chanical function

$$F(\mathbf{p},\mathbf{q})=\int dx\int dyf(x,y)e^{i\mathbf{p}x}\cdot e^{i\mathbf{q}y},\qquad f(x,y)=\tfrac{1}{2}(1+e^{-i\hbar xy})f^c(x,y). \tag{3}$$

The operator $F(\mathbf{p},\mathbf{q})$ has the following properties: (1) it is hermitian; (2) it is "well ordered," i.e., it may be written as a sum of terms where the $\mathbf{p}$ are on the left of the $\mathbf{q}$; (3) for $\hbar$ going to 0, F is reduced to F^c, which expresses the condition of correspondence. All these properties are invariant under the operations of derivation or integration.[5]

The infinitesimal canonical transformation generated by the real function $F^c(p,q)$ is described by the following equations:

$$\begin{cases} p'=p+\alpha\{F^c(p,q),p\}=p-\alpha\partial F^c/\partial q \quad (\alpha \text{ real}),\\ q'=q+\alpha\{F^c(p,q),q\}=q+\alpha\partial F^c/\partial p.\end{cases} \tag{4}$$

By application of Taylor's formula, we get the transform $G^c(p',q')$ of any real function $G^c(p,q)$, neglecting terms in $\alpha^2,\alpha^3,\cdots,$

$$G^c(p',q')=G^c(p,q)+\alpha\{F^c(p,q),G^c(p,q)\} \tag{4a}$$

with the definition for the poisson bracket

$$\{F^c(p,q),G^c(p,q)\}=\frac{\partial F^c}{\partial p}\frac{\partial G^c}{\partial q}-\frac{\partial G^c}{\partial p}\frac{\partial F^c}{\partial q}.$$

Now we want to extend the notion of a canonical infinitesimal transformation to the case where the commutable variables p,q are replaced by the noncommutables $\mathbf{p},\mathbf{q}$, and, at the same time, the functions $G^c(p,q)$, $F^c(p,q)$, etc., replaced by $G(\mathbf{p},\mathbf{q})$, $F(\mathbf{p},\mathbf{q}),\cdots,$ etc. This extension must satisfy the following conditions: (1) the new variables $\mathbf{p}',\mathbf{q}'$ must be hermitian; (2) the correspondence $F^c(p,q)\rightarrow F(\mathbf{p},\mathbf{q})$ must be the same as the correspondence $F^c(p',q')\rightarrow F(\mathbf{p}',\mathbf{q}')$ (invariance of the law of correspondence (2), (3) with respect to a canonical infinitesimal transformation). The only possible extension of (4) satisfying these conditions is

$$\begin{cases}\mathbf{p}'=\mathbf{p}-\alpha\partial F/\partial\mathbf{q},\\ \mathbf{q}'=\mathbf{q}+\alpha\partial F/\partial\mathbf{p}.\end{cases} \tag{5}$$

The condition (2) gives the transform $G(\mathbf{p}',\mathbf{q}')$ of $G^c(p',q')$:

$$\begin{aligned}G(\mathbf{p}',\mathbf{q}')&=\int dx\int dyg(x,y)\exp\{i\mathbf{p}'x\}\exp\{i\mathbf{q}'y\}\\ &=\int dx\int dyg(x,y)\exp\{ix(\mathbf{p}-\alpha\partial F/\partial\mathbf{q})\}\\ &\qquad\times\exp\{iy(\mathbf{q}-\alpha\partial F/\partial\mathbf{p})\}.\end{aligned} \tag{6}$$

Easy calculation gives, neglecting terms in $\alpha^2,\alpha^3,\cdots,$

$$\exp\{ix(\mathbf{p}-\alpha\partial F/\partial\mathbf{q})\}=\exp\{ix\mathbf{p}\}+\alpha(i/\hbar)\exp\{i\mathbf{p}x\}\times(\exp\{-x\hbar\partial/\partial\mathbf{q}\}-1)F(\mathbf{p},\mathbf{q})\cdots.$$

