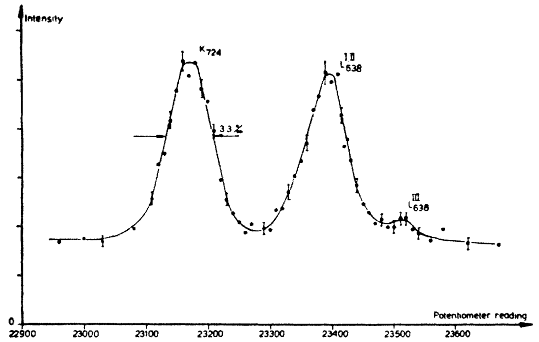

FIG. 2. The K_{724} and L_{638} photolines from a uranium converter.

presence of coincidences between the 80-kev γ-ray the 607-kev β-component definitely rules out the third decay scheme of Cork *et al.*[7] It seems probable that there is a small coincidence effect also between the 80-kev γ- and the 255-kev β-component.

Earlier measurements indicate that the energy difference between the two high energy γ-rays (724 and 638 kev) is approximately 80 kev, i.e., the energy of a third γ-ray. A high precision determination of this energy difference can be obtained by measuring the photoelectron lines from a uranium converter, since the K_{724} line and L_{638} line appear as a narrow doublet. Such a measurement was carried out in the double focusing spectrometer using a 250-mC I^{131} sample and a uranium radiator (1.3 mg/cm^2). The doublet is shown in Fig. 2. The energies of the two γ-rays were found to be 638.0±0.6 kev and 723.9±0.7 kev. The relative intensity, estimated from the K photo lines, is 7.8:1. The energy difference as determined from the doublet lines is 85.7±0.4 kev. A difference of 80.1 kev would give a doublet with approximately twice as large a distance between the lines. The value 85.7 kev definitely excludes the possibility of a cross-over transition for the 724-kev γ-ray as assumed by Zeldes *et al.*[8] and Bell *et al.*,[2] which, in fact, had seemed to be a satisfactory suggestion.

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A Photographic Study of the $\pi^+\to\mu^+\to\beta^+$ Decay Process and the Energy Spectrum of the β^+

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(Received June 29, 1951)

An investigation of meson decay has been undertaken using thick electron sensitive emulsions and the beam of mesons produced by the 385-Mev Nevis cyclotron.

The emulsions were exposed in a collimating chamber[1] which yields plates of low background while restricting entrance of π-mesons to a nearly monoenergetic monodirectional beam of π^+ mesons. The μ^+ mesons entering these plates can, except in exceedingly rare instances[1] be readily distinguished from the main π^+ beam. In addition, the appearance of the decay particle at the terminus of the μ^+ meson can be regarded as a final check of its identification.

Ilford G5 emulsions, approximately 0.6 mm and 1 mm thick, were used in order to investigate the energy spectrum of the charged particle from the decay of the μ^+ meson. To date, 1158 cases of π^+ decay have been observed where the μ's have ended in the emulsion. In each of these cases, an attendant minimum ionization track was always visible, attesting to the uniform minimum ionization achieved throughout the film. Steep angles of the decay track did not introduce difficulties in their identification.

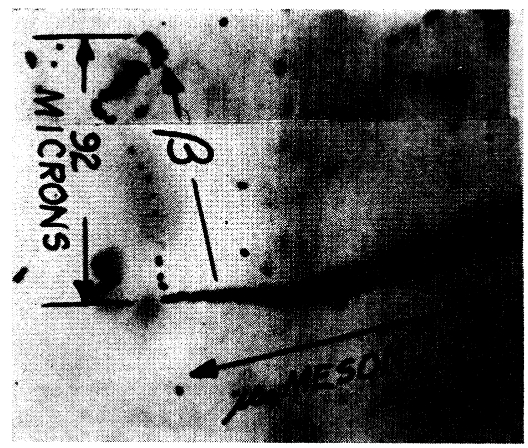
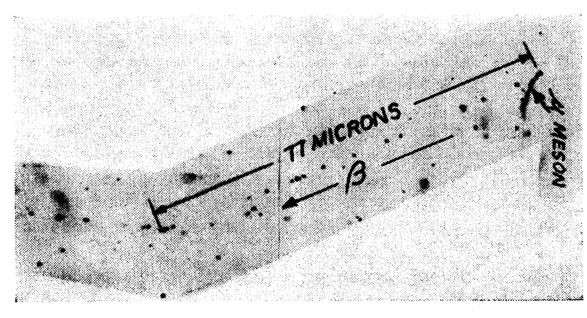


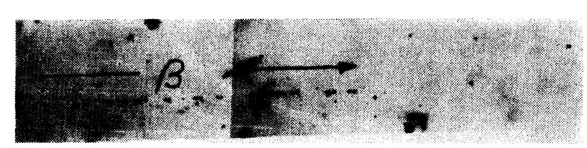
FIG. 1. Slow positron of a $\pi\to\mu\to\beta$-sequence in 1-mm emulsion.

