was ruled from the edges of the Al-covered Nylon backing to ground.

The counter window was about 45 $\mu g/cm^2$ of Nylon whose transmission properties were known.[5] Figure 1 shows that the Fermi plot of the uncorrected data is linear down to about 48 kev, and the data corrected for window absorption is linear to about 32 kev. It must be realized that back scattering of source mounts which are thicker than about 10 $\mu g/cm^2$ may give an excess of low energy betas which in turn may be absorbed or scattered by the counter window to produce a partial canceling of the two effects (source backscattering and counter window absorption).

The present results constitute additional evidence that the beta-spectrum of S^{35} observed with favorable experimental techniques is an allowed Fermi spectrum.

* This document is based on work performed under contract with the AEC.

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The Effect of Bohr Orbit Binding on Negative μ-Meson β-Decay*

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(Received June 26, 1951)

THE phenomenon of the β-decay of μ-mesons is currently attracting both experimental and theoretical interest.[1,2] In the present note we have calculated the effect of binding the negative μ-meson into its lowest Bohr orbit[3] on the negative electron momentum spectrum characterizing the process: $\mu^- \to e^- + 2\nu$;[2] the orbital motion of the μ^- meson induced by the binding results in a "Doppler smearing" of the e^- spectrum. Similar Doppler smearing calculations have been previously made assuming a two-particle mode of decay of the μ^- meson.[4]

The principal result of our investigation is the prediction of a tailing of the e^- spectrum beyond the cut-off momentum appropriate to a meson which is free and stationary before decay, i.e., effectively, a tailing beyond the cut-off momentum of the e^+ spectrum from μ^+ mesons. This is shown in Fig. 1. A simple estimate based on the conservation laws gives the order of magnitude of the calculated Doppler smearing of the e^- spectrum:

$$\mathbf{p}_\mu = \mathbf{p}_e + \mathbf{n}_1 + \mathbf{n}_2; \quad \mu c^2 = c([p_e^2 + (mc)^2]^{\frac{1}{2}} + n_1 + n_2) \tag{1}$$

with $\mathbf{n}_1$ practically parallel to $\mathbf{n}_2$ when $|\mathbf{p}_e| \cong \frac{1}{2}\mu c$. We then have, from Eq. (1),

$$(p_{e^-})_{\max} \cong \tfrac{1}{2}\mu c[1 + (p_\mu)_{\mathrm{Av}}/\mu c] \cong (p_{e^+})_{\max}(1 + Z\alpha) \tag{2}$$

for lowest Bohr orbit binding about a nucleus of charge Ze (here, $\alpha = 1/137$), in qualitative agreement with the more detailed results below.

In our complete calculations we have considered explicitly the "antisymmetric charge exchange" tensor coupling and the "charge retention" axial vector coupling;[2] these constitute the two extreme cases of e^+ momentum spectra with regard to shape near cutoff,[2] and have the further desirable property that, upon neglect of terms in $m/\mu \cong 1/210$, all other coupling types yield e^+ and e^- momentum spectra which are linear combinations of the two in question.

To a sufficient approximation in the relatively small parameter $Z\alpha$, we then find the following e^+ and e^- momentum spectra:

$$\{P(p_e)dp_e/\mu c\}_{e^+} = (96/\tau)(p_e/\mu c)^2(1 - 2p_e/\mu c)S(2p_e/\mu c)(dp_e/\mu c),$$

antisym. ch. ex.—tensor, (3a)

$$\{P(p_e)dp_e/\mu c\}_{e^+} = (96/\tau)\tfrac{1}{3}(p_e/\mu c)^2[1 - \tfrac{2}{3}(2p_e/\mu c)]S(2p_e/\mu c)(dp_e/\mu c),$$

ch. ret.—axial vector, (3b)

where $S(x) = 1$ for $0 \leqq x \leqq 1$, $S(x) = 0$ for $x > 1$, τ = meson decay mean life,[5,6] and

