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The Interaction of the Dirac Magnetic Monopole with Matter*

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It has been previously determined that an electron has no bound states in the field of the magnetic monopole. Seeking to establish the character of the monopole's interaction with the more complex fields of atoms and molecules, this paper investigates charged particles, of arbitrary magnetic moment, moving simultaneously in the field of the monopole and an external electric field. It is concluded that the monopole can be coupled to matter with energies comparable to, but not significantly greater than, the chemical bond, reservations being made in the case of hydrogen where the lowest energy state depends upon the mass of the monopole.

Speculation regarding the creation of monopoles by primary cosmic radiation and their consequent motion in the earth's magnetic field instigated an experimental attempt to arrive at an upper limit for the rate of such creation. The results of this experiment determine that the number of monopoles arriving at the surface of the earth is less than 10^{-10} per cm² per sec.

INTRODUCTION

PAST work[1] has shown that in the absence of an external electric field, an electron cannot be bound to the magnetic monopole.[2] Normal matter, however, is built upon such external electric fields. Hence, this work was undertaken to determine whether the presence of the magnetic monopole could reduce the energy of the electronic structure of an atom and so lead to bound states. The problem is clearly quite complex and is approached by consideration of the extreme cases. The total electric energy of an atom is therefore determined with the monopole first at or very near the nucleus, and second, removed to considerable distance. Conclusions may then be drawn concerning the magnitude of binding energy for intermediate cases.

The program of this investigation is: (I) to determine the eigenstructure of a charged particle with arbitrary magnetic moment in the field of a magnetic monopole and, using these results, to discuss the interaction energy of the atomic nuclei and the monopole; (II) neglecting, for the moment, the foregoing interaction energy, to find the eigenstructure of an electron in the combined field of a monopole and an atomic nucleus both situated at the origin; (III) to perform a variation-perturbation calculation on the many-electron problem built with the eigenfunctions found in (II) in order to determine whether the resulting total electronic energies are greater or less than that of the corresponding normal atom; (IV) to consider the approximate diamagnetic and paramagnetic energies of an electronic structure at some distance from a magnetic monopole, and to summarize these various findings in a conclusion regarding the monopole's interaction with matter.

In the last section, consideration of the possibility of monopole creation by cosmic radiation and the consequent motion of these particles in the earth's magnetic field leads to the description of an experiment which has set an upper limit on the arrival of monopoles at the earth's surface.

* This paper is based on a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Chicago.

[1] P. Banderet, Helv. Phys. Acta **19**, 503 (1946).

[2] E. Teller has observed that this conclusion is an immediate consequence of the Dirac equation for an electrically charged particle, since in the absence of an external electric field, yet with any magnetic field describable by a vector potential, Dirac's equation can have no energy eigenvalue whose absolute magnitude is less than mc².

I. THE DETERMINATION OF THE EIGENSTRUCTURE OF A SPIN 1/2 PARTICLE OF CHARGE $Z|e|$ WITH ARBITRARY MAGNETIC MOMENT IN THE FIELD OF A MAGNETIC MONOPOLE

1. A vector potential of a monopole of charge M, situated at $r=0$, satisfying $\mathrm{div}\mathbf{A}=0$, $\mathrm{curl}\mathbf{A}=\mathbf{H}=M\mathbf{r}/r^3$, is

$$A_\phi=M/r(1-\cos\theta/\sin\theta)$$

and

$$A_\theta = A_r = 0. \qquad \text{(I 1.1)}$$

2. The separated hamiltonian, to the Pauli approximation including an arbitrary radial electric field, but neglecting the spin orbit interaction and terms of higher order, is:

$$H = \frac{1}{2\mu}\left(\mathbf{P} - \frac{Z|e|}{c}\mathbf{A}\right)^2 + V(r) - B_z \frac{|e|\hbar}{2m_z c}\frac{M}{r^2}\sigma_r, \qquad \text{(I 2.1)}$$

where B_z is the number of magnetons carried by the particle of charge $Z|e|$ and mass m_z. $B_z = Z$ for a Dirac particle. H may be written

$$H = -\frac{\hbar^2}{2\mu r^2}\left\{\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{\sin^2\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \left\{\frac{\partial}{\partial\phi} - i\kappa_z(1-\cos\theta)\right\}^2\right] + \gamma\sigma_r\right\} + V(r), \qquad \text{(I 2.2)}$$

in which

$$\gamma = B_z(\mu/m_z)\kappa_{z=1} \qquad \text{(I 2.3)}$$

and

$$\kappa_z = Z|e|M/\hbar c \qquad \text{(I 2.4)}$$

where, as first concluded by Dirac,[3] $2\kappa_z$ must be an integer.

