



FIG. 1. Birefringence pattern of a noncounting diamond.

with the aid of a petrographic microscope with a Federov rotation stage, in an investigation largely resembling that of Ramachandran. This disclosed a mosaic structure of most of the noncounting diamonds, instead of the laminated structure found by Raman and Ramachandran.<sup>4</sup> The counting diamonds displayed a very poor pattern of birefringence, contrary to the noncounting specimens. This is illustrated by Figs. 1 and 2. The first photograph gives a birefringence pattern of a noncounting specimen. This photograph was taken with the polarizer and analyzer of the microscope set in the directions of the bisectors of the angle of 70° between the streaks of birefringence. The second picture shows a few streaks of birefringence in a counting diamond. Using this experience to eliminate noncounting diamonds from the collection (already selected as stated by transmission in the ultraviolet) it is found that 75 percent of the remaining crystals are counters.

From this investigation the following conclusions can be drawn:

(1) The counting property is a property of the crystal itself and not a consequence of the imperfection of the crystal, as was suggested by Lonsdale.<sup>5</sup> This is proved by the high counting efficiency of some of the diamonds and by the (relative) perfection of the counters.

(2) The laminations observed by Raman, Rendall, and Ramachandran<sup>4</sup> are in fact a well-ordered mosaic superstructure of small octahedrons. This view is supported by another experiment with two diamonds in which the variation of birefringence was observed along the four directions normal to the octahedron surfaces.

(3) The selection of counters is improved from about 30 percent to about 75 percent if the absence of streaks of birefringence is used as a criterion together with the ultraviolet transparency. This improvement is obtained with a Federov rotation stage to obtain the correct inclination of the crystal and a petrographic microscope.

The investigation was completed with the comparison of luminescence of the 36 diamonds. A large variety of intensity and of color (blue in most cases) was observed. In general this result



FIG. 2. Birefringence pattern of a counting diamond (with a few laminations).

supported the view taken by Pringsheim<sup>6</sup> that luminescence in diamonds will be a consequence of impurities. However, this is contrary to the experience of Raman and co-workers who found absence of luminescence in this kind of diamond (of type II) and presence of blue luminescence only in ultraviolet nontransparent specimens (of type I).<sup>4</sup> Moreover, the suggestion of Frerichs<sup>7</sup> on the correlation between counting and luminescence obtained with irradiation at 2250 Å was not confirmed.

A detailed report of this investigation will be published elsewhere<sup>8</sup> in the near future. The authors are indebted to Professor Milatz for his highly valued and stimulating interest in this investigation.

<sup>1</sup> G. Stetter, *Verhandl. deut. phys. Ges.* **22**, 13 (1941); W. Jentschke, *Phys. Rev.* **73**, 77 (1948); D. E. Wooldridge, *et al.*, *Phys. Rev.* **71**, 913 (1947); L. F. Curtiss and B. Brown, *Phys. Rev.* **72**, 643 (1947); A. J. Ahearn, *Phys. Rev.* **73**, 1113 (1948); D. R. Corson and R. R. Wilson, *Rev. Sci. Instr.* **19**, 1467 (1948); H. Ess and J. Rossel, *Helv. Phys. Acta* **23**, 484 (1950); and **24**, 247 (1951).

<sup>2</sup> Friedman, Birks, and Gauvin, *Phys. Rev.* **73**, 186 (1948).

<sup>3</sup> G. P. Freeman and H. A. van der Velden, *Physica* **17**, 565 (1951).

<sup>4</sup> C. V. Raman, *et al.*, *Proc. Indian Acad. Sci., Symposium on the Properties of Diamond (I)* **19A** (1944); *Symposium II*, **24A** (1946).

<sup>5</sup> K. Lonsdale, *Phys. Rev.* **73**, 1467 (1948).

<sup>6</sup> P. Pringsheim, *Fluorescence and Phosphorescence* (Interscience Publishers, Inc., New York, 1949), p. 648.

<sup>7</sup> R. Frerichs, *J. Opt. Soc. Am.* **40**, 219 (1950).