$$\left\{\frac{P(p_e)dp_e}{\mu c}\right\}_{e^-} = \left\{\frac{P(p_e)dp_e}{\mu c}\right\}_{e^+}\left[S\left(\frac{2p_e}{\mu c}\right)\right]^{-1} S\left(\frac{p_e}{\mu c}\right) \times\left\{\frac{1}{\pi}\left[\tan^{-1}\left(\frac{1-2y}{Z\alpha}\right) + \tan^{-1}\left(\frac{1}{Z\alpha}\right)\right] + Z\alpha\left(\frac{2}{\pi}\right)\frac{1-y}{1-2y}\left[1 - \frac{1}{3}\left(\frac{1-y}{y}\right)\frac{(Z\alpha)^2}{(Z\alpha)^2 + (1-2y)^2}\right]\right\},$$

antisym. ch. ex.—tensor, (4a)

$$\left\{\frac{P(p_e)dp_e}{\mu c}\right\}_{e^-} = \left\{\frac{P(p_e)dp_e}{\mu c}\right\}_{e^+}\left[S\left(\frac{2p_e}{\mu c}\right)\right]^{-1} S\left(\frac{p_e}{\mu c}\right) \times\left\{\frac{1}{\pi}\left[\tan^{-1}\left(\frac{1-2y}{Z\alpha}\right) + \tan^{-1}\left(\frac{1}{Z\alpha}\right)\right] + Z\alpha\left(\frac{2}{\pi}\right) \times\frac{1-y}{(Z\alpha)^2 + (1-2y)^2}\left[(1-2y) - \frac{2}{3}\frac{(Z\alpha)^2(1-3y/2)}{y(1-4y/3)}\right]\right\},$$

ch. ret.—axial vector, **(4b)**

with $y \equiv p_e/\mu c$.

The expressions in Eqs. (3) and (4) are obtained by standard perturbation calculations of the corresponding transition $|\mathrm{M.E.}|^2$; in these $|\mathrm{M.E.}|^2$ we use Schrödinger plane-wave and bound state wave functions for the μ^+ and μ^-, respectively, Dirac plane-wave wave functions for the e^+ and e^-, and Majorana plane-wave wave functions for the neutrinos. As a result, the momentum conservation quantity $\delta(\mathbf{p}_\mu - \mathbf{p}_e - \mathbf{n}_1 - \mathbf{n}_2)\delta(\mathbf{p}_\mu)$ appropriate to μ^+ decay, is replaced to a good approximation by the corresponding quantity

$$\delta(\mathbf{p}_\mu - \mathbf{p}_e - \mathbf{n}_1 - \mathbf{n}_2)\{(8/\pi^2)(Z\alpha\mu c)^{-3}[1 + p_\mu^2/(Z\alpha\mu c)^2]^{-4}\}$$

in the case of μ^- decay, where in the last expression, the second factor represents the normalized momentum probability distribution of the bound μ^- moving about the point charge nucleus Ze. The integration over $\mathbf{p}_\mu$ in the calculation of the μ^- decay transition probability is then immediate; the integration over $\mathbf{n}_1$ and $\mathbf{n}_2$, however, appears possible only upon use of the integral representation,

$$[1 + (\mathbf{x} + \mathbf{x}_1 + \mathbf{x}_2)^2]^{-4} = (192\pi)^{-1} \times \int d\mathbf{q} q^3(\pi/2iq)^{\frac{1}{2}} H_{5/2}^{(1)}(iq)\ \exp(i\mathbf{q}\cdot[\mathbf{x} + \mathbf{x}_1 + \mathbf{x}_2]),$$

and even then is both difficult and tedious.

