nitudes can be estimated by observing that if the scattering length approaches infinity conditions of equivalence for scattering cross section at zero energy and for rate of change with energy give

$$r_0=r_0'+r_1',$$

where r_0 is the radius of the square well without core, while r_1' is the core radius and r_0' is the outer radius of the modified square well. If $r_0=2.7\times10^{-13}$ cm and $r_1'=1\times10^{-13}$ cm, then $r_0'=1.7\times10^{-13}$ cm. A shortening of range in the ratio 1.7/2.7 gives an increase of meson mass by the factors $\sim$2.7/1.7, which corresponds to a mass $\sim$500 m. Unpublished calculations of M. H. Hull and A. Herschman support the conclusion that the apparent meson mass is too high. They also find that the p-p and p-n interactions are in slightly poorer agreement with the core than without. An exact agreement with the measured mass is hardly to be expected, and reasons for differences of the order of 10 or 20 percent can be given.[2] The connection with meson theory becomes forced, however, if the disagreement is by a factor of about 2.

The approximate independence of the p-p scattering cross section on the scattering angle is usually made to appear as a result of the superposition of angle dependent effects. For a few Mev bombarding energy it is very probable that one deals with pure S-scattering. No definite effects have been observed which would indicate the setting in of p or d waves between 0.2 Mev and 30 Mev. There is an uninvestigated gap of energies between 30 Mev and 75 Mev. From here on[3] the scattering is again spherically symmetric. The standard reason for assuming that at the higher energies superposition of angle dependent effects takes place is that the cross section is too large for pure S-scattering.

There appears to be no compelling reason for supposing that at energies comparable with the rest mass energy of the π-meson the collision process does not change the nature of the protons. If, after collision, the protons are not identical, they can exist in ^3S as well as ^1S states. A statistical factor 4 is gained and the explanation of angular independence can again be reduced to dominance of S-scattering. The changed state of the proton need not be one of different mass. Any change, even that of an otherwise unobservable internal coordinate, would be satisfactory. It would be of interest to see whether scattered protons are identical with other protons in all respects. Effects depending on symmetry properties of the wave-function would be most decisive. In principle there is the possibility of rescattering. Polarization effects would have to be separated from effects sought for in such tests.

If collisions produce isomeric states it would be natural to suppose that the formation occurs through an intermediate state of the two-nucleon system, requiring for its formation a relative kinetic energy of the order of the meson mass energy. The observed flatness of the cross-section energy curve for p-p scattering would be the result of compensation of the decrease in scattering of identical protons and an increase in scattering of nonidentical ones. The view discussed above has been presented in part at the Chicago International Conference on Nuclear Physics and Fundamental Particles.

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Limitations on Mass Changes of Scattered Nucleons*

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In connection with the preceding note[1] it is of interest to determine the degree to which scattering experiments performed by the coincidence method exclude changes in mass of protons on scattering. The collision of two particles of equal mass M is considered. After the collision the mass of one of the particles is taken to be M and of the other M_1. In the reference system of zero momentum (rest system) the scattering angle for the particle of mass M is taken to be θ. The angle between the directions of motion of the two particles when observed in the reference system in which one of them is at rest before the collision (laboratory system) is denoted by

$$\chi=(\pi/2)-\delta.$$

For small values of $\Delta M/M=(M_1-M)/M$ and small velocities, conservation of energy and momentum gives the approximate value

$$\tan\delta=(\beta^2/2)\sin\theta+\cdots$$
$$+(\Delta M/2M)(1-\beta^2)^{\frac{1}{2}}[(\beta^{-2}/\sin\theta)+\cot\theta-\tfrac{1}{2}\sin\theta]+\cdots,$$

where $\beta=v/c$ and v is the velocity in the rest system before the collision. The first term is an approximation to the relativistic effect which makes χ differ from $\pi/2$ even if the masses are equal. The second term shows the effect of changing one of the masses. The incident energy in the laboratory is $2Mc^2\beta^2/(1-\beta^2)=T$. Introducing $\epsilon=T/(2Mc^2)$, one has $\beta^2=\epsilon/(1+\epsilon)$ and the effect of ΔM on $\tan\delta$ is

$$\delta'=[\Delta M/(2M\epsilon\sin\theta)]\{1+\epsilon[\cos\theta+\tfrac{1}{2}\cos^2\theta]\}.$$