3. Solution of the angular operator when $\gamma = 0$. Consider

$$\mathfrak{L}^2 = \frac{1}{\sin^2\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \left\{\frac{\partial}{\partial\phi} - i\kappa_z(1-\cos\theta)\right\}^2\right], \qquad \text{(I 3.1)}$$

which commutes with $L_z = -i\hbar(\partial/\partial\phi)$ (but not L^2).

Hence, if it is assumed that

$$\mathfrak{L}^2\Theta\Phi = -\beta_0\Theta\Phi,$$

then

$$\Phi = e^{im\phi}$$

where

$$m = 0, \pm 1, \pm 2, \cdots. \qquad \text{(I 3.2)}$$

Therefore

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\{m - \kappa_z(1-\cos\theta)\}^2\right]\Theta = -\beta_0\Theta. \qquad \text{(I 3.3)}$$

Now let

$$x = \sin^2\tfrac{1}{2}\theta = (1-\cos\theta)/2. \qquad \text{(I 3.4)}$$

Then

$$\left[\frac{\partial}{\partial x}x(1-x)\frac{\partial}{\partial x} - \frac{(m/2)^2}{x(1-x)} - \frac{\kappa_z(\kappa_z - m)}{(1-x)} + \kappa_z^2\right]\Theta = -\beta_0\Theta. \qquad \text{(I 3.5)}$$

[3] P. Dirac, Proc. Roy. Soc. (London) **A133**, 60 (1931); Phys. Rev. **74**, 817 (1948).

The form of the two middle terms suggests trying

$$\Theta = x^{\frac{1}{2}|m|}(1-x)^{\frac{1}{2}|m-2\kappa_z|}u, \qquad \text{(I 3.6)}$$

which leads to

$$x(1-x)\partial^2 u/\partial x^2 + [(1+|m|) - 2(1+P)x]\partial u/\partial x - [P(P+1) - \kappa_z^2]u = -\beta_0 u, \qquad \text{(I 3.7)}$$

where

$$P = \tfrac{1}{2}(|m| + |m - 2\kappa_z|). \qquad \text{(I 3.8)}$$

If u is written as a series in x,

$$u = \sum_n C_{n+s}x^{n+s} \qquad \text{(I 3.9)}$$

then $s = 0, -|m|$, the latter indicial being unacceptable and

$$\frac{C_{n+1}}{C_n} = \frac{[n(n-1) + 2(1+P)n + P(P+1) - \kappa_z^2 - \beta]}{[n(n+1) + (1+|m|)(n+1)]}. \qquad \text{(I 3.10)}$$

The condition that this series terminates is

$$\beta_0 = l'(l'+1) - \kappa_z^2, \qquad \text{(I 3.11)}$$

where

$$l' = n + P = |\kappa_z|, |\kappa_z| + 1, |\kappa_z| + 2\cdots. \qquad \text{(I 3.12)}$$

This eigenvalue has previously been obtained by Tamm[4] and Fierz.[5]

4. The solution of the angular operator for arbitrary γ. Since

$$\sigma_r = \begin{vmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{+i\phi} & -\cos\theta \end{vmatrix},$$

consider

$$\mathfrak{J}^2 = \begin{vmatrix} \mathfrak{L}^2 + \gamma\cos\theta & \gamma\sin\theta e^{-i\phi} \\ \gamma\sin\theta e^{+i\phi} & \mathfrak{L}^2 - \gamma\cos\theta \end{vmatrix} \qquad \text{(I 4.1)}$$

where $\mathfrak{J}^2$ commutes with $J_z = -\hbar(\partial/\partial\phi) + \frac{1}{2}\hbar\sigma_z$.

Hence, if it is assumed that $\mathfrak{J}^2\Theta\Phi = -\beta\Theta\Phi$, then

$$\Phi = \begin{vmatrix} e^{i(m-1)\phi} \\ e^{im\phi} \end{vmatrix}, \quad m = 0, \pm 1, \pm 2, \cdots. \qquad \text{(I 4.2)}$$

Therefore,

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\{(m-1) - \kappa_z(1-\cos\theta)\}^2 + \gamma\cos\theta\right]\Theta_1 + \gamma\sin\theta\Theta_2 = -\beta\Theta_1$$

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\{m - \kappa_z(1-\cos\theta)\}^2 - \gamma\cos\theta\right]\Theta_2 + \gamma\sin\theta\Theta_1 = -\beta\Theta_2. \qquad \text{(I 4.3)}$$

[4] J. Tamm, Z. Physik **71**, 141 (1931).

[5] M. Fierz, Helv. Phys. Acta **17**, 27 (1944).

