Form of Transient Currents in Townsend Discharges with Metastables

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The form of the current is calculated for a Townsend discharge stimulated by a pulsed light beam, with particular reference to the current component initiated by metastable effects. The calculation is directed particularly to the development of methods for quantitative interpretation of current patterns observed experimentally.

A. INTRODUCTION

THE electrtical currents caused by the action of metastable atoms in a Townsend discharge are characterized by their relatively slow time of build-up to a steady-state value. In a typical experiment this time is of the order of one millisecond and is set essentially by the lifetime of the metastables in the space between the electrodes. Because of this slow build-up, a study of the transient character of Townsend currents initiated by pulsed light-illumination of the cathode permits separating currents attributable to the action of metastables from those caused by ions and photons, which build up much more rapidly. Such experiments are described in an accompanying paper (hereafter designated as II). Our purpose here is to develop a theoretical basis for interpreting the current forms obtained in these experiments.

This problem has been discussed by Engstrom and Huxford[1] and Newton.[2] The treatment here follows closely that of Engstrom and Huxford and represents an extension of their analysis to include higher order terms and the volume destruction of metastables, as well as the diffusion loss they considered.

B. STATEMENT OF PROBLEM

We assume we have a gas-filled tube with two plane-parallel electrodes, an anode and cathode, and that beginning at time $t=0$, a current of electrons, i_0, leaves the cathode as a result of photoeletric action of light shone on the cathode.[3] This electron current we assume is amplified to a value, i_f, in a time of the order of ten microseconds, which we take to be instantaneous on our time scale. The value i_f is given by the well-known Townsend equation,

$$i_f = i_0 \exp[\alpha_i(X-x_0)]/1-\gamma_f(\exp[\alpha_i(X-x_0)]-1), \quad (1)$$

where α_i = the first Townsend coefficient for ionization, defined specifically as the number of electron-ion pairs generated per electron per cm of path in a direction perpendicular to the electrode surfaces; X = the electrode separation; x_0 = the electrode separation at which ionization begins,[4] γ_f = the number of electrons released at the cathode and entering the discharge stream (i.e., not diffusing back) per ion generated in the gas by the fast processes, which include both ion and photon bombardment of cathode (γ_f is defined and discussed in greater detail in II).

The action of metastable atoms in the discharge results in a slowly rising current component, $i_s(t)$ (see Fig. 1); and it is the form and magnitude of this component which we wish to calculate.

We assume that the i_s-component originates either from (1) the emission of electrons from the cathode caused by metastables striking the cathode directly, or (2) the conversion of metastables into radiating atoms from which the emitted photons can photoelectrically eject electrons from the cathode.[5]

We shall see that the form and magnitude of $i_s(t)$ can quite readily be computed for any assumed set of parameters describing the efficiencies of metastable production and of electron emission by the different processes. The inverse problem of establishing these parameters from observed current patterns is less straightforward. For this purpose various approximations are introduced into the analysis.

C. INTEGRAL EQUATIONS

Following Engstrom and Huxford, we start by considering the flow of metastables produced by one electron leaving the cathode, crossing the gap, and causing ionization and excitations. The metastables generated by this electron and its progeny (resulting from ionizations) will diffuse to the containing walls (the electrodes in this case) and be destroyed there unless they are first "destroyed" by special types of collision with normal

[1] R. W. Engstrom and W. S. Huxford, Phys. Rev. **58**, 670 (1940).
[2] R. R. Newton, Phys. Rev. **73**, 570 (1948).
[3] Our analysis is for a "step-function" cycle of light stimulation, and furthermore we consider only the rising part of the transient. The falling part which starts at the removal of the stimulating light has a form exactly the inverse of the rising part. The general method of analysis can, of course, be applied equally well to any form of the stimulating light. For more complicated forms, say, that of a sinewave-modulated light beam, the analysis is probably most easily carried through by first calculating the current form resulting from a "delta-function" type of stimulation, and then integrating a series of these currents modulated according to the assumed form of the light modulation.
[4] This definition of x_0 is rough. In practice x_0 is adjusted to a value such that the experimental values of i_f for $X>x_0$ fit closely Eq. (1).
[5] Metastables can probably also produce electrical effects in pure gases by such processes as a metastable-metastable collision yielding an ionized and a neutral atom plus an electron, or an electron-metastable collision yielding an ion and another electron. These processes come into importance only at high metastable densities, and they are easily reduced to negligible importance by using sufficiently small values of current.

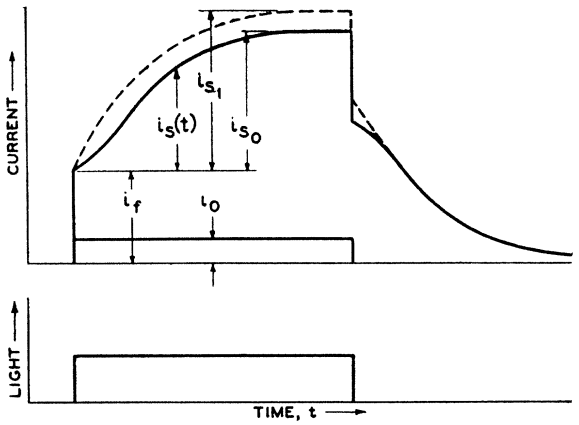

FIG. 1. The current form (schematic) in a pulsed Townsend discharge with metastables. The light pulse is assumed here to have a duration of about 0.005 second. The fast component of current, i_f, is shown here to rise instantaneously. Actually it rises with a time constant of 1 to 30μsec in a typical experiment. The dashed line indicates the form the current would have with a $P(t)$-function described by a single exponential. The effect of higher terms, having a net negative amplitude, is to give the solid curve, although the difference is exaggerated here for purpose of clarity.

atoms.[6] We define a function $P_K(t)$ as the rate of arrival at the cathode of the metastables produced by such an electron leaving the cathode at $t=0$. Similarly, $P_A(t)$ is the rate of arrival at the anode, while $P_G(t)$ is the rate of disappearance of metastables in the volume. Hence, the total rate of flow of the metastables, created by one single electron leaving the cathode, out of the gap is the sum of these three P-functions. We shall calculate their individual forms in Sec. D.

