)
40	2.403(1)	1.606(2)	5.600(2)	9.300(2)	1.123(3)
50	1.085(1)	3.741(1)	8.317(1)	8.112(1)	5.926(1)
60	5.190(0)	1.081(1)	1.623(1)	1.035(1)	4.834(0)
70	2.713(0)	3.604(0)	3.721(0)	1.598(0)	5.479(−1)
80	1.636(0)	1.334(0)	9.516(−1)	3.152(−1)	8.995(−2)
88	9.989(−1)	6.622(−1)	3.551(−1)	1.094(−1)	2.977(−2)
96	6.691(−1)	3.661(−1)	1.848(−1)	4.991(−2)	8.452(−3)

Z	β_1	β_2	β_3	β_4	β_5
10	4.222(2)	2.219(5)	3.472(7)	2.587(9)	1.124(11)
20	1.087(2)	1.415(4)	5.471(5)	9.874(6)	1.084(8)
30	5.075(1)	2.893(3)	4.867(4)	3.919(5)	1.841(6)
40	3.072(1)	8.942(2)	8.830(3)	3.892(4)	1.003(5)
50	2.123(1)	3.881(2)	2.374(3)	6.466(3)	1.032(4)
60	1.737(1)	2.033(2)	8.215(2)	1.484(3)	1.626(3)
70	1.548(1)	1.225(2)	3.393(2)	4.257(2)	3.158(2)
80	1.528(1)	8.304(1)	1.605(2)	1.436(2)	7.637(1)
88	1.764(1)	6.582(1)	9.549(1)	6.576(1)	2.714(1)
96	2.313(1)	5.608(1)	6.060(1)	3.394(1)	1.034(1)

The results are given in Table I. The notation is that of reference 1. Since screening has been ignored, the threshold energies for which these results were computed were those obtained from the relativistic single-electron model, given by $k=1-(1-[\alpha Z]^2)^{\frac{1}{2}}$. Figures in parentheses indicate the power of 10 by which the number must be multiplied.

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Primary Specific Ionization of Cosmic Rays in Hydrogen*

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MANY attempts have been made to test the dependence of primary specific ionization upon momentum for high energy particles. Such measurements have been made by Kunze,[1] Corson and Brode,[2] J. G. Wilson,[3] Sen Gupta,[4] and Hazen[5,6] by the use of cloud chamber techniques. Except for Sen Gupta, who reported an increase for electrons but not for mesons, the other observers were unable to support the relativistic increase in ionization beyond the minimum as predicted in the theory of collision loss given by Bethe.[7]

Low efficiency counters have been employed by Danforth and Ramsey,[8] Cosyns,[9] and most recently by Hereford.[10] Of these, Hereford obtained results in substantial agreement with theory upon comparing the primary specific ionization of 1-Mev electrons with that of the sea-level cosmic radiation. This technique makes use of the unique dependence of the efficiency of a counter, operating in the Geiger region, upon the primary specific ionization,

$$\text{efficiency}=1-\epsilon^{-JLP/76}.$$

The present experiment makes use of this technique to compare the primary specific ionization in hydrogen of two groups of cosmic-ray particles of different average momenta. The efficiency of a low pressure (2.0 cm Hg) hydrogen-filled counter was measured at sea level and under ~140 feet of rock. These measurements were made with a fourfold coincidence telescope which included the hydrogen counter and 20 cm of lead. The average momentum of the sea-level cosmic radiation (presumed to be principally μ-mesons because of the Pb filter), computed on the basis of the combined data of Wilson, Blackett, and Jones,[11] is ~3500 Mev/c, while the average momentum under 140 feet of rock (effective thickness ~13,800 g/cm^2) is ~48,000 Mev/c. The ratio of the primary specific ionization underground (J_{140}) to that at sea level (J_0), computed from the measured efficiencies, is

$$(J_{140}/J_0)_{\exp}=1.17\pm0.03,$$

where the error indicated is the standard statistical error. The use of the ratio of the specific ionizations avoids the need for determining the average path length (L) through the counter

The theoretical values of J were determined from the expression,

$$\langle J\rangle=\int_{p\min}^{\infty} J(p)\cdot S(p)dp\bigg/\int_{p\min}^{\infty} S(p)dp,$$

where $J(p)$ is obtained from Bethe's theory, $S(p)$ is the momentum distribution of the mesons, and $\langle J\rangle$ is consequently the expected average ionization. The sea-level momentum distribution $S_0(p)$ was obtained directly from the data of Wilson, Blackett, and Jones,[11] while the underground spectrum $S_{140}(p)$ was computed from the sea-level data by taking into account the absorption in the rock. The integrations were performed numerically, and $p_{\min}$ represents the cutoff resulting from the 20 cm of Pb in the telescope. This was selected to be at ~400 Mev/c, which is at the minimum of the $J(p)$ distribution. The theoretical value of the ratio so obtained is

$$(J_{140}/J_0)_{\text{th}}=1.20$$

if the underground spectrum is determined on the basis of Bethe-Bloch absorption in the rock, and becomes 1.18 if the Halpern-Hall[12] correction is applied.

The agreement with theory obtained here is in direct contrast with the results obtained by Hazen,[5] who measured the primary ionization at sea level and under 100 feet of rock with the aid of a cloud chamber. It is our feeling that the cloud chamber technique has inherent limitations which prevent the accurate measurement of the primary ionization produced by extremely high energy particles.