As in (I 3.3) to (I 3.7), the form of solution attempted is

$$\Theta_1 = x^{\frac{1}{2}|m-1|}(1-x)^{\frac{1}{2}|m-1-2\kappa_z|}u,$$
$$\Theta_2 = x^{\frac{1}{2}|m|}(1-x)^{\frac{1}{2}|m-2\kappa_z|}v \qquad \text{(I 4.4)}$$

leading to

$$x(1-x)\frac{\partial^2 u}{\partial x^2}+[(1+|m-1|)-2(1+\bar{P})x]\frac{\partial u}{\partial x}-\bar{\alpha}u$$
$$+\gamma(1-2x)u+2\gamma(1-x)^{1-A}x^{1-B}v=-\beta u$$
$$x(1-x)\frac{\partial^2 v}{\partial x^2}+[(1+|m|)-2(1+P)x]\frac{\partial v}{\partial x}-\alpha v \qquad \text{(I 4.5)}$$
$$-\gamma(1-2x)v+2\gamma(1-x)^A x^B u=-\beta v,$$

with $\alpha=P(P+1)-\kappa_z^2$ where P is defined in Eq. (I 3.8) and $\bar{P}$ and $\bar{\alpha}$ are similar expressions with (m) replaced by $(m-1)$, and

$$A=1 \text{ for } (m-2\kappa_z)\leqslant 0, \quad B=1 \text{ for } m\leqslant 0$$
$$A=0 \text{ for } (m-2\kappa_z)>0, \quad B=0 \text{ for } m>0. \qquad \text{(I 4.6)}$$

The symbol ϵ_{AB} is used to distinguish the four cases, where

$$\epsilon_{10}=\begin{cases}1 \text{ when } (m-2\kappa_z)\leqslant 0,\ m>0\\ 0 \text{ all other cases}\end{cases}$$

$$\epsilon_{01}=\begin{cases}1 \text{ when } (m-2\kappa_z)>0,\ m\leqslant 0\\ 0 \text{ all other cases}\end{cases}$$

$$\epsilon_{00}=\begin{cases}1 \text{ when } (m-2\kappa_z)>0,\ m>0\\ 0 \text{ all other cases}\end{cases}$$

$$\epsilon_{11}=\begin{cases}1 \text{ when } (m-2\kappa_z)\leqslant 0,\ m\leqslant 0\\ 0 \text{ all other cases.}\end{cases}$$

This notation permits writing all four cases at once.

If the further substitution

$$u=(-1)^{\epsilon_{01}+\epsilon_{00}}x^{\epsilon_{00}}u' \quad \text{and} \quad v=x^{\epsilon_{11}}v' \qquad \text{(I 4.7)}$$

is made, then

$$x(1-x)\partial^2u'/\partial x^2+[\epsilon_{00}2(1-x)+(1+|m-1|)$$
$$-2(1+\bar{P})x]\partial u'/\partial x-[\bar{\alpha}+\epsilon_{00}\{2(1+\bar{P})$$
$$-(1+|m-1|)/x\}]u'+\gamma[(1-2x)u'$$
$$+2(-1)^{\epsilon_{01}+\epsilon_{00}}(1-x)^{\epsilon_{01}+\epsilon_{00}}x^{\epsilon_{11}+\epsilon_{10}}v']=-\beta u' \qquad \text{(I 4.8)}$$

and

$$x(1-x)\partial^2v'/\partial x^2+[\epsilon_{11}2(1-x)+(1+|m|)$$
$$-2(1+P)x]\partial v'/\partial x-[\alpha+\epsilon_{11}\{2(1+P)$$
$$-(1+|m|)/x\}]v'-\gamma[(1-2x)v'$$
$$-2(-1)^{\epsilon_{01}+\epsilon_{00}}(1-x)^{\epsilon_{11}+\epsilon_{10}}x^{\epsilon_{01}+\epsilon_{00}}u']=-\beta v', \qquad \text{(I 4.9)}$$

where it is to be noted that in no case (ϵ_{AB}) does an x^2 term actually appear in the brackets multiplied by γ. In subtracting the foregoing equations,

$$x(1-x)\frac{\partial^2(u'-v')}{\partial x^2}-2(1+P+\epsilon_{11})x\frac{\partial(u'-v')}{\partial x}$$
$$-(\alpha+2\epsilon_{11}(1+P))(u'-v')+(-1)^{\epsilon_{11}+\epsilon_{10}}\gamma(u'-v')$$
$$+(1+|m|+2\epsilon_{11})\frac{\partial(u'-v')}{\partial x}+\epsilon_{00}\frac{|m|}{x}u'-\epsilon_{11}\frac{1+|m|}{x}v'$$
$$+\{2(\epsilon_{01}+\epsilon_{00})-1\}\frac{\partial u'}{\partial x}=-\beta(u'-v'), \qquad \text{(I 4.10)}$$

since

$$1+P+\epsilon_{11}=1+\bar{P}+\epsilon_{00}; \quad \alpha+2\epsilon_{11}(1+P)=\bar{\alpha}+2\epsilon_{00}(1+\bar{P}).$$