Let us first restrict ourselves to the case in which $i_s(t)$ arises from the release of electrons at the cathode caused only by direct bombardment by metastables. The component $i_s(t)$ will then consist of an electron current at the cathode, initiated by the metastables, enhanced by the same processes which amplify the primary photocurrent i_0 to i_f. Thus, we have

$$i_s(t)=i_f/i_0 \text{ (rate of release of electrons at the cathode by metastables).} \quad (2)$$

The term in the parenthesis is given by the product of the rate of arrival of metastables at the cathode and their efficiency of electron emission. The latter quantity is given $\gamma_m f_{esc}$, where γ_m is the number of electrons released per metastable striking the surface and f_{esc} is the fraction of these electrons which actually enter the discharge stream and do not diffuse back to the cathode. The rate of arrival is given by

$$\int_0^t P_K(t-t')(\text{current of electrons leaving the cathode at } t=t')dt'.$$

[6] Assuming there are no impurities present, such "destruction" is usually assumed to be caused by the conversion of the metastables into atoms excited into nearby radiating states, which for all the noble gases, except helium, lie within 0.1 electron volt. In the case of helium this spacing is much larger. Here the destructive mechanism is less well understood and probably involves the formation of helium molecules. See R. Meyerott, Phys. Rev. **70**, 671 (1946).

The current of electrons leaving the cathode at time t' is given by the total current, $i_f+i_s(t)$, divided by the gas amplification factor, $\exp[\alpha_i(X-x_0)]$. Hence, we have

$$i_s(t)=\left(\frac{i_f}{i_0}\right)(\gamma_m f_{esc})\int_0^t P_K(t-t')\frac{i_f+i_s(t')}{\exp[\alpha_i(X-x_0)]}dt'. \quad (3)$$

By making use of Eq. (1) and introducing the notation $R_f\equiv\gamma_f(\exp[\alpha_i(X-x_0)]-1)$, we can rewrite Eq. (3) as

$$\frac{i_s(t)}{i_f}=\frac{\gamma_m f_{esc}}{1-R_f}\int_0^t P_K(t-t')\left(1+\frac{i_s(t')}{i_f}\right)dt'. \quad (4)$$

This is the fundamental equation describing the problem we are here discussing.

If we assume instead that metastable atoms give rise to a current by being converted into radiating atoms in the gas from which the emitted photons cause photoelectric emission at the cathode, then by analogous reasoning we come out with the equation,

$$\frac{i_s(t)}{i_f}=\frac{\gamma_r f_{rk}' f_{esc}}{1-R_f}\int_0^t P_G(t-t')\left(1+\frac{i_s(t')}{i_f}\right)dt', \quad (5)$$

where γ_r = the number of electrons emitted per gas photon striking the cathode; f_{rk}' = the fraction of the photons emitted by the converted metastables which reach the cathode. For infinite plane-parallel geometry and in the absence radiation imprisonment effects, $f_{rk}'=0.5$. In a practical situation, the lack of perfect plane-parallel geometry and imprisonment effects will tend to make f_{rk}' somewhat smaller. If both processes are acting, then obviously $i_s(t)/i_f$ is given by sum of the right-hand sides of Eqs. (4) and (5).

D. CALCULATION OF P-FUNCTIONS

To calculate the functions $P_K(t)$, $P_A(t)$, and $P_G(t)$, defined earlier, we assume the metastables have a diffusive motion, and so we proceed by solving the diffusion equation. This we write in the form,

$$\partial\rho/\partial t=D_m(\partial^2\rho/\partial x^2)-G\rho, \quad (6)$$

where ρ = the linear density of metastables created by a single electron leaving the cathode at $t=0$, D_m = the diffusion coefficient of metastable atoms in the parent gas; x = the distance from the cathode, and G = the probability of a metastable being destroyed in the gas per second.

We assume that the metastables are destroyed when striking either electrode, so that our boundary conditions are

$$\rho=0 \text{ for } x=0 \text{ and } x=X. \quad (7)$$

It is readily shown that the following solution fulfills our requirements:

$$\rho=\sum_n a_n \sin(n\pi x/X)\exp\{-[(\pi^2n^2D_m/X^2)+G]t\}, \quad n=1, 2, 3, \cdots, \quad (8)$$

where the a_n's are coefficients determined by having ρ as given by Eq. (8) describe the initial distribution of

metastables. Since at $t=0$ the exponential factors are all unity, it is evident that the a_n's are coefficients in a fourier expansion of the initial distribution function. For this we assume that metastables are initially formed in the space between electrodes in a density α_m/α_i times that at which the ions are formed, where α_m is the first Townsend coefficient for metastable excitations and is specifically the number of metastable excitations per electron per centimeter.[7] Since the ion distribution is zero for $x<x_0$ and $\alpha_i \exp[\alpha_i(X-x_0)]$ for $x>X$, we can write the initial metastable distribution as

$$\rho=0, \qquad 0<x\leq x_0,$$
$$=\alpha_m \exp[\alpha_i(X-x_0)], \quad x_0<x\leq X. \tag{9}$$

Using the standard formula for fourier coefficients, we have

$$a_n=(2/X)\int_{x_0}^{X} \alpha_m \exp[\alpha_i(X-x_0)]\sin(n\pi x/X)dx. \tag{10}$$

The integration is readily carried out. We shall not take space here to do so, but simply note that a_n is positive for all odd values of n, and negative for all even values.

