# Measurement of the Magnetic Moment of the Proton in Bohr Magnetons*

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An experimental determination of the ratio of the precession frequency of the proton, $\omega_p=\mu_p H_0$, to the cyclotron frequency, $\omega_e=eH_0/mc$ of a free electron in the same magnetic field has been carried out. The result, $\omega_p/\omega_e$, is the magnitude of the proton magnetic moment, $\mu_p$, in Bohr magnetons, since $\gamma_p=2\mu_p/\hbar$.

Possible sources of systematic error were carefully investigated and in view of the results of this investigation and the high internal consistency of the data it is felt that the true ratio, uncorrected for diamagnetism, lies within the range $\omega_e/\omega_p=657.475\pm0.008$. If the diamagnetic correction to the field at the proton for the hydrogen molecule is applied here the proton moment in Bohr magnetons becomes $\mu_p=(1.52101\pm0.00002)\times10^{-3}(e\hbar/2mc)$.

## 1. INTRODUCTION

In this experiment the magnetic moment of the proton, $\mu_p$, is obtained in terms of the Bohr magneton, $\mu_0=e\hbar/2mc$. The cyclotron frequency in the microwave region of the free electron in a magnetic field, $\omega_e=eH/mc$, is measured by the resonance method, and the magnetic field is measured in terms of the proton moment, $\omega_p=2\mu_p H/\hbar$, by the nuclear resonance absorption method.

In addition to the importance of the ratio $\mu_p/\mu_0$ as a fundamental physical constant its significance as here measured is increased by its ability to provide further experimental confirmation to recent developments in quantum electrodynamics and by its utility in the calculation of the constants $\alpha$, $e/mc$, and $M/m$.

In 1947 Kusch and Foley[1] measured, by the method of atomic beams, the ratios of the $g$-factors of various atomic states and showed that the spin $g$-factor is not precisely twice the orbital $g$-factor. This anomaly was explained by Schwinger[2] who showed that a radiative correction to the magnetic moment of the electron in an external magnetic field was necessary. This correction gives the value

$$g_s/g_0=2(1+\alpha/2\pi-2.973\alpha^2/\pi^2)=2(1.001147)$$

to fourth order in $e$. Kusch and Foley's experimental value of this quantity is $g_s/g_0=2(1.00119)$.

More recently Taub and Kusch compared the proton $g$-factor, $g_p$, with the $g$-factors of the ground states of indium and of cesium, which in turn were known in terms of the $g$-factor of the ground state of sodium from the experiments of Kusch and Foley and of Kusch and Taub.[3] Since the $g$-factor of the ground state ($^2S_{\frac{1}{2}}$) of sodium is simply the spin $g$-factor of the electron $g_s$[3] one can calculate the ratio of the proton moment to the Bohr magneton by using either the experimental or theoretical value of $g_s/g_0$ as follows:

$$\frac{g_p}{(g_J)_{\mathrm{In}}}\times\frac{(g_J)_{\mathrm{In}}}{(g_J)_{\mathrm{Na}}}\times\frac{(g_J)_{\mathrm{Na}}}{g_0}=[g_s]=\frac{g_p}{g_0}=\frac{\mu_p}{(\frac{1}{2}e\hbar/mc)}. \quad (1)$$

The experiment described in this paper permits the direct measurement of $g_p/g_0$ using free electrons rather than electrons bound in atoms. One can therefore calculate $g_s/g_0$ where $g_0$ here refers to free electrons, to give an additional experimental confirmation of the electron spin magnetic moment anomaly.

## 2. THE ELECTRON RESONANCE METHOD

The general problem is to measure the cyclotron frequency $\omega_e$, of electrons in a magnetic field by subjecting them to an alternating electric field transverse to the magnetic field. In order to obtain a sharp resonance effect as the frequency, $\omega$, of the electric field is varied through the frequency $\omega_e$, the electrons must have long transit times through the field and $\omega$ and hence $\omega_e$ must be as high as possible. Long transit times correspond to slow electrons and long paths. This suggests the use of thermal velocity electrons, or some means of velocity selection to obtain even lower velocities.

The cyclotron frequency, $\omega_e$, is limited by the magnetic fields obtainable and the electric field frequency, $\omega$, is limited by the equipment and techniques available. We chose 9400 mc/sec for $\omega$, largely because of the immediate availability of equipment working at this frequency. The corresponding magnetic field is about 3300 gauss.

To detect the resonance one could use some means of observing the reaction of the electron resonance on the microwave circuit. We chose the more direct method of observing the effect of the resonance on the current through a circuit involving the electron beam.

A ribbon-shaped beam of slow electrons drifting parallel to the magnetic field is formed by a slit. The beam is kept from expanding by the field inasmuch as the helical trajectories of the individual electrons have axes parallel to the field and radii determined by their initial transverse velocities, assuming no interaction

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[1] P. Kusch and H. M. Foley, Phys. Rev. 74, 250 (1948).
[2] J. Schwinger, Phys. Rev. 73, 416 (1948); R. Karplus and N. M. Kroll, Phys. Rev. 77, 536 (1950).
[3] P. Kusch and H. Taub, Phys. Rev. 75, 1477 (1949).

between the electrons. A ribbon rather than a pencil beam is used to minimize the effect of this interaction on the resonant frequency. This will be discussed later. This beam is subjected to a weak transverse microwave electric field. At resonance the beam expands since the effect of the alternating electric field is then to increase the transverse velocity of the electrons cumulatively so that the radii of their orbits increase.

The beam is selected by a rectangular slit on one side of the short dimension of a $1\times\frac{1}{2}$-in. rectangular wave guide. The wave guide propagates only the $TE_{10}$ mode so that inside the guide the beam is expanded by the transverse electric field. A second slit on the opposite side of the guide permits the unexpanded beam to pass through and be collected at an anode but fails to admit all the expanded beam. Hence one could recognize the resonance condition by a decrease in the current at the anode.