Substituting in (6) and using Fourier's development of $F(\mathbf{p},\mathbf{q})$, we have

$$G(\mathbf{p}',\mathbf{q}')=G(\mathbf{p},\mathbf{q})+\alpha(i/\hbar)\int dx\int dy\int dx_1\int dy_1 g(x,y)f(x_1y_1)\times\begin{bmatrix}\exp\{-i\hbar xy_1+i(x+x_1)\mathbf{p}\}\exp\{i(y+y_1)\mathbf{q}\}\\-\exp\{-i\hbar x_1y+i(x+x_1)\mathbf{p}\}\exp\{i(y+y_1)\mathbf{q}\}\end{bmatrix}$$

or, finally, still neglecting terms on $\alpha^2,\alpha^3,\cdots,$ we have

$$G(\mathbf{p}',\mathbf{q}')=G(\mathbf{p},\mathbf{q})+\alpha(i/\hbar)[F(\mathbf{p},\mathbf{q}),G(\mathbf{p},\mathbf{q})],\qquad [A,B]=AB-BA. \tag{7}$$

In other words, the extension of the infinitesimal canonical transformation (4, 4a) generated by the classical function $F^c(p,q)$ is the infinitesimal unitary transformation

$$\begin{cases}\mathbf{p}'=\mathbf{p}+\alpha(i/\hbar)[F(\mathbf{p},\mathbf{q}),p]=U\mathbf{p}U^{-1},\\ \mathbf{q}'=\mathbf{q}+\alpha(i/\hbar)[F(\mathbf{p},\mathbf{q}),q]=U\mathbf{q}U^{-1},\end{cases} \tag{5'}$$

$$G(\mathbf{p}',\mathbf{q}')=G(\mathbf{p},\mathbf{q})+\alpha(i/\hbar)[F(\mathbf{p},\mathbf{q}),G(\mathbf{p},\mathbf{q})]=UG(\mathbf{p},\mathbf{q})U^{-1} \tag{5a'}$$

"generated" by the hermitian operator $F(\mathbf{p},\mathbf{q})$ or also performed by the unitary operator $U=1+\alpha(i/\hbar)F(\mathbf{p},\mathbf{q})$.

On the whole, the infinitesimal unitary transformations appear here as a suitable generalization of the infinitesimal canonical transformations for noncommutable variables, preserving chiefly the hermitian character of the suitably defined function of these noncommutable variables. Obviously, there is a one-to-one correspondence between an infinitesimal canonical transformation and its unitary extension.

* Fellow of the Swiss Commission for Scholarships in Mathematics and Physics.

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[4] To simplify the notation, we consider only the case of two canonical variables; generalization to $2n$ variables causes no trouble.

[5] The correspondence given here between F^c and F is a slight modification of that proposed by H. Weyl (reference 6), using the notion of well-ordered function introduced by P. Jordan (see reference 2).

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Neutron Crystal Monochromators*

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A POLYCHROMATIC beam of thermal neutrons from the Brookhaven reactor, collimated to about one-minute divergence (slit geometry), was diffracted from a single crystal and the contour of the diffracted beam was determined with a slit mounted on a comparator in front of a BF_3 counter. Coherent regions of the crystal tilted with respect to one another will diffract through slightly different angles according to the Bragg law. It was found that the diffracted beams invariably exhibited several maxima for crystals grown from the melt, and these have been attributed to the lineage structure of such crystals. Lineages are large regions of the crystal tilted with respect to one another but not to such an extent as to call them grains.[1] Within each lineage region there are coherent domains (commonly called mosaic blocks) tilted with respect to one another to a lesser degree than lineages and about 5000A in size[2] in annealed specimens. The existence of lineage is