When ρ, the metastable density, is calculated by the method outlined above, the P-functions are obtained from the following relations,

$P_K(t)\equiv$(diffusion current to the cathode)
$$=D_m(\partial\rho/\partial x)_{x=0};$$
$P_A(t)\equiv$(diffusion current to the anode)
$$=-D_m(\partial\rho/\partial x)_{x=X}; \tag{11}$$
$P_G(t)\equiv$(rate of destruction in the gas)
$$=G\int_0^X \rho dx.$$

With ρ in the form given by Eq. (8), the formulas in Eq. (11) yield

$$P_K(t)=D_m\sum\left(\frac{n\pi}{X}\right)a_n \exp\left[-\left(\frac{\pi^2n^2D_m}{X^2}+G\right)t\right],$$
$$n=1, 2, 3, \cdots, \tag{12}$$

$$P_A(t)=D_m\sum\left(\frac{n\pi}{X}\right)a_n(-1)^{n+1}$$
$$\times\exp\left[-\left(\frac{\pi^2n^2D_m}{X^2}+G\right)t\right],$$
$$n=1, 2, 3, \cdots, \tag{13}$$

$$P_G(t)=G\sum\left(\frac{2X}{n\pi}\right)a_n \exp\left[-\left(\frac{\pi^2n^2D_m}{X^2}+G\right)t\right],$$
$$n=1, 3, 5, \cdots \text{(only odd)}. \tag{14}$$

[7] This assumption is obviously approximate. On the other hand, for the experiments we are here trying to explain, only the gross exponential form of the initial distribution is important.

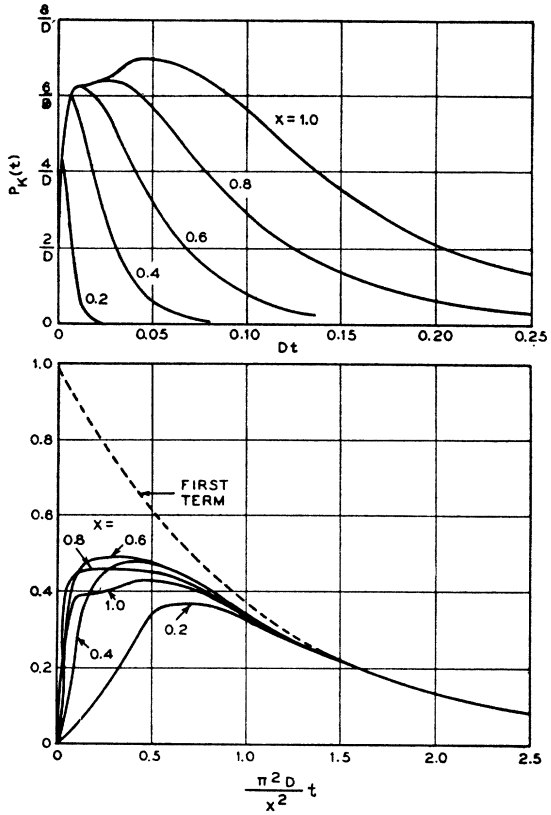

FIG. 2. Typical plots of $P_K(t)$ *vs* t for various values of the electrode spacing, X, computed for an initial distribution of metastables given by

$$\rho=0, \qquad 0\leq x\leq 0.1,$$
$$\rho=e^{3(x-0.1)}, \quad 0.1\leq x\leq X,$$

and with $G=0$. The lower set of curves gives $P_K(t)$ for the same conditions, but plotted here in a manner such as to permit direct comparison with the first term of each series. The scales of ordinates and abscissas are adjusted in each case so that the first term falls on the dashed curve. Note that for large X the curves have roughly the same shape when plotted in this form.

We note from the foregoing relations that all the P-functions are described by a series of exponentials with the same time constants but different constant multipliers. Since, as indicated, a_n alternates in sign, the constant multipliers alternate in sign for $P_K(t)$, while for $P_A(t)$ they have the same magnitude, but are all positive. In $P_G(t)$ only the odd terms are present; and, furthermore, because of the appearance of n in the denominator, the constant multipliers get small rapidly with n.

P-functions calculated by the equations are shown in Figs. 2 and 3 for various assumed values of the various parameters. On examination of these curves, it can be seen that $P_G(t)$ is closely represented by a single exponential. The first term of the expansion for $P_G(t)$ given by Eq. (14) contains 95 percent of the area under the $P_G(t)$ curve for a variety of typical cases we calculated. For $P_K(t)$ a single exponential approximation is somewhat worse. Except for the very small values of