Now expressing u' and v' as descending series in x,

$$u'=x^n+C_1x^{n-1}+\cdots; \quad v'=x^n+C_2x^{n-1}+\cdots \qquad \text{(I 4.11)}$$

then the coefficient of x^n in (I 4.9) is

$$-n(n-1)-2(1+P+\epsilon_{11})n-[\alpha+2\epsilon_{11}(1+P)]$$
$$-(-1)^{\epsilon_{11}+\epsilon_{10}}-2\gamma(C_1-C_2)=-\beta, \qquad \text{(I 4.12)}$$

while the coefficient of x^{n-1} in (I 4.10) is

$$(C_1-C_2)[(n-1)(n-2)+2(1+P+\epsilon_{11})(n-1)$$
$$+\{\alpha+2\epsilon_{11}(1+P)\}-(-1)^{\epsilon_{11}+\epsilon_{10}}-\beta]$$
$$=[1-2(\epsilon_{11}+\epsilon_{10})]n+|m|(\epsilon_{00}-\epsilon_{11})-\epsilon_{11}. \qquad \text{(I 4.13)}$$

Eliminating (C_1-C_2) between these last two equations, and letting $l''=n+P+\epsilon_{11}$, then

$$\beta=l''^2-\kappa_z^2\pm[l''^2+2\gamma\{|m|(\epsilon_{11}-\epsilon_{00})$$
$$-P(-1)^{\epsilon_{11}+\epsilon_{10}}\}+\gamma^2]^{\frac{1}{2}},$$

which reduces to

$$\beta=l''^2-\kappa_z^2\pm[l''^2-\kappa_z^2+(\kappa_z-\gamma)^2]^{\frac{1}{2}}. \qquad \text{(I 4.14)}$$

Since ϵ_{11} is implicitly contained in β, it is here represented in terms of m and κ_z,

$$\epsilon_{11}=\tfrac{1}{4}(|m-2\kappa_z-1|+1$$
$$-|m-2\kappa_z|)(|m-1|+1-|m|). \qquad \text{(I 4.15)}$$

To understand the lowest roots of β, it is observed that both when $\epsilon_{01}=1$ and $\epsilon_{10}=1$, $P=|\kappa_z|$ and therefore $l''^2-\kappa_z^2=0$ for $n=0$. In these cases $C_1=C_2=0$ when $n=0$ and Eq. (I 4.13) becomes identically zero so that Eq. (I 4.12) alone determines β. Indeed when $\epsilon_{10}=1$, $\beta=\kappa_z-\gamma$ while when $\epsilon_{01}=1$, $\beta=\gamma-\kappa_z$, which are the two possible roots of the general form (I 4.14) when $l''^2-\kappa_z^2=0$. However when $\epsilon_{10}=1$, $\kappa_z>0$ while when $\epsilon_{01}=1$, $\kappa_z<0$, hence this lowest root may be written

$$\beta=|\kappa_z|-(\kappa_z/|\kappa_z|)\gamma \qquad \text{(I 4.16)}$$

in agreement with the results of (I 3.10) as γ approaches zero.

TABLE I. The energy levels and degeneracies of normal hydrogen compared with those of Eq. (II 1.8).

Degeneracy	n'	Hydrogen degeneracy
1	1	2
3	$\sqrt{2}$	
1	2	8
6	$1+\sqrt{2}$	
5	$\sqrt{6}$	
1	3	18
6	$2+\sqrt{2}$	
10	$1+\sqrt{6}$	
7	$\sqrt{12}$	
1	4	32
6	$3+\sqrt{2}$	
10	$2+\sqrt{6}$	
14	$1+\sqrt{12}$	
9	$\sqrt{20}$	
1	5	50

When the monopole is very massive compared to the charged particle, then the reduced mass $\mu\cong m_z$, and if this is true for the electron, where $Z=-1$, $B_z=-1$ and if κ_z has its smallest value $\kappa_{z=-1}=-\frac{1}{2}(M/|M|)$, $\gamma=-\frac{1}{2}(M/|M|)$ then β can be reduced to

$$\beta=l(l+1)\pm[l(l+1)]^{\frac{1}{2}}, \qquad \text{(I 4.17)}$$

where $l=0, 1, 2, \cdots$. This last result has also been obtained by Banderet[1] as the eigenvalue of the angular operator in the relativistic treatment for an electron in the field of an infinitely massive monopole.

5. The lowest states of the atomic nuclei in the field of the monopole.

The radial equation is, in this case

$$(1/r^2)(\partial/\partial r)r^2(\partial/\partial r)R-(k^2+\beta/r^2)R=0, \qquad \text{(I 5.1)}$$

where $k^2=-2\mu E/\hbar^2$ and μ is the reduced mass of the monopole-nucleus system.