<sup>8</sup> G. P. Freeman and H. A. van der Velden, *Physica*, in preparation.

## Angular Distribution of $\gamma$ -Radiation from Polarized Nuclei

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THE intensity of  $\gamma$ -emission from polarized nuclei is expected to show a dependence on the angle between the direction of emission and the axis of polarization. Spiers<sup>1</sup> has given the dependence for a single emission for arbitrary degree of polarization, and considerably simpler formulas valid for low degrees of polarization. These latter have been extended by the present author to apply to a cascade of  $\gamma$ -rays from an oriented  $\gamma$ -emitting nucleus and to a cascade of  $\gamma$ -rays following a  $\beta$ -emission from an oriented  $\gamma$ -emitting nucleus. It is assumed that (a) the half lives of the intermediate states are sufficiently short, (b) only "pure" radiations are involved, i.e., no mixture of electric and magnetic radiation in the case of  $\gamma$ -emission, or, in the case of  $\beta$ -emission, no mixtures of  $j$ -values, where  $j$  is the total angular momentum of the  $\beta$ -neutrino system,<sup>2</sup> (c) the multipole order of any  $\gamma$ -transition  $L = |J_i - J_f|$ , where  $J_i$  = initial spin and  $J_f$  = final spin. A number of methods for polarizing nuclei are discussed by Bleaney.<sup>3</sup>

Let  $w(M_0)$  be the mean population of states of the initial nucleus, of spin  $J_0$  and  $z$ -component  $\pm M_0$ . For low degrees of polarization it can be expanded in powers of  $M_0^2$ .<sup>\*</sup> Taking only the term in  $M_0^2$ ,  $w(M_0)$  is in all cases of the form

$$w(M_0) = \{1 + \frac{1}{2}f[3M_0^2 - J_0(J_0 + 1)]\} / (2J_0 + 1).$$

If the polarization is accomplished by a magnetic field,  $H$ , acting on the nuclear magnetic moment,  $\mu$ ,

$$f = \frac{1}{2} \{H\mu/kTJ\}^2$$

where  $k$  is Boltzmann's constant and  $T$  is the absolute temperature. If it is accomplished by a crystalline field,  $E$ , acting on the quadrupole moment,<sup>4</sup>

$$f = 3eQ \text{ grad} E / [kT2J_0(J_0 - 1)],$$

in the notation of reference 4.

By using this approximation the following simple expressions for angular distribution can be obtained, applying equally to either electric or magnetic radiation. Investigation shows that they should be valid for  $|f| \leq 1/2J_0(J_0 + 1)$ .

Case 1, a single  $2^L$ -pole  $\gamma$ -emission, spin change  $J_0 \rightarrow J_1$ :

The angular distribution,  $I(\theta)$ , is

$$I(\theta) = 1 - \frac{1}{2}f\alpha(L)\hbar(J_0, J_1)(1 - 3\cos^2\theta),$$

where

$$\begin{aligned} h(J_0, J_1) &= J_0(2J_0 - 1) & \text{if } J_0 < J_1, \\ h(J_0, J_1) &= (J_0 + 1)(2J_0 + 3) & \text{if } J_0 > J_1, \\ \alpha(L) &= [3 - L(L + 1)] / [(L + 1)(2L + 3)]. \end{aligned}$$

The asymmetry factor to this order of approximation is

$$\epsilon = [I(0) - I(\pi/2)] / I(\pi/2) = \frac{1}{2} f \alpha(L) h(J_0, J_1).$$

Note that for  $L=1$  (dipole),  $\alpha=1/10$ , for  $L=2$  (quadrupole),  $\alpha=-1/7$ , and for all higher multipoles  $\alpha(L) < 0$ , so that in the case of magnetic polarization ( $H\mu$ ) all emissions have an excess in the equatorial plane with the exception of dipole emission. In the case of polarization by an electric field this effect will depend on the sign of the quadrupole moment.