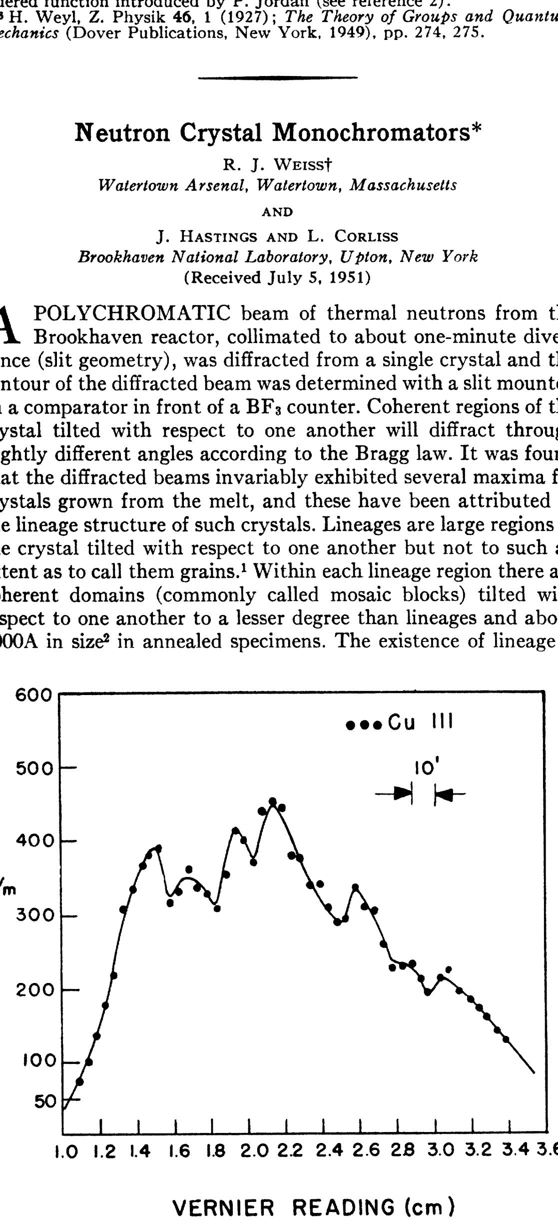

Fig. 1. Contour of Bragg peak from a diffracting plane perpendicular to growth direction, showing lineage.

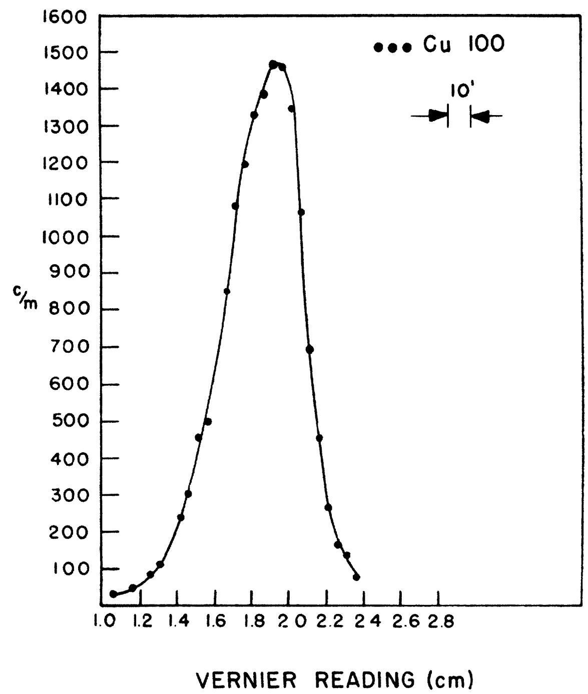

FIG. 2. Contour of Bragg peak from a diffracting plane parallel to growth direction, showing decrease in lineage.

well known to metallurgists, but has not as yet been specifically correlated with the diffraction of neutrons. The lineage structure is a result of the growth process and appears as a fibrous structure in the direction of growth. Figure 1 is a diffraction pattern of a crystal from a diffracting plane approximately perpendicular to the growth direction, with the beam intercepting many lineage regions. Figure 2 is a diffraction pattern from a diffracting plane parallel to the growth direction. In the latter case the beam was parallel to the fibers and intercepted only 2 lineage regions.