For the spin zero nuclei, the lowest value of β is $\beta_1=|\kappa_z|$ from (I 3.11), while for nuclei where $|\kappa_z|\gg\gamma$ (i.e., $1\gg|B_z|$), $\beta_1\cong|\kappa_z|$ from (I 4.16). However, in the case of the proton where $B_{z=1}=2.79$,

$$\beta_1=|\kappa_{z=1}|[1-B_{z=1}(\mu/m_{z=1})]$$

from (I 4.16), (I 2.2), and it is seen that this may be negative for sufficiently large values of the reduced mass.

TABLE II. The total electronic energies of normal atoms and atoms with a monopole in their nucleus, assuming no electronic interaction. Energy in units of $Z^2(-e^2/2a)$.

Z	Atom	Normal	Monopole
1	H	1	1
2	He	2	1.500
3	Li	2.250	2
4	Be	2.500	2.500
5	B	2.750	2.750
6	C	3	2.922
10	Ne	4	3.508
16	S	4.666	4.613
28	Ni	6	5.575
40	Zr	6.750	6.662
60	Nd	8	7.638
80	Hg	8.800	8.690

It has been shown[6] that Eq. (I 5.1) has only solutions of positive energy for $\beta\geqslant-\frac{1}{4}$ but when $\beta<-\frac{1}{4}$ the interaction is such that arbitrarily low energy eigenvalues may be found. Hence, relativistic effects must be considered around this critical value of β. The monopole, then, can be bound to the proton if its mass is comparable to the proton mass, but the exact character of this situation is not clear.[7]

On the other hand, these considerations indicate that in the interaction of the magnetic monopole with all other atomic nuclei, no bound state exists.

II. THE ENERGY LEVELS AND EIGENFUNCTIONS OF AN ELECTRON IN THE FIELD OF A MONOPOLE AND A PARTICLE OF CHARGE $Z|e|$ BOTH SITUATED AT THE ORIGIN

1. The radial equation obtained from separation of the more general hamiltonian of Sec. A is:

$$(1/r^2)(\partial/\partial r)r^2(\partial/\partial r)R-(k^2+U(r)+\beta/r^2)R=0 \qquad \text{(II 1.1)}$$

where $k^2=-2\mu/\hbar^2 E$ and $U(r)=-(2\mu/\hbar^2)(Ze^2/r)$. For the electron then, where $\beta=l(l+1)\pm[l(l+1)]^{\frac{1}{2}}$ when $|\kappa_z|=\frac{1}{2}$, and for positive Z, we consider negative E.

Let $\rho=2kr$ and let $n'=(\mu/\hbar^2)(Ze^2/k)$, exactly as for the normal Laguerre polynomials, then

$$(1/\rho^2)(\partial/\partial\rho)\rho^2(\partial/\partial\rho)R +(-\tfrac{1}{4}+n'/\rho-\beta/\rho^2)R=0 \qquad \text{(II 1.2)}$$

asymptotically $R=e^{-\frac{1}{2}\rho}F$,

$$F''+\left(\frac{2}{\rho}-1\right)F'+\left[\frac{n'-1}{\rho}-\frac{\beta}{\rho^2}\right]F=0 \qquad \text{(II 1.3)}$$

let $F=\rho^s L(\rho)$,

$$s=\frac{-1+(1+4\beta)^{\frac{1}{2}}}{2}=|\tfrac{1}{2}\pm[l(l+1)]^{\frac{1}{2}}|-\tfrac{1}{2} \qquad \text{(II 1.4)}$$

the other root leading to unacceptable origin divergence. Therefore

$$\rho L''+[2(s+1)-\rho]L'+(n'-s-1)L=0, \qquad \text{(II 1.5)}$$

where L is a polynomial of order n'' if

$$n'=n''+s+1=n''+\tfrac{1}{2}+|\tfrac{1}{2}\pm[l(l+1)]^{\frac{1}{2}}|. \qquad \text{(II 1.6)}$$

Hence,

$$R=e^{-\frac{1}{2}\rho}\rho^s L(\rho) \qquad \text{(II 1.7)}$$

[6] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1933), p. 40.

[7] A paper is being prepared which considers a monopole with spin in its interaction with the proton. The results indicate that the monopole mass which gives rise to the critical value, $\beta=-\frac{1}{4}$, is equal to the proton mass.