The radius of the electron orbits under the influence of the magnetic field alone is given by

$$r=cmv/eH. \tag{2}$$

For a field of 3300 gauss and thermal velocity electrons ($\sim 2\times 10^7$ cm/sec) this is approximately 0.003 mm. The width of the slit is 0.1 mm or approximately 30 electron-orbit radii. In the presence of the microwaves if this initial radius is neglected the radius of the orbit after the electron has traveled across the guide of width $a$ with a velocity $u$ parallel to the field is given by

$$r_a=\left|\frac{eEa}{m\omega\pi u}\frac{\cos(\frac{1}{2}a\delta\omega/u)}{1-(a\delta\omega/\pi u)^2}\right|, \tag{3}$$

where $\delta\omega=\omega-\omega_e$ is the difference between the microwave frequency and the cyclotron frequency and $E$ is the maximum amplitude of the microwave electric field. This formula assumes standing waves in the guide excited in the $TE_{10}$ mode. At resonance ($\delta\omega=0$), for the radii to expand to approximately the original width of the beam upon traversal of the guide, an electric field of 0.03 esu is required. This corresponds to a power of 20 milliwatts propagating into the guide if the beam is at a voltage maximum. For $a\approx 2.3$ cm, thermal electrons revolve about 1000 times during their transit through the guide.

If one assumes that the only factor serving to diminish the number of electrons arriving at the anode as resonance is passed through is the action of the second slit which discriminates against electrons with large radii one obtains for the current at the anode as a function of the initial velocities of the electrons,

$$I(\alpha)=I_0(\alpha)\left(1-\frac{AeE}{m\omega w}\left|\frac{\cos(\frac{1}{2}\pi\delta\omega/\alpha)}{\alpha(1-(\delta\omega/\alpha)^2)}\right|\right), \tag{4}$$

where

$$I_0(\alpha)=2N(\alpha)eLw;\quad \alpha=\pi u/a. \tag{5}$$

$2w$ is the width of the slit, $L$ the length, and $N(\alpha)$ is the number of electrons with velocity between $u$ and $u+du$. $A$ is either 1 or $1/\pi$, depending upon whether one assumes that no electron striking the wall of the slit is collected or all electrons entering the slit are collected, respectively. The full width at half-value, $I_{\frac{1}{2}}$, of the dip in current as resonance is passed through may readily be calculated for electrons all having the same velocity to be

$$I_{\frac{1}{2}}=3.30\alpha/2\pi. \tag{6}$$

For thermal velocity electrons (approximately $2\times 10^7$ cm/sec) and a guide width of 2.3 cm, this is of the order of 14 mc or 4.3 gauss. If one were to take into account the maxwellian distribution of initial velocities in calculating $I_{\frac{1}{2}}$ this value would be somewhat reduced.

It may be noted that the formula for the current admits the possibility of side lobes on the current dip and these were observed in the experiment. One would expect the first return of the current to full value after passing resonance to occur for that frequency at which the electrons gain energy from the field during the first half of their journey across the guide but completely lose it again by the time they reach the second slit. Of course, these side lobes would be smoothed out by a maxwellian velocity distribution in the beam.

In the preceding discussion the effect of the interaction of the electrons on each other was completely neglected. Under certain conditions of operation this space charge effect becomes of considerable importance and in fact provides a means of detection of the resonance which depends to a large extent upon the more slowly moving electrons in the beam. In this sense it provides velocity selection. By making use of this effect we were able to make measurements with resonance widths of the order of 0.3 gauss. This will be discussed after the apparatus is described.

The effect of the space charge on the resonant frequency must be considered. Each electron is moving in the average field due to the other electrons. If one assumes that the only electrons contributing to a change in the resonant frequency are those inside a pencil beam whose circumference is defined by the orbit of the electron, the effect of the others being averaged out over many cycles, then the force on this electron, neglecting the microwaves, is

$$F=mr\omega^2=(Hev/c)-eE. \tag{7}$$

The field due to this cylindrical distribution of charge is

$$E=2\pi r\rho, \tag{8}$$

where $\rho$ is the space charge density and $r$ is the radius of the electron orbit. Then

$$m\omega^2=(He\omega/c)-e2\pi\rho. \tag{9}$$

Solve for $\omega$

$$\omega=\frac{eH}{2mc}\pm\frac{1}{2}\left(\left(\frac{eH}{mc}\right)^2-\frac{8\pi\rho e}{m}\right)^{\frac{1}{2}}. \tag{10}$$

If we assume the space charge effect to be small, we

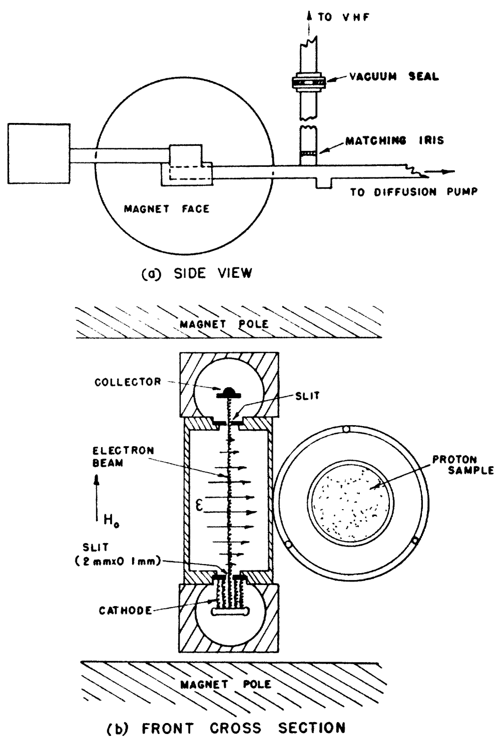



FIG. 1. Electron resonator.

have approximately

$$\omega=\frac{eH}{mc}\left(1-\frac{2\pi pe/m}{(eH/mc)^2}\right)=\omega_0\left(1-\frac{2\pi pe/m}{\omega_0{}^2}\right). \quad (11)$$

### 3. ELECTRON RESONANCE APPARATUS

The electron resonance apparatus consisted essentially of $1\times\frac{1}{2}$-in. shorted wave guide with a 0.080 $\times$0.004-in. slit in the narrow dimension of the guide on either side located at a voltage maximum $\lambda/4$ (0.440 in.) back from the short. An indirectly heated oxide-coated cathode was located just outside of the slit on one side and a collector plate was similarly located near the other slit. These elements were contained in a copper sheath and a $\frac{3}{8}$-in. hole between the sheath and the guide on either side allowed rapid evacuation. The holes were located $\lambda/2$ back from the short to avoid disturbing appreciably the rf currents in the guide (see Fig. 1). The flat rectangular cathode was heated by a tungsten filament folded back and forth several times and stuffed inside the cathode sheath. Batteries supplied the filament current.