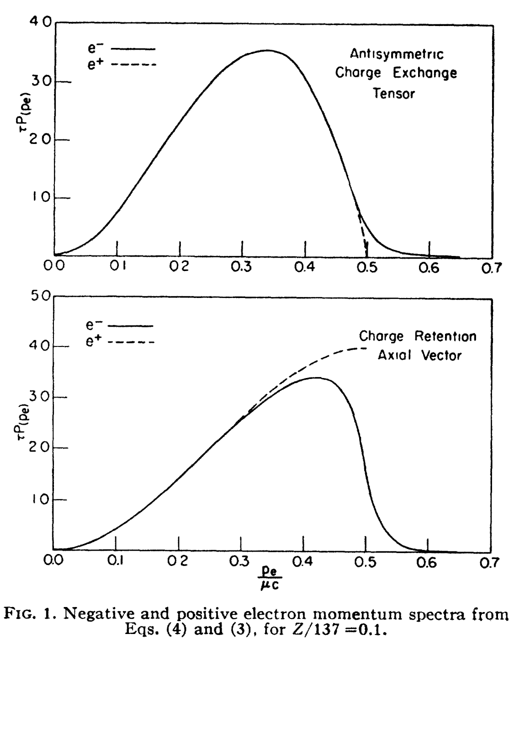

FIG. 1. Negative and positive electron momentum spectra from Eqs. (4) and (3), for $Z/137 = 0.1$.

Plots of the e^- momentum spectra given in Eq. (4) are shown in Fig. 1 for $Z/137=0.1$ (μ^- decay in Si) along with plots of the e^+ spectra given in Eq. (3) where effectively $Z/137=0$ (μ^+ decay).[6] It will be seen that e^+ momentum spectra with a relatively large number of electrons near cutoff give an enhanced e^- tailing effect. It is interesting to note that the recent work of Sagane *et al.*[1] seems to indicate the tensor coupling spectrum of Eq. (3a); in the perhaps analogous phenomenon of nucleon β-decay, recent work also seems to favor tensor coupling.[7]

* Assisted by the joint program of the ONR and AEC.
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[3] It will be recalled that practically all μ^- mesons stopped in matter are captured, previous to decay, into lowest Bohr orbits about the nuclei of the slowing-down material. See E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947).

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[5] With neglect of terms in m/μ, (3a) and (3b) agree with the results of reference 2. (The corresponding formulas of Tiomno and Wheeler (their Eqs. (27) and (32)) seem, however, to contain misprints.) The relation between τ and the dimensionless field interaction coupling constant, $G^2/\hbar c$, is: $\tau^{-1}=\{(9/16\pi^3)(1/96)(\mu/m)^5(mc^2/\hbar)\}(G^2/\hbar c)_{\text{tensor}}$; $\tau^{-1}=\{(1/2\pi^3)(1/96)\times(\mu/m)^5(mc^2/\hbar)\}(G^2/\hbar c)_{\text{ax. vector}}$.

[6] It may be noted that the Bohr orbit binding of the μ^- causes a slight difference, $\approx(Z\alpha)^2\times100$ percent, between the mean lives of the μ^- and μ^+.

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Multiple Coulomb Scattering of High Energy Protons in Photographic Emulsions*

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MEASUREMENTS of the multiple coulomb scattering of fast charged particles in photographic emulsions reported in the recent literature have been confined to groups of cosmic-ray particles,[1] and to electrons produced by gamma-rays[2] or accelerated by a synchrotron.[3] Their results, when expressed in terms of a universal "scattering constant" for the photographic emulsion, show some disagreement. Because of the value of a precise knowledge of this quantity for practical applications such as mass and energy determinations, an investigation was therefore undertaken of the multiple scattering of artificially accelerated protons. In order to obtain sufficient accuracy, it was essential to investigate a group of monoenergetic particles whose energy was precisely known. We had an opportunity of doing this by analyzing proton tracks in an Ilford G5 plate that had been exposed by being dropped through the proton beam of the Berkeley cyclotron, which had an energy $E=340\pm1$ Mev.