TABLE III. The unnormalized eigenfunctions of an electron in the field of a charged particle and a monopole, from Eqs. (I 4.4) and (II 1.7).

$$\psi_{100}=(-\sin\tfrac{1}{2}\theta e^{-i\phi}\uparrow+\cos\tfrac{1}{2}\theta\downarrow)e^{-zr/a}$$

$$\psi_{\sqrt{2}0+1}=(-\cos\tfrac{1}{2}\theta[\sin^2\tfrac{1}{2}\theta+(3-\beta)]\uparrow+\sin\tfrac{1}{2}\theta\cos^2\tfrac{1}{2}\theta e^{+i\phi}\downarrow)e^{-zr/\sqrt{2}a}r^{\sqrt{2}-1}$$

$$\psi_{\sqrt{2}10}=\left(\sin\tfrac{1}{2}\theta\left[\cos^2\tfrac{1}{2}\theta-\frac{1}{\beta}\right]e^{-i\phi}\uparrow+\cos\tfrac{1}{2}\theta\left[\sin^2\tfrac{1}{2}\theta-\frac{1}{\beta}\right]\downarrow\right)e^{-zr/\sqrt{2}a}r^{\sqrt{2}-1}$$

$$\psi_{\sqrt{2}0-1}=(\cos\tfrac{1}{2}\theta\sin^2\tfrac{1}{2}\theta e^{-i2\phi}\uparrow-\sin\tfrac{1}{2}\theta[\cos^2\tfrac{1}{2}\theta+(3-\beta)]e^{-i\phi}\downarrow)e^{-zr/\sqrt{2}a}r^{\sqrt{2}-1}$$

$$\psi_{200}=(-\sin\tfrac{1}{2}\theta e^{-i\phi}\uparrow+\cos\tfrac{1}{2}\theta\downarrow)e^{-\frac{1}{2}zr/a}(1-\tfrac{1}{2}zr/a)\quad\text{where}\quad\beta=2-\sqrt{2}.$$

and

$$E_{n'}=-(e^2/2a)(Z^2/n'^2), \qquad \text{(II 1.8)}$$

where $a=\hbar^2/\mu e^2$.

The multiplicity of each eigenvalue in Eq. (II 1.8) follows from the $(2l+1)$-fold degeneracy of each l value and from the fact that, for a given $l\geqslant 1$, each n' value resulting from (II 1.6) with the upper sign is obtained once more by using the lower sign and replacing n'' by $n''+1$.

2. Table I implicitly compares the energy levels of Eq. (II 1.8) and the degeneracy of each level with those of normal hydrogen. The table actually lists the values of this parameter n' of (II 1.6) which is inversely proportional to the square root of the energy.

Table II compares the total electronic energies of various normal atoms with those containing a monopole on the assumption that there is no electronic interaction. The values tabulated after $Z=6$ are alternately those corresponding to maxima and minima of the energy difference between the normal and monopole atoms. It is to be noted that, in this approximation, the monopole increases the total electronic energy in all cases except hydrogen, beryllium, and boron. A perturbation calculation should indicate whether the monopole-electron interaction can cause binding of the monopole in beryllium and boron, while the total energy in hydrogen depends upon the monopole-proton interaction discussed in (I 5).

Table III lists the unnormalized eigenfunctions, obtained from (I 4.4), (I 4.7), (I 4.11), and (II 1.7), which are necessary in the calculation mentioned above. The vectors $\uparrow$ and $\downarrow$ indicate the orthogonal eigenfunctions of σ_z. The total eigenfunction is identified by the three indices n', n, and m, where n' is the energy parameter of (II 1.6), while n and m are the parameters defining the angular part of the wave function as in (I 4.4) and (I 4.11).

3. The perturbation variation computation of the total electronic energies of the beryllium-like and boron-like atoms proceeds in a conventional and somewhat tedious fashion. The technique and results were checked by performing the identical manipulations to approximate the energies of the normal atoms as well. These computed energies are listed in Table IV and compared with the observational values for the normal atoms. These results demonstrate that a monopole at or near an atomic nucleus markedly increases the total electronic energy. However one might have expected that normally paramagnetic atoms would attract the monopole. While such attraction is overcompensated by diamagnetic effects when the monopole is at $r=0$, the behavior of the monopole at other distances must still be investigated.

III. APPROXIMATE CONSIDERATIONS WITH THE MONOPOLE AT SOME DISTANCE FROM THE NUCLEUS

1. The previous sections have determined the energetic consequences of a magnetic monopole near the nucleus of the atom. The problem of the monopole at an arbitrary distance from the nucleus is far more complex and is dealt with here in a most approximate way.

When the monopole is at considerable distance from an electronic structure, the change in its energy, ΔE, is the sum of the normal paramagnetic and diamagnetic terms,

$$\Delta E=E_p+E_d=-(e\hbar/2\mu c)\mathcal{H}\textstyle\sum_i(m_z{}^i+2s_z{}^i) + (e^2/\mu c^2)\mathcal{H}^2\textstyle\sum_i(x_i^2+y_i^2) \qquad \text{(III 1.1)}$$

where the magnetic field is $\mathcal{H}=M/R^2$ and R is the distance of the monopole of charge M from the structure, where i is summed over the various electrons and $m_z{}^i$, $s_z{}^i$, x_i^2, and y_i^2 are the eigenvalues of the usual operators of the z component of angular momentum, spin, and coordinate position, respectively.