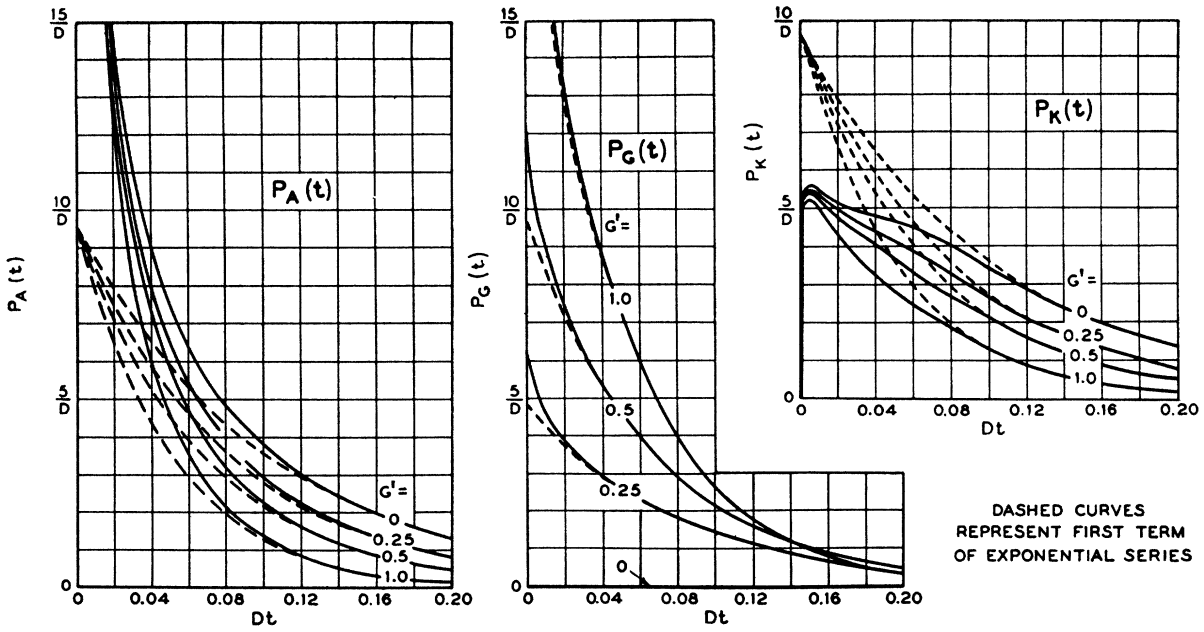

FIG. 3. Plots of $P_A(t)$ and $P_K(t)$ for various values of the gas destruction parameter. Here $G' \equiv GX^2/\pi^2 D_m$, and the initial distribution was taken to be

$$\rho = 0, \qquad 0 \leq x \leq 0.1,$$
$$\rho = e^{2(x-0.1)}, \quad 0.1 \leq x \leq 1.0.$$

The curves for $\rho_G(t)$ are seen to be closely approximated by the first term of their series expansions, whereas for $P_K(t)$ that approximation is less good and for $P_A(t)$ very poor.

X, the area under the $P_K(t)$ curve was 75 to 90 percent of the area under the first term of the expansion as given by Eq. (12), for a variety of typical cases which we computed. Here a two-term approximation, however, is quite satisfactory, as illustrated in Fig. 4.

E. SOLUTION OF INTEGRAL EQUATION

Having outlined the methods of calculating the P-functions, we are now in a position to discuss the solution of the integral Eqs. (4) and (5).

For simplicity let us use for the P-functions the notation

$$P(t) \equiv P_1 e^{-t/\tau_1} + P_2 e^{-t/\tau_2} + \cdots = \sum_1^N P_j e^{-t/\tau_j}. \quad (15)$$

Although $P(t)$ in the strict sense requires an infinite number of terms, we have seen that both $P_K(t)$ and $P_G(t)$ can be closely approximated by a limited number of terms; and, in fact, $P_G(t)$ can be described very well by a single term, while for $P_K(t)$ two terms provide a satisfactory approximation.

With such a series representation of $P(t)$, it is readily shown that $i_s(t)$ can be described by a similar series of exponentials,

$$i_s(t) = i_{s0} - i_{s1} e^{-t/T_1} - i_{s2} e^{-t/T_2} - \cdots = i_{s0} - \sum_1^N i_{sl} e^{-t/T_l}, \quad (16)$$

having the same number of exponential terms as the $P(t)$ series. If this $i_s(t)$ series is substituted into Eq. (4), then we find the following relations between the time constants and coefficients:

$$i_{s0} = i_f \left(\frac{\gamma_m f_{esc}}{1-R_f}\right) \sum_1^N P_j \tau_j \Big/ 1 - \left(\frac{\gamma_m f_{esc}}{1-R_f}\right) \sum_1^N P_j \tau_j, \quad (17)$$

$$1 = \left(\frac{\gamma_m f_{esc}}{1-R_f}\right) \sum_{j=1}^N \frac{P_j}{(1/\tau_j) - 1/T_l} \text{ for each value of } l, \quad (18)$$

$$0 = \sum_{l=1}^N \frac{i_{sl}}{(1/\tau_j) - 1/T_l} - \frac{i_f + i_{s0}}{1/\tau_j} \text{ for each value of } j. \quad (19)$$

In addition we have

$$i_{s0} = i_{s1} + i_{s2} + i_{s3} + \cdots = \sum_{l=i}^N i_l$$

simply from the physics of the situation.[8]

A similar set of equations apply in the case of Eq. (5) with γ_m replaced by $\gamma_r f_{rk}'$.

The accuracy with which $i_s(t)$ can be calculated obviously depends on the number of terms used to express $P(t)$, and the labor involved in solving Eq. (18) and (20) correspondingly depends on the number of terms involved. If, for example, four terms are used, then Eq. (18) is a quartic algebraic equation, from which T_1, T_2, T_3, and T_4 can be obtained. The coefficients i_{s1},

[8] In his paper R. R. Newton (see reference 2, p. 580) derives these relations by another method and discusses them briefly.

i_{s2}, i_{s3}, amd i_{s4} are obtained by solving four linear equations given by Eq. (19).