The tube was evacuated to a pressure of $2\times 10^{-6}$ mm Hg by an oil diffusion pump (Distillation Products, Inc., VMF-20-W) backed by a Cenco Hyvac. This was attached at the end of the guide opposite to the location of the short. The microwaves were introduced between these through a $T$. These were kept out of the vacuum pump end by means of a stub designed to present an apparent short circuit adjacent to the $T$ on the pump side. A Corning 707 glass window sealed to Kovar was soldered to the guide between the $T$ and the oscillator by means of an asymmetrical $E$ plane iris. The pressure was measured by means of an ionization gauge made from a 24-C(3-C-24) triode.

The rf source was a 723-A klystron in a TS-13/AP test set. An attenuator in the output provided a convenient control of the power available at the position of the electron beam.

The wave guide was maintained at some voltage $V_a$ relative to the cathode. This is an important parameter in the operation of the tube. The collector plate was maintained at approximately 20 volts positive relative to the guide so as to collect all of the electrons that came through the second slit. The anode current could be measured by means of a galvanometer or be amplified and displayed on a CRO. A simple audio amplifier was used for this latter purpose.

The magnetic field was provided by a water-cooled electromagnet. The magnetizing current was furnished by storage batteries. The pole faces of the electromagnet were 8 in. in diameter and were separated by a 2-in. gap. The gap was uniform aside from 0.062-in. thick by 0.320-in. wide raised rims at the perimeters of the pole faces which were provided to correct for edge effects. Coarse control of the magnetic field was effected by a sliding rheostat made from strips of Advance alloy connected in series with the magnet. Fine control was provided by an auxiliary coil around a pole cap.

A second auxiliary coil on the pole cap of the magnet provided 60-cycle modulation of the magnetic field when the resonance was to be displayed on the CRO. The gross magnetic field was set approximately at the resonance value and this small modulation carried the field back and forth through the resonance. The same variac that supplied the current to this coil provided the CRO sweep. A phase-shifting network in this circuit made it possible to adjust the phase of the sweep relative to that of the modulation.

### 4. OPERATION OF THE ELECTRON RESONATOR

The electron apparatus was first operated with the voltage $V_a$ between guide and cathode at approximately 5 volts and above (guide positive with respect to cathode) and the anode current was observed with a sensitive galvanometer (sensitivity approximately 0.04 microampere per millimeter deflection). Under these circumstances a dip in the current was observed as the magnetic field passed through resonance and the width of the dip turned out to be approximately that predicted by the foregoing equations. Side lobes were observed as predicted by Eq. (4). The current was of the order of magnitude of 0.4 microampere and the dip brought it to approximately 0.3 microampere.

When the anode current, $i_p$, was amplified and displayed on the CRO it was found possible to obtain peaks in the current rather than dips if $V_a$ was less than about 3 volts. These peaks were considerably narrower than the dips, being of the order of magnitude of 0.5 gauss as opposed to approximately 5 gauss for the dips. The anode current under these circumstances was something less than 0.0005 microampere when away from resonance and could be made to go several hundred times this value at resonance.

This behavior can be understood by noting that owing to the presence of the magnetic field to a first approximation one has in effect two infinite plane diodes in series. The magnetic field confines the electrons to a cylindrical sheath thus making it possible to obtain an approximate one-dimensional treatment of the space charge problem. Since the static electric field inside the guide is due to the electrons alone a potential minimum is present in this region and space charge limitation may exist here. As one passes through reonance the space charge limitation, and the current through the tube, $i_p$, shows a peak. This is evidently very sensitive to the character of the potential minimum due to the space charge. The electrons contributing most to the potential minimum are those moving with small velocities at the minimum, and these are the ones which are acted on for a long time by the microwaves, with the result that the resonance width is very small.

For the plane one-dimensional case[4] space charge limitation may exist for $V_a$ between about 0.7 and 2 volts with an anode current of $10^{-9}$ ampere which is roughly the value obtained. This checks as well as could be expected with the range of $V_a$ over which the peak in anode current may be observed.

Inasmuch as this space charge detection was made use of in the measurements the remainder of the discussion will have reference only to this type of detection.

The important parameters studied in connection with the resonance were the effect of the microwave power level and the effect of the voltage, $V_a$.

The microwave power input could be varied over a wide range. It was observed that if one started with the smallest power for which the resonance was observable and increased the power the magnitude of the resonance peak would increase very rapidly. The width would remain constant for low power but would eventually beging to broaden. Side lobes would develop. In some cases the region in the neighborhood of the peak would begin to invert for very large power. This effect may be due to the fact that some electrons getting through the potential minimum were spiraling to large enough radii to miss the second slit. This explanation is not entirely satisfactory because the dip in the peak was less than one gauss wide and one would expect a width of about 3 gauss in this case. It is very likely possible, however, to narrow the width of the dip for this sort of situation by misaligning the two slits so that the expanded beam does not completely fill the second slit. Then, of those electrons reaching the second slit, only those with the smaller velocities would be effective in changing the current through the tube. This type of arrangement would amount to velocity selection for those electrons getting past the potential minimum.

[4] Fay, Samuel, and Shockley, Bell System Tech. J. **17**, 49 (1938).