Case 2, two  $\gamma$ -rays in cascade,  $2^{L_1}, 2^{L_2}$ -pole,  $J_0 \xrightarrow{\gamma_1} J_1 \xrightarrow{\gamma_2} J_2$ :

The angular distribution of  $\gamma_1$  is calculated as for single emission. The angular distribution of  $\gamma_2$  is

$$I^{(2)}(\theta) = 1 - \frac{1}{2} f g(J_0, J_1) \alpha(L_2) h(J_1, J_2) (1 - 3 \cos^2 \theta),$$

where

$$\begin{aligned} g(J_0, J_1) &= [J_0(2J_0 - 1) / J_1(2J_1 - 1)] & \text{if } J_0 < J_1, \\ g(J_0, J_1) &= [(J_0 + 1)(2J_0 + 3) / (J_1 + 1)(2J_1 + 3)] & \text{if } J_0 > J_1. \end{aligned}$$

The observed angular distribution of both  $\gamma$ -rays will be  $I(\theta) = \delta_1 I^{(1)}(\theta) + \delta_2 I^{(2)}(\theta)$ , where  $\delta_1$  and  $\delta_2$  are the relative efficiencies of the detecting device for the two  $\gamma$ -rays. The observed asymmetry for both  $\gamma$ -rays is

$$\epsilon = \frac{1}{2} f [ \{ \delta_1 / (\delta_1 + \delta_2) \} \alpha(L_1) h(J_0, J_1) + \{ \delta_2 / (\delta_1 + \delta_2) \} \alpha(L_2) g(J_0, J_1) h(J_1, J_2) ].$$

Case 3, three  $\gamma$ -rays in cascade,  $2^{L_1}, 2^{L_2}, 2^{L_3}$ -pole,

$$J_0 \xrightarrow{\gamma_1} J_1 \xrightarrow{\gamma_2} J_2 \xrightarrow{\gamma_3} J_3:$$

These formulas readily extend to three or more emissions.  $I^{(1)}(\theta)$  and  $I^{(2)}(\theta)$  are calculated without regard to  $\gamma_3$  and

$$I^{(3)}(\theta) = 1 - \frac{1}{2} f g(J_0, J_1) g(J_1, J_2) \alpha(L_3) h(J_2, J_3) (1 - 3 \cos^2 \theta).$$

Case 4, a single  $2^L$ -pole  $\gamma$ -ray following a  $\beta$ -particle, spin change,

$$J_0 \xrightarrow{\beta} J_1 \xrightarrow{\gamma} J_2:$$

The angular distribution of the  $\gamma$ -ray is

$$I(\theta) = 1 - \frac{1}{2} f k(J_0, J_1) \alpha(L) h(J_1, J_2) (1 - 3 \cos^2 \theta)$$

where  $k(J_0, J_1) = g(J_0, J_1)$  if  $j = |J_0 - J_1|$ ,  $j$  being the total angular momentum of the  $\beta$ -neutrino system, and

$$k(J_0, J_1) = \frac{(2J_0 - 1) [(J_1 + 1)^2 - j(J_1 + 4)]}{(2J_1 - 1) [J_1(J_1 + 1)]} \quad \text{if } j = J_1 - J_0 + 1.$$

$$k(J_0, J_1) = \frac{(2J_0 + 3) [J_1^2 + j(J_1 - 3)]}{(2J_1 + 3) [J_1(J_1 + 1)]} \quad \text{if } j = J_0 - J_1 + 1.$$

The asymmetry factor is  $\epsilon = \frac{1}{2} f k(J_0, J_1) \alpha(L) h(J_1, J_2)$ . Note that the only effect of the  $\beta$ -emission on the asymmetry is to modify it by the factor  $k$ .