We carried out such a solution for the case corresponding to an initial metastable distribution given by

$$\rho=0, \quad 0\leq x\leq 0.1,$$
$$\rho=e^{2(x-0.1)}, \quad 0.1\leq x\leq 1.0, \tag{20}$$

and G assumed to be zero. Then we approximated $P_K(t)$ by four-, three-, and two-term series (see Fig. 4). In each case the coefficients of the exponentials were made the same as those corresponding to an infinite series representation, except for the term with the smallest value of τ. For this the coefficient was adjusted so that the value of $P\tau$ for this term was equal to the sum of the $P\tau$'s for this and all higher terms in the infinite series representation. Thus, the quantity $\sum_{j=1,N} P_j\tau_j$ was the same for all three cases, and hence i_{s0} as calculated from Eq. (17) was not dependent on the number of terms used in $P(t)$.

Values of i_{sl} and T_l as calculated from Eq. (18) and (19) for the four-term series are plotted in Figs. 5 and 6 as a function of the quantity,

$$R_m/1-R_f\equiv(\gamma_m f_{esc}/1-R_f)\sum P_j\tau_j.$$

Over the range of this plot i_{s1} differed less than 0.2 percent for the three series representations, while T_1 differed by less than 0.6 percent.[9] Similarly, the sum of the amplitudes of the higher terms, i_{s2}, i_{s3}, and i_{s4} was maintained within 10 percent.

Note that for small values of $R_m/(1-R_f)$ the relative amplitudes of the various terms remain fixed, while the values of T_l are close to those of corresponding τ_j's in the $P(t)$ series. As $R_m/(1-R_f)$ is made larger, the first terms shows a marked increase in both amplitude and time constant, while the higher terms deviate little from the earlier trends. Note also that the total amplitude of the higher terms, $i_{s2}+i_{s3}+i_{s4}$, is always negative.

These observations suggest that a fairly satisfactory description of $i_s(t)$ is obtained with a two-term description of $P_K(t)$, yielding a two-term description of $i_s(t)$ in which the fundamental or first term becomes the increasingly dominant fraction as $R_m/(1-R_f)$ is made larger. Such a description fits well the experimental situation, in which case it turns out to be feasible to describe any observed $i_s(t)$ curve by only two terms.[10]

For the case in which metastables are converted into radiating atoms which emit light and thereby give rise to photoelectric emission at the cathode [Eq. (5)], the calculation is even simpler. Here $P_G(t)$ is closely represented by a single exponential. If a second term is added, however, it will have an amplitude with the same sign as the first, and so the higher term in $i_s(t)$ will *add* to the first term. If both electron emission processes are simultaneously active, then Eqs. (4) and (5) can be combined in an obvious way, and a solution carried out in a manner as outlined for Eq. (4).

F. CALCULATION OF FUNDAMENTAL TIME CONSTANT, τ_1, FROM EXPERIMENTAL DATA

In the preceding section a method is outlined for calculating $i_s(t)$ for given values of $\gamma_m f_{esc}/(1-R_f)$ or $\gamma_r f_{rk} f_{esc}/(1-R_f)$ and $P_K(t)$ or $P_G(t)$. In the experimental situation we are faced with the inverse problem, namely, that of calculating these various quantities from data describing $i_s(t)$. The quantity most easily obtained in this way is τ_1, the fundamental time constant of decay of the metastables, and we discuss this problem first.

The experimental arrangements described in the accompanying paper permit fairly accurate determination of i_f, i_{s1}, T_1, and i_{s2}. No accurate time constant corresponding to i_{s2} can be measured. We proceed by applying Eq. (19), which for this case reduces to

$$0=\frac{i_{s1}}{(1/\tau_1)-1/T_1}+\frac{i_{s2}}{(1/\tau_1)-1/T_2}-\frac{i_f+i_{s0}}{1/\tau_1}. \tag{21}$$

If now we recall that $i_{s0}=i_{s1}+i_{s2}$, and further that T_2 does not deviate much from τ_2, so that we can set

$$\beta=\tau_1/\tau_2\approx\tau_1/T_2, \tag{22}$$

where β is a constant, then by rearranging Eq. (21) we have

$$\tau_1=T_1\left[\frac{i_f+(\beta/\beta-1)i_{s2}}{i_f+i_{s1}+(\beta/\beta-1)i_{s2}}\right]. \tag{23}$$

The value of β will depend on whether the function $P_k(t)$ or $P_G(t)$ is involved, and what the relative values

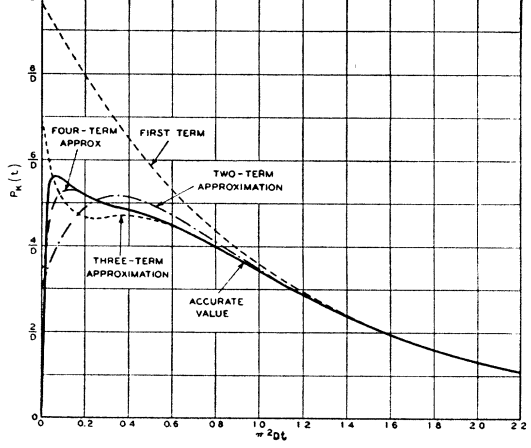

FIG. 4. Various approximations to $P_K(t)$ used in the calculation of $i_s(t)$. $P_K(t)$ here corresponds to an initial metastable distribution same as that in Fig. 3 with $G=0$.

[9] These differences were too small to show up on the scale used in Fig. 5.