The voltage $V_a$ between the wave guide and the cathode controls the initial velocities of the electrons in the beam and also the beam current. The value of this voltage was known only with an additive constant since the contact potential was not known. A plot of the current to the collector, $i_p$, (anode current) and of the current entering the first slit as a function of $V_a$ appears in Fig. 3. The values of the current entering the first slit are only approximate and relative. It was assumed that the ratio of this current to the anode current was approximately proportional to the height of the resonance curve for the power high enough to produce saturation, a situation in which space charge limitation is not present and all the electrons passing the first slit are collected at the anode. The values in the graph are really the heights of the resonance curve, measured under these limiting conditions.

The range of $V_a$ used was about 0 to 3 volts. The resonance width did not vary appreciably with $V_a$, and in fact the resonance appeared to be at its narrowest for high $V_a$. It is possible that this is due to the fact that the potential minimum is sharper and more critical for large $V_a$.

An interesting effect occurs as one increases $V_a$ (refer to Fig. 2). With the magnetic field due to the filament opposing the gross magnetic field, increasing with small $V_a$ of less than 1.0 volt merely increases the height of the resonance peak but on going from 1.0 to 1.2 volts the magnetic field at which the peak occurs shifts over by about 1 part in 10,000. No further shift takes place upon increasing beyond 1.2 volts. The height of the peak does not at first change appreciably, and in fact decreases to some extent, till it eventually begins to rise again and the resonance broadens. The locus of the peaks on the CRO display as one increases $V_a$ is shown in Fig. 2. The oscilloscope picture becomes unsteady and the effect cannot be studied much beyond where the rise begins again. If the variation in $V_a$ is repeated with the field due to the filament current aiding the gross magnetic field the peak begins to shift slightly in the opposite direction by something less than one part in 50,000 and then shifts back again with no further shift above 1.2 volts.

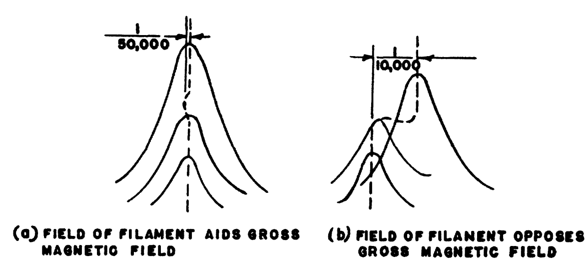



FIG. 2. Effect of changing bias voltage upon frequency at which resonance peaks occur. Dotted line shows locus of peaks as one increases $V_a$.

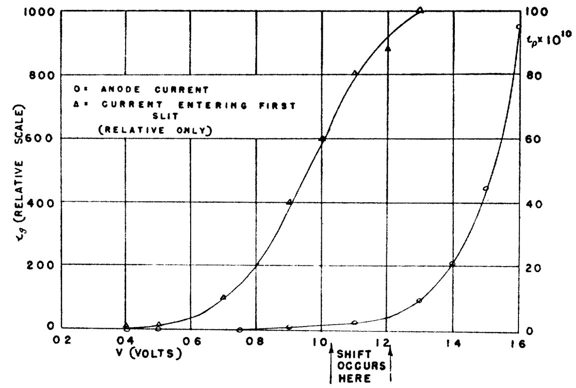



FIG. 3. Current-voltage relationships for electron tube.

This behavior is attributed to the combination of two effects: the change from a maxwellian velocity distribution to one for which a finite minimum velocity exists with a resultant shift of the region of the beam at which the most important contribution to the resonance takes place towards the cathode and hence further into the magnetic field of the heater current; and the accompanying increase in space charge density of this region with a resultant space charge shift in the resonance.

The fact that the heater current actually produced a magnetic field sufficiently large to affect the resonance was verified by momentarily shorting out this current. If the voltage $V_a$ was under 1.0 volt, a shift in the position of the electron resonance of the order of 1 part in 60,000 took place. Furthermore, this shift took place in the opposite direction if the same thing was done with the filament current reversed. For $V_a$ above 1.2 volts the shift is of the order of 1 part in 15,000.

The space charge shift is the same independent of the direction of the filament current and hence in the one case the two effects add and in the other case they compensate one another.

That part of the shift which is attributed to a change in the space charge density in the critical part of the beam is in magnitude about 1 part in 35,000. It has the correct sign to be associated with an increase of space charge density as $V_a$ is increased from 1.0 to 1.2 volts.

The magnetic field of the cathode may be roughly estimated from a knowledge of the filament current and this, in conjunction with the known shift in resonance upon reversal of the filament current may be used to predict the position of the important region of the beam as far as resonance is concerned. The filament current was 1.6 amp and the area of the portion of the filament which was doubled back and forth and pushed inside the cathode was about 0.5 cm$^2$. This situation can be approximated by a current loop of this area and the magnetic moment of an equivalent dipole may be calculated. This type of calculation gives the result that the important region is about $\frac{2}{5}$ of the distance across the guide from the cathode side, where the dipole magnetic field is 0.05 gauss, for $V_a$ below 1.0 volt; and about $\frac{1}{4}$ of the same distance where the dipole field is 0.165 gauss, for $V_a$ above 1.2 volts.

If the space charge density in the neighborhood of the potential minimum were known one could readily calculate the shift in resonant frequency one would expect due to the space charge effect. Inasmuch as the calculation of the space charge has not been done it is difficult to obtain an estimate of what this shift should be. One can obtain a value for this space charge at the potential minimum by assuming that equilibrium conditions exist there so that the Boltzmann equation can be used. In this case

$$\tfrac{1}{4}n\bar{c} \approx \tfrac{1}{4}ne(kT/m)^{\frac{1}{2}} = J_0, \tag{12}$$

so

$$\rho = 4J_0(2m/\pi kT)^{\frac{1}{2}}, \tag{13}$$

where $\bar{c}$ is the average velocity of the electrons, $J_0$ is the current density at the anode, and $n$ is the number of electrons/unit volume. Taking $J_0$ as approximately $10^{-6}$ ampere/cm$^2$ one obtains a value for the shift in resonance of approximately one part in a million.