Terms of relative order ϵ have been kept. For $T=300$ Mev the factor in braces is close to 1, and one has the approximation

$$\delta'=c^2\Delta M/(T\sin\theta).$$

If δ' is allowed to be 1°, then at 100-Mev bombarding energy one may suppose $c^2\Delta M=0.87$ Mev; while at 300-Mev, bombarding energy $c^2\Delta M=\pm2.6$ Mev would not be excluded for $\theta=30°$. This scattering angle corresponds to $\Theta=15°$. It is not clear from the publications on high energy scattering whether the relativistic angle relation is obeyed to better than 1°. The more significant tests are those having to do with small scattering angles. Presumably these are the least favorable for good geometry. Tests at $\Theta\cong45°$, $\theta=90°$ to 1° would limit the mass within $\sim\pm1.7$ Mev for 100-Mev incident energy. In either case the change in mass is not so small as to discourage tests.

* Assisted by the joint program of the ONR and AEC.

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Thermal Expansion and Specific Heat of Tungsten Oxide at High Temperatures

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We have previously reported[1] that tungsten oxide has ferroelectric properties and that its Curie point seems to be situated at about 710°C; these results followed from our observations of many of its properties, i.e., domain structure, hysteresis loop, permitivity, thermal expansion, specific heat, etc. Afterwards it was ascertained, by x-ray analysis,[2] that the crystal structure changes from orthorhombic to tetragonal ($a=b$) at 700$\sim$750°C. In addition to our previous investigations, we shall now report briefly the quantitative results of our observations of thermal expansion and specific heat above room temperature.

The thermal expansion was measured by a dilatometer of the rotating-rod type with a sensibility of $\Delta l/l=4\times10^{-6}$. Figure 1 shows the linear thermal expansion coefficient, measured by heating at a rate of about 2°C/min. A rather gradual expansion is observed at 330°C and a remarkable contraction at 755°C, the latter amounting to about $\Delta l/l=1.2\times10^{-3}$. On cooling, a temperature hysteresis exists for one anomaly but not for the other; that is, the anomalies of the thermal expansion coefficient in the cooling process are observed at 725°C and 330°C, respectively. We measured the specific heat by a vacuum calorimeter of the same type as the ones used by Bantle[3] for KH_2PO_4

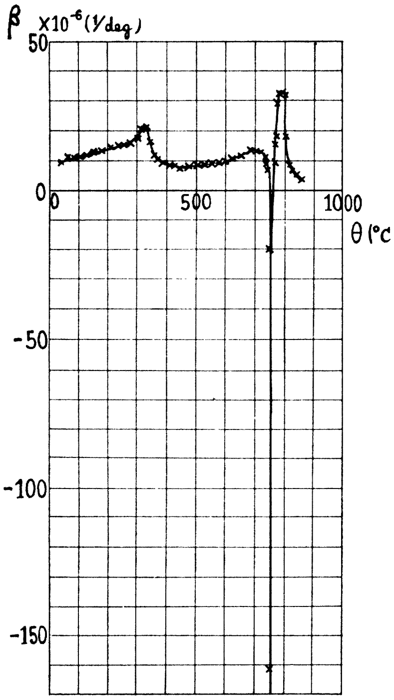

FIG. 1. Linear thermal expansion coefficient *vs* temperature.

and by Blattner *et al.*[4] for BaTiO$_3$. The specific heat was found to be normal near 330°C, and its behavior near 740°C, measured by heating at a rate of about 1°C/min, is shown in Fig. 2. The amount of anomalous heat capacity near 740°C is about 450 cal/mol, the corresponding entropy change being about 0.2 R, and the peak value is observed at 728°C. This temperature is lower than the one at which the thermal expansion coefficient shows a negative peak. Although a part of this discrepancy will be eliminated by the c_p-c_v correction, the residual part may be caused by defects of our apparatus. Previously the anomalies were observed at a lower temperature, i.e., at 710°C, probably owing to the impurity of our previous sample.