In an attempt to judge the minimum energy a normally paramagnetic substance may have in the field of the monopole, the minimum of ΔE will be found for the case of the hydrogen atom where $m_z=0$, $s_z=\frac{1}{2}$, and $x^2=y^2=(\hbar^2/\mu e^2)^2=a_0^2$. Then

$$\partial\Delta E/\partial R=0=2eM\hbar/2\mu cR^3-4e^2M^2\cdot 2a_0^2/8\mu c^2R^5 \qquad \text{(III 1.2)}$$

TABLE IV. The computed total electronic energies of helium, beryllium, and singly ionized boron for normal and monopole atoms. Energy in units of $(-e^2/2a)$.

		Perturbation computation		Observed
Z	Atom	Normal	Monopole	Normal
2	He	5.70	4.63	5.81
4	Be	28.4	25.2	29.35
5	B^+	47.2	43.5	48.76

but $eM/\hbar c = \frac{1}{2}$, so $(\hbar^2/2\mu R^3)(1-a_0^2/2R^2)=0$. Hence,

$$R = a_0/\sqrt{2} \qquad \text{(III 1.3)}$$

and the minimum energy

$$\Delta E_{\min} = -\hbar^2/2\mu a_0^2 + \hbar^2/4\mu a_0^2 = -6.76 \text{ ev.} \qquad \text{(III 1.4)}$$

2. This last result is drawn in a region of R outside the valid range of the approximation (III 1.1). However the use of a trial wave function and the complete hamiltonian corresponding to (III 1.1) permits the establishment of both an upper and lower limit for the binding energy. This has been done for a trial function similar to ψ_{100} in Table III and, after lengthy computation leads to a maximum value for $\Delta E_{\min} \simeq -7$ ev at $R \simeq 0.56 a_0$, quite similar to the approximation (III 1.4). For the many-electron configuration, one sees from Eq. (III 1.1) that while each electron still contributes a diamagnetic term, only the few electrons in unfilled shells or excited states may be paramagnetic. Hence, one would anticipate that the monopole could be coupled to matter with energies comparable to the chemical bond but not significantly greater.

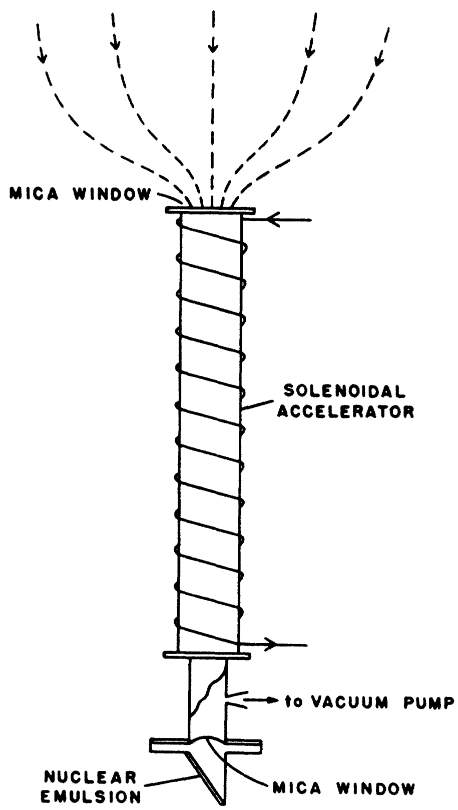

FIG. 1. Schematic diagram of an instrument to detect magnetic monopoles arriving at the earth's surface.

Of course, no estimate of nuclear forces is possible and these may be attractive, repulsive, or neutral. However, a sizable barrier confronts the monopole before it can be influenced by the nucleus.

IV. SOME CONSEQUENCES OF THE CREATION OF MAGNETIC MONOPOLES BY PRIMARY COSMIC RADIATION AND EXPERIMENTAL CONSIDERATONS WHICH SET AN UPPER LIMIT FOR SUCH CREATION

1. The energy needed for the production of a monopole pair is available in the primary cosmic radiation even if the monopole is considerably heavier than the proton.[8] Once created[9] in the atmosphere the initially energetic monopole would reach a low terminal velocity in the earth's magnetic field in a few meters because of its large and velocity independent ionization loss (about 5 Mev/cm N.T.P.). No conventional cosmic-ray techniques would have detected these monopoles moving along the field lines at terminal velocity while the relatively few energetic monopoles created near a nuclear emulsion or a cloud chamber would probably stop in the protective covering of these instruments. For example, one-fourth of a millimeter of brass would stop a monopole with 1 Bev of energy.

If the monopole were strongly bound to matter by unknown nuclear forces or, if more massive than the proton, bound to it by the forces discussed in Sec. I-5, then it would be possible for monopoles to depolarize the earth. Indeed, if this is so, a monopole arrival rate at the surface of the earth of one per cm^2 per sec would cancel the earth's magnetic field in a month. Hence one could tentatively conclude that if monopoles have been accumulating in the earth's crust for the last billion years, then their arrival rate must be less than 10^{-10} per cm^2 per sec, for there is no measurable magnetic charge associated with surface matter.