The important region of the beam for the resonance is likely a little away from this minimum on the cathode side where the density is greater so this shift might be expected to be somewhat larger than that calculated. For large values of $V_a$ it is evident from the increase in the effect of the field of the heater current that this important region has shifted further toward the cathode to a considerably more dense space charge region.

The proper calculation of the space charge density would shed considerable light on the situation and give more confidence in the theory of operation of the tube and the measurements obtained.

In making the final measurements the voltage $V_a$ was kept below 1.0 volt to avoid the space charge shift. Figure 3 shows the region in which this shift takes place. Inasmuch as no shift took place as $V_a$ was increased up to 1.0 volt even though the current entering the first slit shows a large increase in this region with a consequent large increase in space charge it was felt that the space charge had little influence on the position of the resonance below 1.0 volt.

## 5. THE PROTON RESONANCE

The Larmor frequency, $\omega_p = 2\mu_p H/h$, of a sample of protons in the magnetic field is measured by the reonance absorption method. This proton resonance technique is essentially the same as that used by Purcell and collaborators and has been described adequately in the literature.[5] The source of rf was the oscillator stage of a surplus SCR-522 transmitter.[6] This unit is a modified Pierce circuit using a 7.12-mc crystal and gives an output at 14.24 mc. The signal for the proton resonance was coupled off from the tank coil by a single

[5] Bloembergen, Purcell, and Pound, Phys. Rev. **73**, 679 (1948).
[6] War Department Tech. Manual, TM 11-590 (1945).

loop of wire. The proton sample was mineral oil which occupied a volume of about 0.13 $cm^3$ inside a coil consisting of 8 turns of number 20 copper wire $\frac{1}{8}$-in. long by $3\frac{1}{4}$-in. inside diameter. This was supported by a Teflon form and shielded with $\frac{3}{4}$-in. brass with $\frac{1}{8}$-in. wall. The bridge was similar to that described by Bloembergen, Purcell, and Pound.[5] The receiver was a Signal Corp B.C. 312 N.

The proton resonance widths have been studied in some detail by Purcell and others[5] and will not be discussed here. It is sufficient to note that in a liquid the perturbing fields at the nuclei are very small due to the thermal motion of the molecules and consequently very narrow lines are predicted. The line widths actually observed are largely due to the inhomogeneity of the magnetic field. The magnetic field used was sufficiently homogeneous to render the proton line width well below the electron resonance width thus making the latter the limiting factor.

An effect called the "wiggles" [7] was of importance in the measurements. A series of damped oscillations occur after resonance has been passed. These are due to the fact that the proton moments remain coherent for a short time after the resonance and come in and out of phase with the applied rf. An analysis of this effect shows that the persistence of this coherence after resonance causes the main peak of the resonance curve to appear after the resonance has been passed. This would lead to some difficulty if it were not for the fact that it is a time effect, or "before and after resonance" effect, which may be made to take place in exactly the same way going up through resonance as coming down through resonance. Hence the picture on the oscilloscope of the line going up may be made to be the mirror image of that coming down and the point of symmetry of such a scope picture is the true resonance position.

To get an idea of how important symmetry was for locating the true resonance position in this CRO display, the radio frequency bridge was balanced for perfect symmetry ("up" trace mirror image of "down" trace) and then adjusted for noticeable asymmetry and the shift in the cross-over point of the two traces was noted. It was found that this would shift by less than 1 part in 100,000 for readily detectable asymmetry. Great care was taken to adjust the bridge and receiver for symmetry in the measurements. Some of the spread in the numbers taken may be due to slight asymmetry. It should be noted that errors due to this are of no great importance since they are random provided no inherent asymmetry is present.

The proton signal was displayed on the oscilloscope simultaneously with the electron signal by means of an electronic switch (Dumont type 150).

### 6. FREQUENCY COMPARISON SYSTEM

If the Larmor frequency of the proton is observed in the same magnetic field in which the cyclotron frequency of the electron is measured, the magnetic field may be eliminated by taking the ratio of these two quantities:

$$\frac{\omega_e}{\omega_p}=\frac{eH/mc}{2\mu_p H/\hbar}=\frac{e\hbar/2mc}{\mu_p}=\frac{\mu_0}{\mu_p}. \tag{14}$$

The result is the ratio of the Bohr magneton to the magnetic moment of the proton.

In order to obtain a cyclotron frequency in the $X$-band region (9365 mc) the proton resonance was chosen to occur at 14.24 mc.

A precise comparison of these frequencies was obtained by multiplying the 14.24-mc signal 657 times and comparing this with the $X$-band signal on a spectrum analyzer. A spectrum analyzer[8] is an electronic instrument which provides a plot of input signal power as a function of signal frequency on a cathode-ray tube. It consists essentially of a narrow band superheterodyne receiver which is electronically tuned in frequency by a linear modulating voltage applied to the local oscillator. This same sawtooth voltage is applied to the horizontal deflecting plates of a cathode-ray tube and the output of the receiver is applied to the vertical deflecting plates.