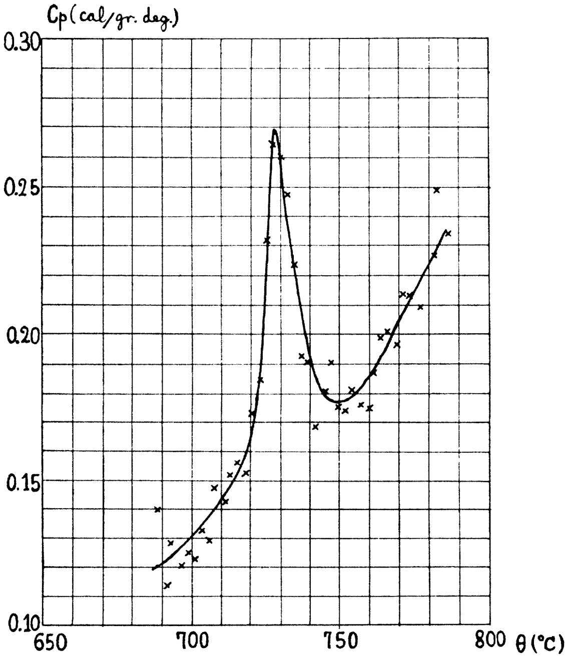

FIG. 2. Isobaric specific heat *vs* temperature.

Although the apparent behavior of the 740°C transition of tungsten oxide thus resembles closely that of the 120°C transition of barium titanate, its essential nature will be revealed only by further thorough investigations. Our observation of an anomaly in the thermal expansion near 330°C, on the other hand, agrees with an anomaly in the dc resistance observed by Nagasawa and Fukui,[5] but it seems yet to be doubtful whether this temperature is a transition point which is closely related to the ferroelectricity of the substance.

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Isotope Shifts in Erbium

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An investigation of the isotope shifts in erbium ($Z=68$) is being conducted in this laboratory using a Fabry-Perot interferometer for the necessary resolving power. The oxide is excited in a hollow cathode discharge cooled with liquid nitrogen, similar to the type described by Arroe and Mack.[1]

Three components of the isotope shift are clearly resolved in more than fifty lines in the region between 4250A and 6000A. The three components can be unambiguously attributed to the isotopes Er^{166}, Er^{168}, and Er^{170} on the basis of the intensity of the components compared with the relative abundances of the isotopes. Natural erbium contains six isotopes: Er^{162} (0.136 percent), Er^{164} (1.56 percent), Er^{166} (33.4 percent), Er^{167} (22.9 percent), Er^{168} (27.1 percent), and Er^{170} (14.9 percent). The odd isotope, Er^{167}, is reported[2] to have a nuclear spin of 7/2. Despite its relatively large abundance, the components caused by the odd isotope are not resolved, and this may be attributed to the large number of components into which the 167 component is split as a result of its magnetic hyperfine structure. In only a few lines do the intensities deviate from the expected values sufficiently to be attributed to the presence of the odd isotope.

In a few overexposed lines a fourth component has been detected; on the basis of intensity and position it could be attributed to Er^{164}, although it is also possible that it is the result of Er^{167}. Measurements have not yet been completed on this component. Table I shows the results of measurements on 11 lines using four different spacer sizes. The individual measurements are probably accurate to within ±0.0015 cm^{-1}, and the mean is accurate to within ±0.0010 cm^{-1}. These limits do not include the possibility of disturbing effects as a result of the odd isotope. The ratio $(\nu_{168}-\nu_{170})/(\nu_{166}-\nu_{168})$ is in most cases quite close to unity, and as far as the measurements have proceeded, there is no evidence that the ratio is other than unity.

The shifts have been taken as positive where the heaviest isotope has the smallest wave number, and negative where the converse is true. To the best of our knowledge, the occurrence of both positive and negative shifts in rare earth spectra has not been reported before. According to the nuclear volume picture of isotope shifts,[3] s-electrons are more tightly bound in light isotopes than in heavy isotopes. Since electrons with greater orbital angular momentum (except $p_{\frac{1}{2}}$) are not affected appreciably by the nuclear charge distribution, observable isotope shifts appear only in transitions between levels which have different numbers of