However, if the conclusions of Sec. III are correct and the monopole is bound only weakly to paramagnetic material, then monopoles could diffuse through the earth and would have little effect on the magnetic field. In this case as many positive monopoles would be moving one way as negative monopoles the other, and once beyond the earth's atmosphere, the monopoles would be hurled free of the earth's dipole due to their inertia. The only effect then is to dissipate part of the energy stored in the earth's magnetic field which could be restored by the internal regenerative processes hypothesized in recent literature.

2. A simple experiment has been performed to detect those monopoles arriving at the surface that can diffuse through the earth at any rate greater than one kilometer in a billion years. Figure 1 is a schematic diagram of

[8] Although postulated as a particle completely analogous to the electron, the magnetic monopole has a "fine" structure constant $M^2/\hbar c = 137/4$, and consequently such processes as monopole pair production cannot be dealt with by the conventional weak coupling approximation. However, a computation based on the assumption of weak coupling would conceivably be correct within several orders of magnitude. Such a computation, for monopole masses comparable to the proton mass, implies that the arrival of monopoles at the surface of the earth would be roughly one per cm^2 per sec.

[9] The monopole, to conserve its charge, will continue to exist indefinitely since the probability of an annihilation collision will be vanishingly small.

the instrument constructed. A long solenoid draws monopoles moving at terminal velocity along the earth's field lines through a thin window into its evacuated core. The monopoles are then accelerated to several hundred Mev and pass through a second window to strike a photographic emulsion. The experimental requirements and the conclusions drawn from operating the instrument are described in the following paragraphs.

The monopole, of charge $M=(137/2)e$ emu, gains $(137/2)300H$ ev/cm in free fall in a field of H gauss. Hence in a field of 250 gauss, one meter long, the monopole gains 500 Mev. As indicated in the figure, monopoles moving along the earth's field lines would be drawn to the upper surface of the evacuated[10] brass tube which forms the core of the solenoid. Here they diffuse through a 10 mg/cm^2 mica window and attain 500 Mev in the one-meter fall. On passing through a second mica window at the lower end of the solenoid the monopole will lose less than 50 Mev, while its loss in the photographic emulsion which it then strikes will be roughly 1 Mev per micron.

The effective cross-sectional area, A_E, of the earth's field lines drawn into the solenoid can be found by observing that all the flux drawn into the upper end of a solenoid comes from the earth's field in any solenoid for which

$$H_a=H_s\cdot A_s/4\pi a^2<H_E, \qquad \text{(IV 2.1)}$$

where H_a is the external field due to the solenoid alone near its midpoint; H_s is the internal field of the solenoid; A_s is the cross-sectional area of the solenoid and a its half-length; H_E is the earth's field. In the solenoid constructed this inequality (IV 2.1) holds, hence

$$A_EH_E=A_sH_s. \qquad \text{(IV 2.2)}$$

It should be noted that no ferromagnetic material which might distort the magnetic field was placed near the solenoid and that it was directed along the local field lines in an exposed place.

[10] The pressure was kept well below one micron.

Careful scanning of the emulsions exposed during the two week period of operation showed no heavy tracks other than the few short and randomly oriented tracks of alpha-particles. (The monopole tracks should be heavier than those of alpha-particles, several hundred microns long, and oriented in only one direction.)

The upper limit[11] of the monopole arrival rate set as a consequence of this negative result is $1/A_ET$, where $A_E=H_sA_s/H_E$ from (IV 2.2) and T the time of observation, which was 1.2×10^6 sec. In this equipment $H_s=250$ gauss; $A_s=20$ cm^2, while $H_E=0.6$ gauss. Therefore $A_E\cong 8300$ cm^2 and

$$1/A_ET\leqslant 10^{-10} \text{ monopoles per cm}^2 \text{ per sec.} \qquad \text{(IV 2.3)}$$

The corresponding cross section for monopole production by primary cosmic radiation is

$$\sigma_{\max}\leqslant 1\bigg/A_ET\int CNdx \qquad \text{(IV 2.4)}$$

where C is the number of primary cosmic particles crossing a cm^2 per sec at altitude x and N is the number of atmospheric nuclei per cm^3 at altitude x. The value[12] of $\int CNdx$ is approximately 3.8×10^{24} per cm^2 per sec for primary protons alone; hence,

$$\sigma_{\max}\leqslant 3\times10^{-35} \text{ cm}^2. \qquad \text{(IV 2.5)}$$

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[11] The statistical significance of the phrase "upper limit" is that an arrival rate n times as great as that established has a probability of e^{-n}.

[12] B. Rossi, Revs. Modern Phys. **20**, 566 (1948).