The proton signal from the tank circuit of the oscillator was first multiplied nine times by the SCR 522 transmitter itself by means of two tripler stages. This transmitter was designed to give high power output (about 10 watts) at the frequency of this ninth harmonic. The signal was then passed through a selenium crystal 1N21 which excited harmonics to the 73rd and beyond. The crystal was contained in a $1\times\frac{1}{2}$-in. wave guide crystal holder of Radiation Laboratory design[8]

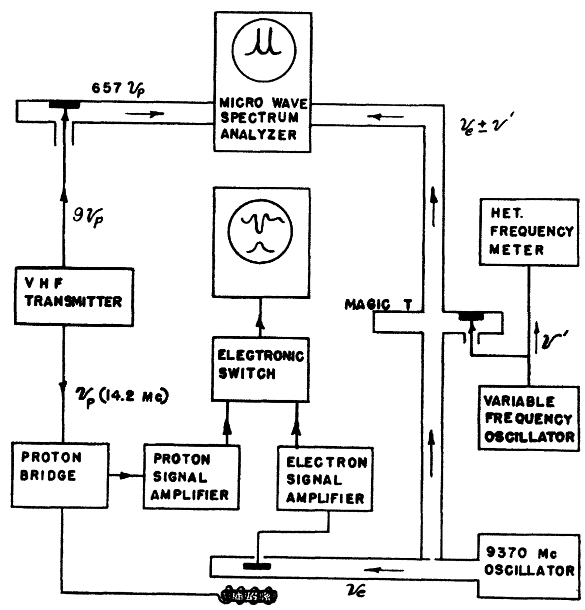



FIG. 4. Block diagram showing method of frequency comparison.

[7] See reference 8, and B. A. Jacobsohn and R. K. Wangsness, Phys. Rev. **73**, 942 (1948).

[8] C. G. Montgomery, *Technique of Microwave Measurements*, Rad. Lab. Series (McGraw-Hill Book Company, Inc., New York, 1947).

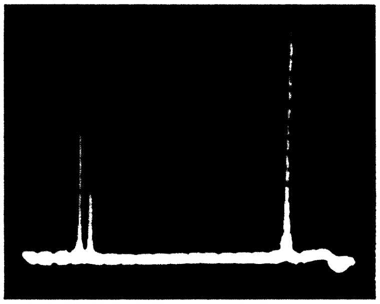


FIG. 5. Scope picture of frequency comparison on spectrum analyzer. Large pips are signal from electron oscillator and side band. Small pip is 657th harmonic of proton oscillator signal.

and the wave guide served as a filter. This signal then passed directly into the spectrum analyzer mixer (see Fig. 4).

A portion of the electron signal was coupled out by means of a probe in the wave guide leading to the electron tube. This signal passed into the $H$ plane arm of a magic $T$.[8] Into a crystal on one side of the $T$ was passed the output of the microvolter (Ferris model 18C) whose frequency could be varied over a wide range in the megacycle region. A matched load was placed on the other side of the $T$. The signal taken out at the $E$ plane arm was thus amplitude modulated, at the frequency, $\nu'$, of the microvolter by the change in crystal response, then passed directly into the mixer of the spectrum analyzer. The frequency of the microvolter was varied so that the side band of the electron signal coincided with the 657th harmonic of the proton signal. This frequency was measured with a Navy LM-18 crystal calibrated frequency indicating equipment.

This frequency, $\nu'$, was all that needed to be measured inasmuch as the ratio of the two frequencies was then given by

$$\omega_e/\omega_p = 9\times 73 \pm (\nu'/14.24). \tag{15}$$

### 7. EXPERIMENTAL PROCEDURE AND RESULTS

The process of making a measurement consisted merely in adjusting the frequency of the $X$-band oscillator until the center of the electron resonance coincided with the center of the proton resonance on the oscilloscope, then adjusting the modulation frequency, $\nu'$, of the signal coupled off from the electron oscillator till the pip on the spectrum analyzer of the first side band coincided with the pip due to the 657th harmonic of the proton signal. The frequency of the oscillator providing the modulation was then measured. Then either the magnetic field of the cathode heater (heater current) was reversed or the gross magnetic field was reversed to eliminate the effect of the heater current, and a second measurement was made. The same process was repeated for the positions of the proton sample and electron resonator interchanged. These four numbers were then averaged to constitute a single measurement. All measurements were taken with the bridge adjusted so that the proton signal for field increasing through resonance was the mirror image of the proton signal for the field decreasing through resonance, also the signals were at the center of the sweep. The electron signal was very nearly symmetrical (see Figs. 5 and 6).

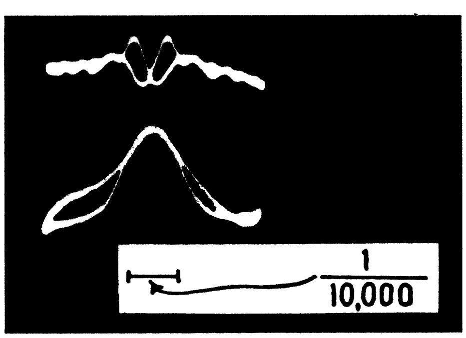



FIG. 6. Scope picture of proton and electron resonances.

For most measurements an 8-mc ($\sim$2.8-gauss) sweep was used although in some cases a 3-mc sweep proved more convenient. The width of the proton resonance was about 0.25 gauss and the width of the electron resonance varied from about 0.3 gauss to 0.75 gauss, depending upon the power used and the voltage $V_a$ between the wave guide and cathode of the electron tube.

The microwave power in the electron tube was turned as low as possible (between 80 and 300 microwatts) for most measurements. If the power was turned very much higher than this the beam began to broaden. This is due to saturation, all the electrons entering the guide being collected at the anode near the peak of the resonance.

A series of nine complete measurements of $\nu'$ (megacycles) were taken over a period of five weeks. Within this period the electron tube was torn down and the cathode was replaced; this resulted in no apparent change in the numbers obtained. A tabulation of the numbers taken, together with a deviation from the mean appears in Table I. These represent all the data that were not thrown out at the time of observation for reasons not having to do with the consistency of the data itself. No selection has been made. In Table I F. C. refers to filament current and M.C. to magnetizing current. When both are positive or both negative the two magnetic fields aid. The starred runs were taken by reversing the gross magnetic field current keeping the filament current in the direction defined as positive. All other runs were with the gross magnetic field current in the direction defined as positive.

The 72nd and 74th harmonics of the 9th harmonic of the proton signal could also be observed on the spectrum analyzer. These signals were just 128.16 Mc apart. A cavity wave meter on the spectrum analyzer measured the 73rd harmonic to be at 9356 Mc which is just 4 Mc away from where it should be. This serves as a good check on whether or not we were looking at the

73rd harmonic. Care was taken to ascertain that pips from the electron and proton signals were both associated with the same side band of the spectrum analyzer response. The same sequence of pips was observed 40 Mc away (spectrum analyzer i-f is 20 Mc).