Throughout these studies anomalous ranges of μ-mesons were searched for without success. Since W. F. Fry[2] has reported four μ-meson ranges which were clearly anomalous, our results were rescanned, specifically in a search for μ-meson ranges less than 420 microns. However, no such anomalous events have been found. Therefore, we believe either that such events are not as common as indicated by Fry's results, or that the statistical fluctuations of scattering measurements and grain counting over such short track lengths can permit such variations in usual μ-meson tracks to account for Fry's results.

Three interesting examples of μ^+ decays are shown here. Figure 1 shows the slowest positron yet observed in this study. From its range, which lies entirely within the emulsion, the energy was estimated to be about 0.1 Mev. Figure 2(a) shows an instance of an unusually sharp scattering of the decay particle shortly



(a)



(b)

FIG. 2. (a) Instance of large scattering of positron (0.6-mm emulsion). (b) Example of positron annihilation in 1-mm emulsion. (Annihilation takes place 4900 microns from μ-meson, 190 microns beneath air surface, 600 microns above glass surface.)

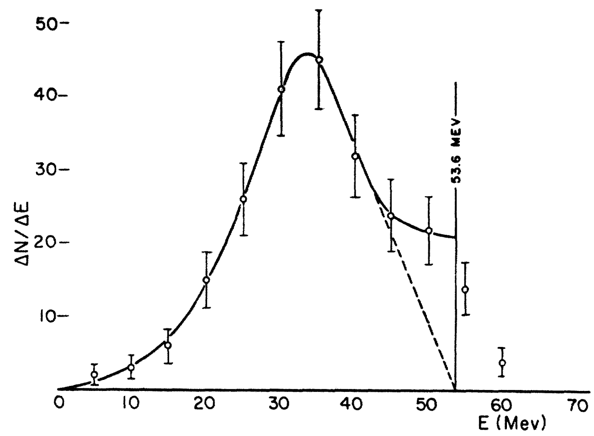


FIG. 3. Spectrum of 117 positrons from μ^+ meson decay. Each point represents the number of positrons per 10-Mev energy interval. The vertical lines indicate the statistical spread. The smooth curve represents the best match to the observed points. The dotted portion represents the best match to the data by a curve approaching zero at the upper energy end.

after leaving the μ-meson. Similar large scatterings (at least eight times the average scattering, including the large scatterings) are found to occur with a frequency of about 10 percent within the first g/cm^2 of emulsion. With a fair bremsstrahlung loss possible in these events, this comprises a possible source of error in techniques employing appreciable "dead" mass to stop the μ-meson. Figure 2(b) shows one of six events in which the decay particle disappears. In view of the uniform minimum ionization achieved, this disappearance is interpreted to be an annihilation process. Hence, in addition to having the same charge as the positron, and a mass not much heavier (if at all),[3] this decay particle is now observed to have a third property in common with the positron, that of annihilation.

The use of nuclear emulsions for investigating the energy spectrum of the positron from μ^+ decay has several advantages. In a few hours of cyclotron running time, $1500\pi\rightarrow\mu\rightarrow\beta$ sequences were obtained. A qualitative examination of the β-tracks can be made at a rate of 40/day/scanner. Positrons of all energies give substantially the same grain density;[4] hence, all energies are recorded with almost equal sensitivity. The complete history of the positron is observed; therefore, there is no possibility of failing to observe appreciable energy losses in any "dead" mass.

However, some disadvantages also exist. The results of multiple scattering measurements must be accurately correlated with energy. A long track is required for good accuracy. Long tracks imply appreciable energy degradation during the measuring process through collision and radiation loss. Long tracks also discriminate against slow positrons because of outscattering. Distortion in the processed emulsion will cause lower estimates of energy. Finally, these scattering measurements are tedious and exacting if precision is desired.

The modification of the stage and drive of a conventional microscope enabled the measurements to be made with greater ease and speed. The effective track length depends upon the shortest "foil" thickness feasible, below which the noise level becomes critical. After some investigation, it was found possible to make Fowler-type measurements of track ordinates with a mean deviation of 0.03 micron. This permitted the use of intervals as short as 50 microns for the fastest particles observed, with a noise error of less than 2 percent in the most unfavorable cases. Therefore, a 1500-micron track length was selected at the minimum acceptable for the desired statistical accuracy: 10 percent standard deviation. Outscattering is then less than 2 percent above 20 Mev, reaching 10 percent at 11 Mev and 38 percent at 5 Mev. The average bremsstrahlung loss is 7 percent for the 2.91-cm radiation length in G5 emulsion. Hence, individual variations from this average loss are much smaller than the statistical uncertainty in energy determination. On all plates no distortion was noted in the central portions for the track lengths used. Taking into account all these sources of error, the standard deviation per track is probably less than 11 percent.