Case 5, two  $\gamma$ -rays following a  $\beta$ -particle,  $2^{L_1}, 2^{L_2}$ -pole, spin change  $J_0 \xrightarrow{\beta} J_1 \xrightarrow{\gamma_1} J_2 \xrightarrow{\gamma_2} J_3$ :

The angular distribution of  $\gamma_1$ ,  $I^{(1)}(\theta)$ , is calculated without regard to  $\gamma_2$ , and the angular distribution of  $\gamma_2$  is

$$I^{(2)}(\theta) = 1 - \frac{1}{2} f k(J_0, J_1) g(J_1, J_2) \alpha(L_2) h(J_2, J_3) (1 - 3 \cos^2 \theta).$$

The asymmetry for the two observed  $\gamma$ -rays is

$$\epsilon = \frac{1}{2} f k(J_0, J_1) [ \{ \delta_1 / (\delta_1 + \delta_2) \} \alpha(L_1) h(J_1, J_2) + \{ \delta_2 / (\delta_1 + \delta_2) \} \alpha(L_2) g(J_1, J_2) h(J_2, J_3) ].$$

It should be noted that in all cases if  $J_0 = \frac{1}{2}$  or 0, all emissions have spherical symmetry; and if  $J_1 = \frac{1}{2}$  or 0, the second, third, and all subsequent emissions have spherical symmetry, and so forth.

It will be seen, from the behavior of  $\alpha(L)$  that high multipole orders favor large asymmetries and that asymmetries for transi-

tions in which the spin decreases are larger than for those in which the spin increases.

The author is indebted to Dr. J. A. Spiers for advice on this problem.

<sup>1</sup> J. A. Spiers, National Research Council of Canada publication No. 1925.

<sup>2</sup> R. J. Blin-Stoyle and J. A. Spiers, Phys. Rev. **82**, 969 (1951).

<sup>3</sup> B. Bleaney, Proc. Phys. Soc. (London) **A64**, 315 (1951).

\* The derivation is straightforward when  $M_0$  is a "good" quantum number. It can be shown that an expansion of this form is also valid in the more complex cases that can arise in paramagnetic salts.

<sup>4</sup> R. V. Pound, Phys. Rev. **79**, 685 (1950).

## Remarks Concerning the Existence of the Foldy-Wouthuysen Transformation\*

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FOLDY and Wouthuysen<sup>1</sup> have shown for a free Dirac particle that there exists a unitary transformation which removes all odd operators<sup>2</sup> from the Hamiltonian. In the case where there is an electromagnetic field present such a transformation was not found explicitly, and a power series expansion in inverse powers of the particle mass was given for the unitary transformation and for the transformed Hamiltonian. These authors raised questions concerning the convergence of these series but did not answer them.

It is the purpose of this note to show that a complete transformation of the Foldy-Wouthuysen type does not in general exist, and thus, in some circumstances these power series cannot converge. This is done by showing that the existence of the Foldy-Wouthuysen transformation is incompatible with known properties of the solution of Dirac's equation in the presence of external electromagnetic fields. To do this we consider the case where the electromagnetic field is present only in a finite portion of space-time. If we consider any solution of the Dirac equation which has only positive (negative) frequency parts at times before there was any electromagnetic field present, then at times after the electromagnetic field has disappeared the solution will have both positive and negative frequency parts. If the Foldy-Wouthuysen transformation exists, the original Dirac equation is decomposed into two sets of uncoupled two-component equations. For times earlier or later than the region in which the electromagnetic field is present, this transformation insures that one of these two sets of equations corresponds to positive, the other to negative frequency solutions of Dirac's equation. Since these two sets of equations are uncoupled, a solution of Dirac's equation which contains only positive (negative) frequency components before the time of interaction would contain only positive (negative) frequency components after the time of interaction. This is not true and one may therefore conclude that the complete Foldy-Wouthuysen transformation does not exist under circumstances where pair production by an external field is possible. This does not, of course, mean that a few terms of the power series of Foldy-Wouthuysen are not useful and valid for the description of the nonrelativistic aspects of the Dirac equation.

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<sup>1</sup> L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).

<sup>2</sup> For definition of odd and even operators see reference 1.

## Penetrating Showers in Copper

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THE penetrating showers produced by cosmic radiation in various materials have been studied extensively with Geiger-Müller counters. Since little or no work on showers produced in copper has been reported, measurements were taken