The number obtained gives the desired ratio of the two resonance frequencies from Eq. (15). The fact that we should add rather than subtract the correction was ascertained by noting that the electron signal lay between the 73rd and 74th harmonics of the 9th harmonic of the proton signal.

## 8. DISCUSSION OF SYSTEMATIC ERRORS

In view of the fact that the measurement is capable of a precision of better than one part in 150,000 if judged solely by internal consistency, it appeared that considerable care was justified in running down small systematic errors. The known possible sources of systematic error were (1) magnetic contamination of the apparatus; (2) magnetic inhomogeneities due to currents; (3) inexact interchange of proton sample and critical portion of electron beam; (4) space charge effects on electron resonance; (5) difference in position of resonances when approaching from below resonance from that in approaching from above; (6) diamagnetic correction to field of proton; (7) relativistic dependence of mass on velocity.

The possibility of magnetic contamination of materials used in the apparatus was carefully explored. Samples of all types of materials that went into the construction of the critical parts were placed in the field in such a way as to have a maximum effect on the resonance. The only material found that did have any observable effect was red Glyptal and this was then removed. To obtain a further check on this after the measurements were completed, a large hole was drilled into the electron tube to permit the proton sample to fit inside at the position at which the electron beam had been. The electron tube was then moved on and off of the proton sample while the proton resonance was observed. No effect was noticed even though a shift of 1 part in 200,000 could have been observed.

Nickel leads were used about 3 inches from the electron beam but moving these leads showed no effect on the resonance.

The cathode base was made of nickel. However, the temperature of the cathode was in the neighborhood of 1600°K which is well above the Curie point for nickel which is 360°. The construction of the cathode was such that the entire nickel base was heated to approximately the same temperature.

The discrepancy in the number due to the magnetic field of the heater current was effectively eliminated to first order at least, by averaging readings for both directions of filament current or for both directions of gross magnetic field.

It was, of course, not possible to make an exact interchange of the proton sample and critical portion of electron beam. The difference between the numbers for the interchange was of the order of one part in fifty thousand. The sample and the beam was $\frac{1}{2}$ in. apart. It should be stated, however, that the center of the beam was interchanged with the sample. It was later found that the critical portion of the beam was something less than $\frac{1}{10}$ in. away from the center towards the cathode. This may have led to some error. The interchange was not precisely the same for each reading and the small spread in the data itself, would lead one to suspect that the error would not be appreciable.

The space charge effects have already been discussed.

The protons used in the experiment were the hydrogen nuclei of the water molecule or of the long chain hydrocarbons which make up mineral oil. The numbers obtained for these two substances were the same within experimental error. The shielding of the protons by the electrons in the molecules is large enough to affect the measurements and must be taken into account. The differences in magnetic shielding for $H_2$, $H_2O$, and mineral oil are found to be small[9] so we may apply the correction for the hydrogen molecule which has been calculated approximately by Ramsey[10] to be $-2.7\times10^{-5}H$.

TABLE I. Summary of measurements.

| Date | Proton above electron F.C. or M.C.+ | Proton above electron F.C. or M.C.− | Proton below electron F.C. or M.C.+ | Proton below electron F.C. or M.C.− | Average | Deviation |
|---|---|---|---|---|---|---|
| June 22 | 6.77 | 6.48 | 7.01 | 6.66 | 6.73 | −0.04 |
| June 27 | 6.85 | 6.40 | 7.07 | 6.43 | 6.69 | −0.08 |
| June 27 | 6.92 | 6.52 | 7.15 | 6.61 | 6.80 | +0.03 |
| June 27 | 6.87 | 6.31 | 6.99 | 6.57 | 6.69 | −0.08 |
| June 27 | 6.98 | 6.66 | 6.99 | 6.66 | 6.82 | +0.05 |
| June 28 | 7.00 | 6.52 | 7.04 | 6.62 | 6.80 | +0.03 |
| July 6 | 6.82 | 6.49 | 7.14 | 6.83 | 6.82 | +0.05 |
| July 6 | *6.79 | *6.99 | *7.00 | *6.56 | *6.83 | +0.06 |
| July 29 | *6.73 | *6.81 | *6.95 | *6.40 | *6.72 | −0.05 |
| | | | | Total | 60.90 | 0.47 |
| | | | | Average | 6.77 | 0.05 |

It may be some inherent asymmetry exists in the resonance observed on the oscilloscope due to the difference in the behavior of the protons or electrons below resonance from their behavior above resonance. The relativistic dependence of mass on velocity would produce such asymmetry in the electron resonance. An asymmetry due to this should be distinguished from an asymmetry due to the difference in behavior before resonance and after resonance. A time effect such as this latter may be eliminated by adjusting the gross magnetic field so that the up and down resonances are equally spaced in time and, in the case of the proton signal, carefully adjusting the receiver and bridge so that the frequency response is the same for both cases.

9 H. S. Gutowsky and R. E. McClure, Phys. Rev. **81**, 276 (1951).
10 N. F. Ramsey, Phys. Rev. **77**, 567 (1950).

A requirement for the fulfillment of this latter condition, however, is that no other causes of asymmetry be present because one uses symmetry to determine whether or not the frequency response of the system has been properly adjusted. It is very unlikely that an appreciable inherent asymmetry could be exactly compensated by tuning the system, and, inasmuch as it was possible to tune for excellent symmetry, it seems unlikely that a difference between behavior above resonance from behavior below resonance which would cause asymmetry was significantly present. Of course one could probably not detect experimentally a shift due to a difference in behavior above and below which did not produce asymmetry.

For electron velocities used the relativistic shift in the resonance frequency is just below where it would be observable.