Average multiple scattering angles were calibrated against energy using the Scott-Snyder[5] theory, which from Corson's[6] work appeared to be best. The energy spectrum uncorrected for outscattering resulting from the first 117 decays is shown measured in Fig. 3. The spectrum appears to have a marked non-zero cutoff at the high energy end, as is indicated by both the experimental points and the straggling beyond cutoff (taking the μ-meson mass as 209 electron masses).

It is interesting to note that if (a) the Scott-Snyder-theory provides a perfect calibration and (b) the relativistic momentum of the positron is, on the average, equal to that of each neutrino, then the average energy: 35.76 ± 0.37 Mev, gives the mass of the μ-meson as 209.3 ± 2.2 electron masses. In the absence of an accurate independent verification of the Scott-Snyder theory in this energy region, this excellent agreement with recent values[7] might best be taken as an indication of the high accuracy of this theory as applied here.

We wish to thank Drs. Friedman and Rainwater for kindly lending us their exposure chamber and some plates for our earlier work. We are also grateful to Miss Marjorie Fogarty and Mrs. Carol Major for their invaluable assistance in locating the events, and to Mr. Aurel Seifert for aiding with some of the measurements.

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On a One-to-One Correspondence between Infinitesimal Canonical Transformations and Infinitesimal Unitary Transformations

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(Received July 2, 1951)

IT is well known that there is an analogy between the group of the canonical transformations of classical mechanics and the group of the unitary transformations of quantum mechanics, insofar as in the latter we exclude degrees of freedom without classical analog.[1] Until now, however, this analogy has been found to be not very simple; more precisely, no general homomorphism has been given between the two groups.[2,3] Among the reasons for that are the ambiguities in the definition of the quantum-mechanical function $F(\mathbf{p}, \mathbf{q})$ of the noncommutable operators $\mathbf{p}$, $\mathbf{q}$ with

$$[\mathbf{p}, \mathbf{q}] = -i\hbar\mathbf{1}, \tag{1}$$

corresponding to the classical function $F^c(p, q)$ of the commutable variables[4] p, q. In order to have an actual significance, such a correspondence must be independent of the choice of the variables p, q, at least within a certain subset of the group of canonical transformations.

The first step in finding a correspondence between canonical transformations and unitary transformations is to give this correspondence between $F^c(p, q)$ and $F(\mathbf{p}, \mathbf{q})$. We propose the following one: to every classical real function $F^c(p, q)$ which may be written as a fourier integral

$$F^c(p, q) = \int dx \int dy f^c(x, y) e^{i(px+qy)}, \quad \bar{f}^c(-x, -y) = f^c(x, y) \tag{2}$$

($^-$ means imaginary conjugate) corresponds the quantum-me-

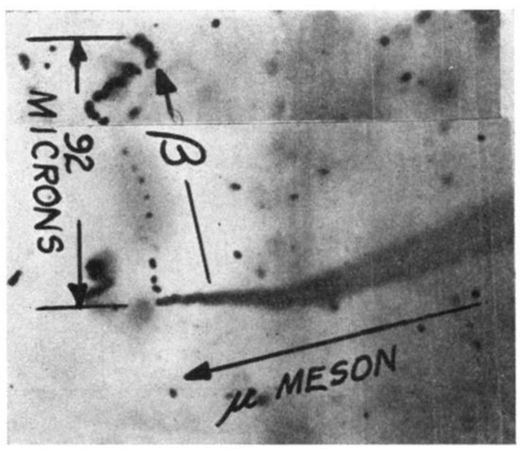
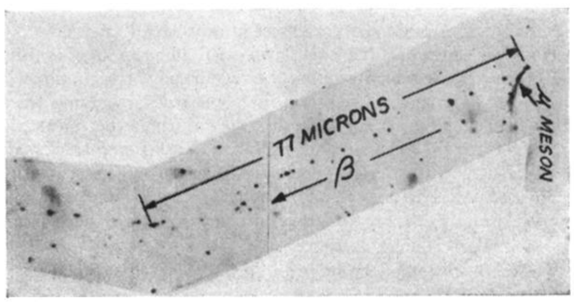
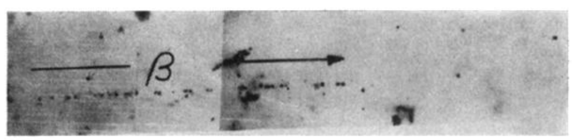


FIG. 1. Slow positron of a $\pi \rightarrow \mu \rightarrow \beta$-sequence in 1-mm emulsion.



(a)



(b)

FIG. 2. (a) Instance of large scattering of positron (0.6-mm emulsion). (b) Example of positron annihilation in 1-mm emulsion. (Annihilation takes place 4900 microns from μ-meson, 190 microns beneath air surface, 600 microns above glass surface.)