[10] Even in a theoretical description, the higher terms depend closely on the assumed spatial distribution of metastables when formed, which we know only approximately. Furthermore, the diffusion equation itself breaks down as a description of the metastable motion for short distances, which in effect correspond to the higher terms of $P_K(t)$. Thus, we lose very little in our knowledge of the actual $i_s(t)$ by approximately $P_K(t)$ is indicated.

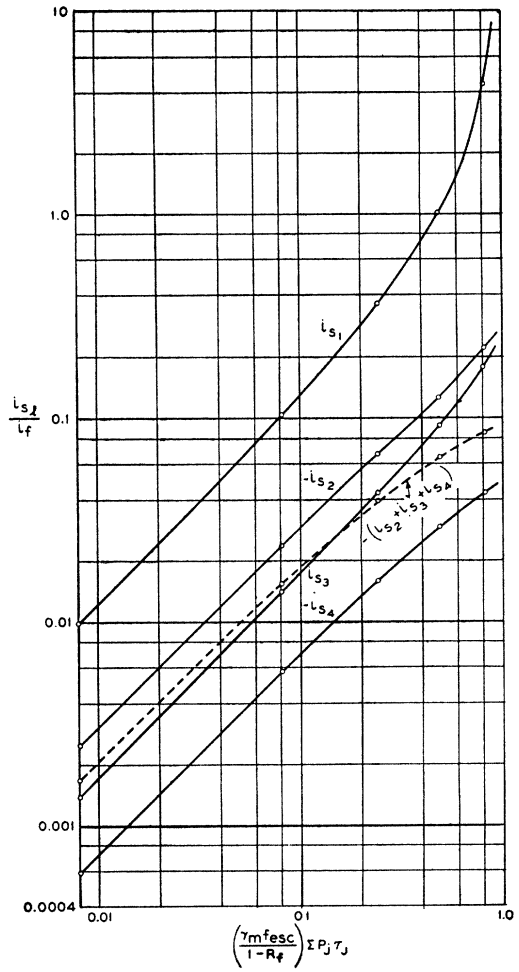

FIG. 5. The amplitudes of the exponential terms in the $i_s(t)$ series *vs* the quantity shown as the abscissas calculated for the case described in the text. In this $(\gamma_m f_{esc})(\Sigma P_j \tau_j)$ gives the number of electrons released at the cathode by the metastables generated by a single electron (and its progeny) leaving the cathode and crossing the gap. The factor $1/(1-R_f)$ gives the multiplication of such electrons by the fast secondary processes. Thus, the abscissa scale gives the total replacement factor by all secondary processes. When it becomes equal to unity, i_{s1} becomes indefinitely large. For small values of the replacement factor, the various i_{si}'s increase linearly, as indicated by the 45° slope of the curves.

of G and D_m are. For many situations β is closely equal to 4. Then Eq. (23) becomes

$$\tau_1 = T_1(i_f + 1.3i_{s2})/(i_f + i_{s0} + 1.3i_{s2}). \tag{24}$$

This is the formula we used to reduce T_1 and τ_1 in the experimental studies described in II.

Note that if β becomes large, then the constant factor multiplying i_{s2} in Eq. (23) approaches unity, and i_{s2} can be considered part of i_f for the purposes here. In the other limit of β approaching unity, Eq. (22) is no longer valid, so Eq. (23) does not hold. This case illustrates the problem of evaluating τ_1 when $P(t)$ is made of exponentials with time constants that differ little from term to term.

In Eq. (24) the quantities on the right side are all easily measurable, and only relative values of the current magnitudes need be known. Such quantities as γ_f, α_i, or i_0, which in general are harder to obtain, are not involved. Thus, in a given experimental situation τ_1 is quite readily evaluated.

G. CALCULATION OF $\alpha_m \gamma_m f_{esc}$ FROM EXPERIMENTAL DATA

Here we make no use of time constant data and use only the value of the total slow component of current, i_{s0}. We rearrange Eq. (17) to give

$$\gamma_m f_{esc} \sum_1^N P_j \tau_j = \frac{(1-R_f) i_{s0}}{i_f + i_{s0}}. \tag{25}$$

The quantity $\sum_{1,N} P_j \tau_j$ is just $\int_0^\infty P_K(t)dt$, in other words, the total number of metastables created by a single electron leaving the cathode which return to the cathode. For our purposes here, we may represent this number by f_{mk} times the total number of metastables created by this electron (and its progeny from ionization), where f_{mk} is the fraction of the metastables which diffuse to the cathode. Now the total number of metastables is found by integrating Eq. (9), which describes the initial distribution, giving

$$\sum_1^N P_j \tau_j = \frac{\alpha_m}{\alpha_i}\{\exp[\alpha_i(X - x_0)] - 1\} f_{mk}. \tag{26}$$