A more detailed account of this experiment is in preparation and will be submitted for publication shortly.

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High Energy Elastic Proton-Deuteron Scattering

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EXPERIMENTAL results on elastic p-d scattering[1,2] at high energies have recently been reported. Since data on n-p and p-p scattering at corresponding energies are also available, the impulse approximation[3,4] may be applied to the analysis of the three-body experiments.

The impulse approximation is most conveniently expressed in terms of the distribution of momentum transfers, κ:

$$d\sigma_{pd}{}^{el}/d\kappa^2=\{|r_{np}{}^0+r_{pp}{}^0|^2+\tfrac{2}{3}|\mathbf{r}_{np}{}'+\mathbf{r}_{pp}{}'|^2\}S(\kappa). \quad (1)$$

$S(\kappa)$ is the "sticking" factor defined and evaluated in reference 4. The quantities, r, are the amplitudes for scattering the proton without spin flip ($r_{np}{}^0$ and $r_{pp}{}^0$) and with spin flip ($\mathbf{r}_{np}{}'$ and $\mathbf{r}_{pp}{}'$) of the target particle. The normalization of these amplitudes is chosen so that the two-body scattering cross sections are

$$d\sigma_{np}/d\kappa^2=|r_{np}{}^0|^2+|\mathbf{r}_{np}{}'|^2,\quad d\sigma_{pp}/d\kappa^2=|r_{pp}{}^0|^2+|\mathbf{r}_{pp}{}'|^2. \quad (2)$$

Note that the relation between κ and the center-of-mass angle in the two-body problem is $\kappa=k_0\sin(\theta_2/2)$ if k_0 is the momentum of the incident proton when the target particle is at rest. In the three-body scattering, the connection between κ and the center-of-mass angle is $\kappa=(4/3)k_0\sin(\theta_3/2)$. Thus $d\cos\theta_2/d\cos\theta_3=16/9$.

Formula (1) is general enough to include any kind of nuclear forces. In the special case of no coupling between spin and orbital angular momentum, the quantities r^0 and $\mathbf{r}'$ are connected to the familiar singlet and triplet amplitudes, r^s and r^t. according to

$$r^0=\tfrac{1}{4}(3r^t+r^s),\quad \mathbf{r}'=\tfrac{1}{4}(r^t-r^s)\boldsymbol{\sigma}_p,$$

where $\boldsymbol{\sigma}_p$ is the spin of the incident proton.

As discussed in reference 4, a knowledge of the two-body cross sections [Eq. (2)] is insufficient to evaluate formula (1) because the relative phases of the n-p and p-p amplitudes are needed, as well as the relative magnitudes of spin flip and nonspin flip amplitudes. These quantities depend on the detailed nature of the forces, which is still unknown. For purposes of illustration, however, we shall here evaluate (1) by making a very crude assumption. We assume S scattering only and no spin dependence except that forced by the Pauli principle in the p-p system. Even though the true picture is much more complicated than this, we may expect to find the right order of magnitude so long as the experimental n-p and p-p cross sections are used to fix the normalization. Also the angular distribution ought not to be far from the truth, since the most rapidly varying function in (1) is $S(\kappa)$, which is independent of the force assumption.

The preceding postulate leads to the following formula for the angular distribution in the center-of-mass system:

$$d\sigma_{pd}{}^{el}/d\omega_3=(16/9)\{d\sigma_{np}/d\omega_2+\tfrac{3}{4}d\sigma_{pp}/d\omega_2 +[(d\sigma_{np}/d\omega_2)(d\sigma_{pp}/d\omega_2)]^{\frac{1}{2}}\cos\Delta\}S(\theta_3), \quad (3)$$

where $d\omega_3=2\pi d\cos\theta_3$, $d\omega_2=2\pi d\cos\theta_2$, and Δ is the difference between n-p and p-p phase shifts. Since only S scattering has been assumed, we must take $d\sigma_{np}/d\omega_2$ and $d\sigma_{pp}/d\omega_2$ to be constants at the energy in question. This may seem unreasonable for n-p scattering, but actually the steepest part of the experimental n-p angular distribution is concentrated in a small solid angle near $\theta_2=180°$. For such large momentum transfers formula (1) is not valid in any case because the "pick-up" mechanism[5] becomes important.

Formula (3) has been evaluated for proton energies of 95 and 240 Mev, corresponding to the Berkeley[2] and Rochester[1] experiments, respectively. Table I gives the values of $d\sigma_{np}/d\omega_2$ and $d\sigma_{pp}/d\omega_2$ which were employed. These numbers are rough averages based on Berkeley measurements.[6] We have taken $\Delta=0$, although strictly speaking, this condition would imply $d\sigma_{np}/d\omega_2=d\sigma_{pp}/d\omega_2$, which is not quite consistent with Table I.

In Fig. 1 our results are compared with experiment. The agreement is about as good as expected. The fact that our theory is consistently too high except at very small angles no doubt means that the phase difference between n-p and p-p amplitudes is not negligible. (The small angle (<20°) theoretical prediction would be larger if we allowed $d\sigma_{np}/d\omega_2$ to increase there, as it actually does experimentally,[6] to twice the average value.) With a more realistic assumption for the fundamental two-particle

TABLE I. Differential n-p and p-p scattering cross sections.

Proton energy (Mev)	$d\sigma_{np}/d\omega_2$(mb/ster)	$d\sigma_{pp}/d\omega_2$(mb/ster)
95	5	4
240	3	4