A total of 150 tracks have been analyzed, and the analyzed section of each track was on the average 5000 microns (μ) long. In order to avoid possible emulsion distortion, measurements were made only in the central section of the plate, and sections of tracks closer than 50μ to the surface of the emulsion were excluded. Owing to energy loss by ionization, the mean proton energy over each track was reduced to 337 ± 1 Mev. The coordinate method of analysis was used, which consists of the measurement of the lateral displacements of the track caused by multiple scattering. The distances $y_1, y_2, \cdots, y_i, \cdots, y_n$ from successive points $P_1, P_2, \cdots, P_i, \cdots, P_n$ on the track to a reference line, very nearly parallel to the track, were measured. The distance s (minimum cell length) between successive points P_i and P_{i+1} was chosen to be 250μ. The measurements were carried out with a precision microscope for which a special stage has been constructed having a rectilinear motion with a deviation of 0.04μ over one cm. The apparatus was located in a basement room kept at very constant temperature, and mounted on a solid base to eliminate vibrations.

The set of second differences of the lateral displacements $\Delta^2y_i=y_{i+2}-2y_{i+1}+y_i$ obtained from a given track provides a measure of the multiple scattering. The mean value $\langle|\Delta^2y|\rangle_{\text{Av}}$ was computed from the Δ^2y_i obtained from all the 150 measured tracks, which are equivalent to a single track 75.3 cm long (with a constant energy of 337 Mev). The amount of multiple scattering is then expressed in the usual manner in terms of a "scattering constant" K_s defined by the relation:

$$\langle|\alpha|\rangle_{\text{Av}}=\langle|\Delta^2y|\rangle_{\text{Av}}(1/s)=K_s(Z/pv),$$

where $\langle|\alpha|\rangle_{\text{Av}}$ is the mean chord angle between successive chords through the points dividing the track into cells; Z, p, and v are the charge, momentum, and velocity of the scattered particle, and s is the cell length. ($\langle|\alpha|\rangle_{\text{Av}}$ is expressed in degrees, (pv) in Mev, and s in microns.)

The experimental results are shown in Table I. It must be mentioned that in order to minimize statistical fluctuations the standard procedure was followed of computing all mean values $\langle|\Delta^2y|\rangle_{\text{Av}}$ so as to include only individual Δ^2y_i with absolute values smaller than four times the experimental mean. This resulted in a loss of approximately 1.4 percent of the data, which appears to verify the presence of an expected non-gaussian tail in the distribution of multiple scattering deflections. It can be seen from Table I that within the limits of error K_s is proportional to $(s)^{\frac{1}{2}}$ for cell lengths in the range from 750 to 250μ. Assuming that this relation is valid down to $s=100\mu$, one obtains by extrapolation K_{100}, which may be compared with the results of other authors:

	K_{100}
Fowler[1] (cosmic-ray particles)	32.7
Recent work at Bristol[4]	26
Voyvodic and Pickup[2] (Li(p, γ) gamma-ray electrons)	21.3 ± 1.0
Corson[3] (40-, 115-, 195-, and 280-Mev electrons)	$26\ \pm1$
BLS (average of values in Table I for 337-Mev protons)	24.1 ± 0.8.

TABLE I. Multiple scattering of (337 ± 1)-Mev protons in Ilford G5 emulsions.

Cell length (microns)	Number of cells	$\langle\lvert\Delta^2y\rvert\rangle_{\text{Av}}$ (microns)	K_s	$K_{100}=K_s(100/s)^{\frac{1}{2}}$
250	3005	0.2880	38.6 ± 1.3	24.4 ± 0.8
500	1480	0.8064	54.1 ± 1.7	24.2 ± 0.8
750	970	1.448	64.7 ± 2.5	23.6 ± 0.9

A more complete description and a theoretical analysis, as well as the results of further measurements at 340 Mev and other proton energies now in progress, will be presented in a forthcoming publication.

The authors would like to express their gratitude to Professors E. O. Lawrence and W. Barkas for their excellent cooperation in exposing plates for us in the Berkeley cyclotron.

* Assisted by the joint program of the ONR and AEC.

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On a Formulation of Quantum Electrodynamics

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DIFFICULTIES arising from the supplementary condition in the formulation of quantum electrodynamics as given by Fermi have been pointed out several times.[1-3] Taking over an idea of Novobátzky,[4] but following a different course, we want to avoid the supplementary condition and the corresponding com-