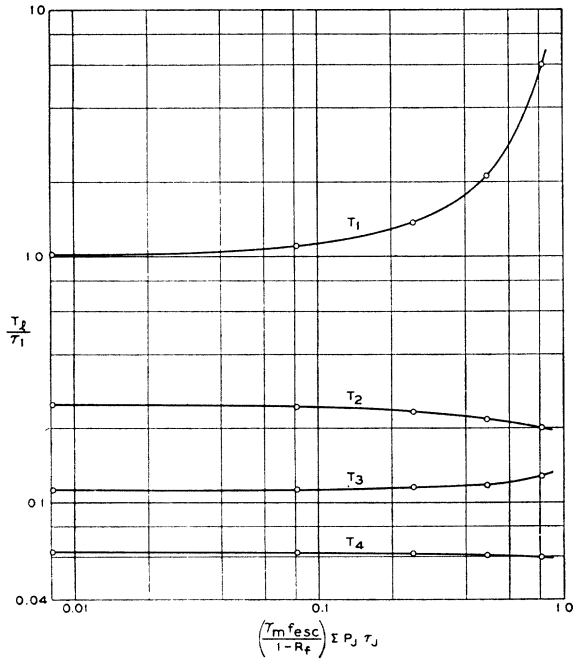

FIG. 6. Time constants, T_i, computed for the same case for which the amplitudes are given in Fig. 5. The time constants start with values in the ratio of $1:\frac{1}{4}:\frac{1}{9}:\frac{1}{16}$, as in the corresponding $P_K(t)$-function. Note that only in the case of T_1 is there a large deviation as the replacement factor approaches unity, while T_2, T_3, and T_4, vary only by a smaller amount. These variations are discussed by Newton [see reference 2, particularly Fig. 2].

Hence, we have finally

$$\alpha_m\gamma_m f_{esc}f_{mk}=\frac{\alpha_i(1-R_f)i_{s0}}{(i_f+i_{s0})\{\exp[\alpha_i(X-x_0)]-1\}}. \quad (27)$$

On the right-hand side, every term can be either directly measured or evaluated by fitting the Townsend equation to i_f/i_0 *vs* X data. Newton[2] showed that f_{mk} is given for the case of $G=0$, by

$$f_{mk}=\frac{1}{\alpha_i X}-\frac{1-x_0/X}{\exp[\alpha_i(X-x_0)]-1}. \quad (28)$$

For the more general case in which $G\neq 0$, we have computed values of f_{mk} by numerically evaluating

$$f_{mk}=\frac{\int_0^\infty P_K(t)dt}{\int_0^\infty P_K(t)+\int_0^\infty P_G(t)+\int_0^\infty P_A(t)dt}. \quad (29)$$

Some typical results are given in Fig. 7. Here we also plot f_{mr} and f_{ma}, which give the fraction of the metastables which are destroyed in the gas and at the anode, respectively.[11] We note that f_{mk} has a maximum value of about 0.4, and if G is not zero, it is less and decreasing at large values of X.

For rough evaluations and under conditions in which G is known to be small, Eq. (27) is probably useful with f_{mk} taken as 0.4. When G is not zero, then the second process of electron emission (i.e., by photoelectric effect of photons from "destroyed" metastables) may also be contributing to i_{s0}. For this situation we outline a method of analysis in the following section.

H. APPROXIMATE ANALYSIS FOR COMBINED PROCESSES

When both emission processes are active, then it is readily shown that Eq. (17) becomes

$$i_{s0}=i_f\frac{\left(\frac{f_{esc}}{1-R_f}\right)\left[\gamma_m\sum_1^N P_{Kj}\tau_j+\gamma_r f_{rk}'\sum_1^N P_{Gj}\tau_j\right]}{1-\left(\frac{f_{esc}}{1-R_f}\right)\left[\gamma_m\sum_1^N P_{Kj}\tau_j+\gamma_r f_{rk}'\sum_1^N P_{Gj}\tau_j\right]}, \quad (30)$$

where the subscripts K and G after P should be interpreted to indicate that the coefficients refer to the $P_K(t)$ and $P_G(t)$ series, respectively.

Now to reduce Eq. (30) to a manageable form, we make the approximation that $P_K(t)$ and $P_G(t)$ are each represented by the first term of their infinite series expansion. Then $f_{rk}'=0.5$, since the first term has a sine-wave distribution in x, and so exactly one-half of the photons go to each electrode. For P_{K1} and P_{G1} we apply Eqs. (12) and (14), and with these substitutions, Eq. (30) becomes

$$f_{esc}\left(\gamma_m+\gamma_r\frac{GX^2}{\pi^2D_m}\right)=\frac{(1-R_f)i_{s0}}{i_{s1}+i_f}\frac{(\pi D_m/X)+GX/\pi D_m}{a_1}. \quad (31)$$

If the integration indicated by Eq. (10) is carried out for a_1, and if we recall the definition of R_f and apply Eq. (1), then Eq. (31) becomes

$$\alpha_m f_{esc}\left(\gamma_m+\gamma_r\frac{GX^2}{\pi^2D_m}\right)=\alpha_i\frac{i_0 i_{s1}}{i_f(i_f+i_{s1})}\left(1+\frac{GX^2}{\pi^2D_m}\right)H, \quad (32)$$

where

$$\frac{1}{H}\equiv\frac{2\alpha_i X}{(\alpha_i X)^2+\pi^2}\left\{1-e^{(\alpha_i x_0-\alpha_i X)}\left[\frac{\alpha_i X}{\pi}\sin\left(\pi\frac{\alpha_i X}{\alpha_i x_0}\right)-\cos\left(\pi\frac{\alpha_i x_0}{\alpha_i X}\right)\right]\right\} \quad (33)$$