These results are important in connection with neutron crystal monochromators, since neutron beams are large enough and penetrate deeply enough to be affected by lineage. In a previous search for good monochromating crystals, double-crystal rocking curves were taken[3] at Oak Ridge, and an attempt was made to interpret the results with the dynamical theory of x-ray diffraction assuming the angular distribution of coherent domains to be gaussian. In most cases the reflectivities were too small by a factor of 2, and it now appears that the discrepancy may be due to lineage (two of the crystals were checked with the present apparatus and were found to contain pronounced lineage). The lineage structure is lost to a great extent in double-crystal rocking curves.

The wavelength resolution, $\Delta\lambda/\lambda$, of a crystal monochromator is related to the angular spread of the coherent domains, $\Delta\theta$, by

$$\Delta\lambda/\lambda=\Delta\theta/\tan\theta_B,$$

where θ_B is the Bragg angle. Twenty assorted crystals grown from the melt (Be, Mg, Cu, Ni, Pb, Bi, NaCl, LiF) were examined, and all contained pronounced lineage with an average $\Delta\theta$ of about 30 minutes. Thermal annealing of lead in general reduced this, and in one case $\Delta\theta$ was reduced to 2.5 minutes. On the other hand, $\Delta\theta$ may be increased by about an order of magnitude by straining crystals below their temperature of recrystallization. In crystals used for monochromators, $\Delta\theta$ should be approximately the same as the geometrical resolution of the instrument. Because of its ease of growth, its high coherent cross section and low absorption, and the ability to vary the lineage to some extent, it appears that lead makes a suitable monochromator for neutrons.

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Radioactivity of F^{17}

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THE gamma-radiation from F^{17} has been re-examined with a scintillation counter and a pulse-height analyzer.

Distilled water was bombarded with deuterons in the internal beam of the 36-in. cyclotron and half-lives were measured at different pulse-height settings. With the increased activity which was produced in this way, and the better energy discrimination, it was found that the previously reported gamma-rays[1] of energy higher than that of annihilation radiation had a half-life distinctly shorter than that of F^{17}.

The large chemical activity of fluorine and of the ozone which is produced when gaseous oxygen is bombarded makes it likely that some impurity was carried along in the previous experiments.

We conclude that F^{17}, in common with other mirror image nuclei which have been investigated with spectrometers, does not emit any nuclear gamma-radiation.

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Measurement of Some Internal Conversion Coefficients*

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THE lifetime-energy relations of Axel and Dancoff[1] lead to the interpretation that the isomeric transitions in Cd^{111m} (48.6 min, 149 kev), Cs^{134} (3.15 hr, 128 kev), and Ta^{182m} (16.4 min, 180 kev) are of multipole order $\Lambda=4$; i.e., electric 2^4-pole ($E4$) and/or magnetic 2^3-pole ($M3$). Measured K/L ratios for these transitions,[2] when compared with the approximately calculated ones of Hebb and Nelson,[3] support the interpretation that these transitions are of the $E4$ type ($\Delta I=4$, no parity change). In addition, the conversion coefficient for Cd^{111m} has been recently reported[4] to agree with that theoretically expected[5] for an $E4$ transition. This is in direct contradiction to the predictions of the strong spin-orbit coupling shell model for Cd^{111m}. The second step (247 kev) of this two-step isomeric transition is an $E2$ transition on the basis of its conversion coefficient.[4] The ground-state spin and magnetic moment of Cd^{111} indicate that it may be designated as an $s_{1/2}$ state. In terms of the shell model the decay of Cd^{111m} would thus be most naturally designated by the transitions

$$h_{11/2}\xrightarrow{149\text{ kev}}d_{5/2}\xrightarrow{247\text{ kev}}s_{1/2}$$

as pointed out by Johansson.[6] The 149-kev transition would be of the $E3$ type ($\Delta I=3$, change in parity). In terms of the classification of Axel and Dancoff,[1] the observed lifetime of the 149-kev transition would then be $\approx 10^7$ times slower than the calculated lifetime.

Weisskopf has recently derived new lifetime-energy relations[7] which give considerably smaller radiation probabilities for all electric transitions with $\Delta I \geq 2$. (The factor is, however, too small to account for the observed discrepancy.) The reclassification of