## 9. CONCLUSION

Using the equipment and technique discussed it was possible to obtain a value of the ratio of the cyclotron frequency of the electron to the precession frequency of the proton with an accuracy approaching one part in a hundred thousand. The mean value of this ratio, uncorrected for diamagnetism, was 657.4752 with a mean deviation of 0.0037 and a maximum deviation of 0.0056. Inasmuch as the accuracy of the result depends on the extent to which systematic errors can be excluded, possible sources of these were carefully investigated as previously discussed. In view of the results of these investigations and the high internal consistency of the values obtained, it is felt that the true ratio, uncorrected for diamagnetism, lies within the range,

$$\omega_e/\omega_p = 657.475 \pm 0.008. \quad (16)$$

If the diamagnetic correction to the field of the proton for the hydrogen molecule[10] of $2.68\times10^{-5}$ is applied here, the proton moment, in Bohr magnetons, becomes

$$\mu_p = (1.52101 \pm 0.00002)\times10^{-3}(e\hbar/2mc). \quad (17)$$

This result may be compared with the value $(1.52106 \pm 0.0007)\times10^{-3}$ obtained by Taub and Kusch.[11] As mentioned in the introduction, this agreement may be regarded as further confirmation of the correction[2] to the spin-moment of the electron, since the experiment discussed here amounts to a comparison of the orbital $g$-factor ($n$ very large) of the electron with the proton $g$-factor, whereas Taub and Kusch compared the proton $g$-factor and the $g$-factor of a $^2S_{\frac{1}{2}}$ state, applying the factor $2[1+(\alpha/2\pi)]$ to obtain the number just quoted.

The result stated here, like that of Taub and Kusch, may be used in conjunction with the hydrogen hyperfine structure splitting[12] and $cR_\infty$ to obtain the fine structure constant $\alpha$, and may be combined with the absolute value of the proton gyromagnetic ratio $\gamma_p$ measured by Thomas, Driscoll, and Hipple[13] to yield $e/mc$. Also, with the ratio of the proton cyclotron frequency to the proton precession frequency $\omega_e/\omega_p$, measured by Hipple, Sommer, and Thomas,[14] this result gives a precise value for $M/m$, the ratio of the mass of the proton to that of the electron.

The value of $\alpha$ may be computed from the Fermi hyperfine structure formula,[15] modified to include Breit's relativistic correction,[16] the effect of the reduced mass,[17] and the radiative correction to the electron magnetic moment:[2,12]

$$\Delta\nu = \alpha^2\left[\frac{16}{3}\frac{\mu_p}{\mu_0}cR_\infty(1+\tfrac{3}{2}\alpha^2)\left(1+\frac{m}{M}\right)^{-3}\left(1+\frac{\alpha}{2\pi}\right)\right]. \quad (18)$$

Since Taub and Kusch measure the ratio $\mu_p/\mu_e$, where $\mu_e$ is the magnetic moment of the electron, the radiative correction (last term in parentheses) must appear twice in this formula when their measurement is used. $\Delta\nu$ is known to 4 parts in a million,[12] $c$ to 3 parts in a million[18] and the result stated here for $\mu_p/\mu_0$ to 12 parts in a million. $\alpha$ computed from the measurements referred to would have a root-mean-square error of $\pm7$ parts in a million. The accuracy is thus improved to the point where the uncertainties in the theoretical formula become important.[19]

It is at present felt that the possibilities of narrowing the electron resonance line by exploiting the velocity selection provided by the space charge effect have not been fully realized. It is also felt that the velocity selection scheme mentioned in connection with the dip in the electron peak for high power would be profitably utilized. It appears that the width of the electron dip would be limited only by the sensitivity of the detection apparatus for this sort of scheme.[20]

The author wishes to express his appreciation to Professor E. M. Purcell, who suggested the possibility of making the measurement discussed here and contributed many ideas which permitted the realization of this possibility.

[11] H. Taub and P. Kusch, Phys. Rev. **75**, 1481 (1949).

[12] J. E. Nafe and E. B. Nelson, Phys. Rev. **73**, 718 (1948).

[13] Thomas, Driscoll, and Hipple, Phys. Rev. **75**, 992 (1949); **78**, 787 (1950).

[14] Hipple, Sommer, and Thomas, Phys. Rev. **76**, 1877 (1949); **80**, 487 (1950).

[15] E. Fermi, Z. Physik **60**, 320 (1930).

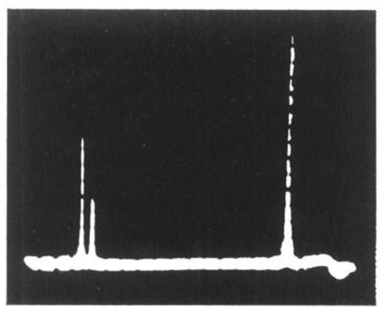
[16] G. Breit, Phys. Rev. **35**, 1447 (1930).

[17] G. Breit and E. R. Meyerott, Phys. Rev. **72**, 1023 (1947).

[18] Kees Bol, Phys. Rev. **80**, 298 (1950).

[19] For recent calculations of these constants see J. A. Bearden and H. M. Watts, Phys. Rev. **81**, 73 (1951).

[20] A more recent measurement using a different means of detecting the resonance has been made by R. W. Nelson. His value falls within the range quoted here.



Fɪɢ. 5. Scope picture of frequency comparison on spectrum analyzer. Large pips are signal from electron oscillator and side band. Small pip is 657th harmonic of proton oscillator signal.

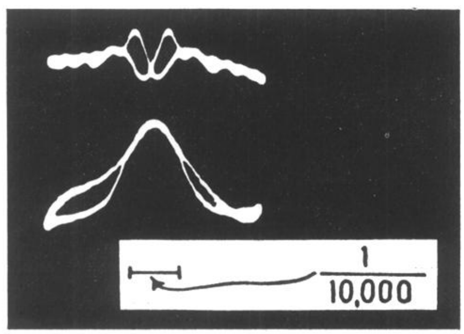



Fig. 6. Scope picture of proton and electron resonances.