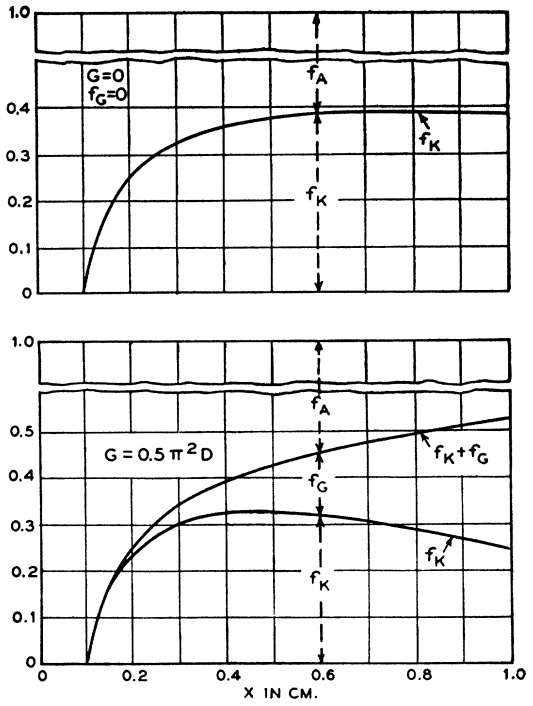

FIG. 7. The fraction of metastables lost at the cathode, f_k, in the gas, f_G, and at the anode, f_A, plotted as function of the electrode spacing for two values of G and an assumed initial distribution of metastables given by

$$\rho=0, \quad x\leq 0.1,$$
$$\rho=e^{(x-0.1)}, \quad x\geq 0.1.$$

Note that in the text the notation f_{mk}, f_{mr}, f_{ma} is used for these quantities.

[11] For the limiting case in which the P-functions are represented by a single exponential, it is readily shown that

$$f_{mk}=0.5\pi^2D_m/(\pi^2D_m+GX^2).$$

The curves in Fig. 7 can be closely approximately at large X by this expression multiplied by 0.76. Note also that if D_m is inversely proportional to the gas pressure, and G is proportional to the pressure, then f_{mk} is a function of pX alone.

Experimentally, we can evaluate $\alpha_i x_0$ and $\alpha_i X$ from studies of i_f/i_0 as a function of X. Thus, we know H. Similarly, we can evaluate D_m and G from a study of the variation of τ_1 with X. Hence, all quantities on the right side of Eq. (32) can be measured experimentally, and so a plot of the right side against the quantity $GX^2/\pi^2 D_m$ should give a straight line with a slope of $\alpha_m \gamma_r f_{esc}$ and a y intercept of $\alpha_m \gamma_m f_{esc}$. This is the method used to interpret some of the experimental data presented in II.

I. DISCUSSION AND SUMMARY

The form of the slow component of current, $i_s(t)$, in a pulsed Townsend discharge is closely described by a single exponential plus second term of higher time constant much smaller in amplitude. This term is negative in amplitude when the metastables initiate a current by releasing electrons in the process of bombarding the cathode. The amplitude is positive (i.e., adds to the first term) when the electrons are released through photoelectric action of photons from metastables converted in the gas.

From a study of experimental current patterns one can get with little trouble the fundamental time constant of decay, τ_1, of the metastables in the gap. This is valuable, because by studying the variation of τ_1 with electrode spacing and gas pressure, the diffusion constant and volume destruction probability of metastables are readily obtained. In addition the quantities $\alpha_m \gamma_m$ and $\alpha_m \gamma_r$ can be obtained. Applications of these methods of analysis are described in II.

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Studies of γ-Processes of Electron Emission Employing Pulsed Townsend Discharges on a Millisecond Time Scale

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The relative amounts of electron emission from the cathode in a Townsend discharge caused by ions, photons, and metastables have been studied experimentally for several cathodes in argon, using pulsed-light stimulation of the discharge. The current initiated by metastables exhibits a slow build-up and decay, thus permitting easy separation from the faster rising effects of gas ionization and electron emission by photons and ions. Time constant studies of the slow component yielded a diffusion constant for metastable argon atoms of 45 cm^2 sec^{-1} at one millimeter pressure. The efficiencies of electron emission by metastables and ions was found to be closely the same, while the quantum yield for photon emission was found to be generally smaller.

I. INTRODUCTION

ELECTRON emission from the cathode in Townsend or glow discharges is generally believed to arise from the bombardment of the cathode by ions, metastable atoms, and photons.[1,2] These processes are commonly called the γ-processes, or sometimes the γ-mechanism, after the coefficient γ used in the Townsend equation to describe the part of the amplification of an electron current between electrodes in a gas attributable to these processes. The experiments in this paper were directed towards an evaluation of the relative amount of electron emission produced by the ions, the metastables, and the photons in the case of argon gas with several cathode materials. An incidental by-product of these studies was a determination of the lifetime of metastable argon atoms.

The method employed was similar to that described by Engstrom and Huxford.[3] A Townsend discharge was stimulated by photoelectrons generated by a shuttered light beam shone on the cathode of a gas-filled tube, and the transient character of the resultant current between the electrodes was observed by an oscilloscopic technique. The current is found to be composed of a component closely in step with the stimulating light pulse and a component which lags by an amount of the order of a millisecond. The second component is initiated by the action of metastables, which have lifetimes in the discharge space of this amount of time. The fast component includes the primary electron current amplified by gas ionization and electron emission from the cathode caused by ion and photon effects, all of which reach a steady-state value in a time of the order of ten microseconds.[4] From an analysis of these patterns the fractions of the electron emission produced by metastables and by ions and photons were obtained.[5]

[1] L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939).
[2] M. J. Druyvesteyn and F. M. Penning, Revs. Modern Phys. **12**, 87 (1940).
[3] R. W. Engstrom and W. S. Huxford, Phys. Rev. **58**, 67 (1940).
[4] A parallel study of pulsed Townsend discharges on a microsecond time scale, in which these effects can be resolved, has been carried out by J. A. Hornbeck of these laboratories.
[5] A theoretical analysis of transient form of the current in the Townsend discharge under these conditions is described in a companion paper (hereafter referred to as I), and reference will be made to the relations derived in that paper.