Precise Determination of the Magnetic Moment of the Deuteron

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A precision measurement of the ratio of the magnetic moment of the proton to that of the deuteron has been carried out by the magnetic resonance method. The measurements were made using mixtures of H_2O and D_2O and also mixtures of H_2 and D_2 gas. The result for the H_2O:D_2O mixture is

$$\mu_H/\mu_D=3.2571999\pm0.0000012,$$

while for the H_2:D_2 mixture

$$\mu_H/\mu_D=3.2571990\pm0.0000010.$$

I. INTRODUCTION

THE hyperfine structure splittings observed in atoms are closely related to the magnetic moments of the nuclei of these atoms. In the case of hydrogen and deuterium, in particular, it should be possible to calculate the ratio of the hyperfine structure splittings from the ratio of the nuclear magnetic moments to a very high degree of accuracy. When accurate measurements of these quantities[1] were made, however, their values were not in complete accord with the Breit-Meyerott formula,[2]

$$\kappa_{\text{theor}}=\nu_D/\nu_H=\tfrac{3}{4}(m_D/m_H)^3\mu_D/\mu_H. \quad (1)$$

Here, ν_D/ν_H is the ratio of the hyperfine splitting in deuterium and hydrogen, m_D/m_H is the ratio of the reduced masses of an electron in these atoms, and μ_D/μ_H is the ratio of their nuclear moments. The measurements of the hyperfine splitting gave a value of ν_D/ν_H which was larger than that given by Eq. (1) by 17 parts in 10^5, whereas the estimated experimental uncertainty was only 1 part in 10^5.

It was shown by Bohr[3] that most of this discrepancy could be accounted for by taking into consideration the structure of the deuteron. He pointed out that when the electron is close to the deuterium nucleus, it moves rapidly compared with the nuclear motion and centers its motion around the proton rather than around the deuteron center of mass. The interaction between the electron and nuclear spins is increased thereby. For the order of magnitude of this effect, Bohr obtained the formula

$$\epsilon=\Delta\kappa/\kappa=-(\mu_N/\mu_D)d/a=1.84\times10^{-4}, \quad (2)$$

where d and a represent the radii of the deuterium nucleus and atom, respectively.

The present measurement was undertaken at the instigation of Professor Rabi as part of a campaign to press to the limit both the experimental and theoretical possibilities of this subject. Accordingly, a careful evaluation of all the known effects of nuclear motion on the hyperfine structure was made by Low[4] with the result:

$$\epsilon_{\text{theor}}=(1.83\pm0.22)\times10^{-4}. \quad (3)$$

The uncertainty given is due mostly to the uncertainties in the knowledge of the electronic wave function of the deuteron. An improved measurement of ν_D/ν_H was made by Prodell and Kusch[5] with the result,

$$\nu_D/\nu_H=0.230486536\pm0.00000010.$$

This paper reports a measurement of μ_D/μ_H with accuracy comparable to that obtained for ν_D/ν_H.

II. MAGNETIC RESONANCE METHOD

The measurement was based on the magnetic resonance method.[6,7] The technique was a further refinement of the one which was used by Anderson,[8] hereafter referred to as **I**, in a measurement of the magnetic moment of He^3. The sample consisted of a mixture of deuterium and hydrogen gases under pressure. Measurements were also made with a mixture of H_2O and D_2O. Two rectangular small coils were placed at right angles to one another but arranged to include the same volume of the gas mixture. Each coil was connected as one arm of its own "Twin T" radiofrequency bridge. The coils were placed between the poles of an electromagnet with their axes at right angles to that of the poles of the magnet.

Under these circumstances, a substance with spin I may orient itself in $2I+1$ ways with respect to the direction of the magnetic field, each having a different energy value. Transitions between neighboring states will be induced where the frequency of the signal applied to the coil satisfies the resonance condition,

$$2\pi f=|\gamma|H, \quad (4)$$

where $\gamma=\mu/\hbar I$ is the ratio of the magnetic moment to the angular momentum of the substance, f is the frequency in cycles per second, and H is the magnetic field in gauss.

The magnetic field is modulated about the resonance value by a small amount of 25 cycles per second. Under

[1] J. E. Nafe and E. B. Nelson, Phys. Rev. **73**, 718 (1948).
[2] G. Breit and E. R. Meyerott, Phys. Rev. **72**, 1023 (1947).
[3] A. Bohr, Phys. Rev. **73**, 1109 (1948).
[4] F. Low, Phys. Rev. **77**, 361 (1950).
[5] A. G. Prodell and P. Kusch, Phys. Rev. **79**, 1009 (1950).
[6] Purcell, Torrey, and Pound, Phys. Rev. **69**, 37 (1946).
[7] Bloch, Hansen, and Packard, Phys. Rev. **69**, 127 (1946); **70**, 474 (1946); F. Bloch, Phys. Rev. **70**, 460 (1946).
[8] H. L. Anderson, Phys. Rev. **76**, 1460 (1949).

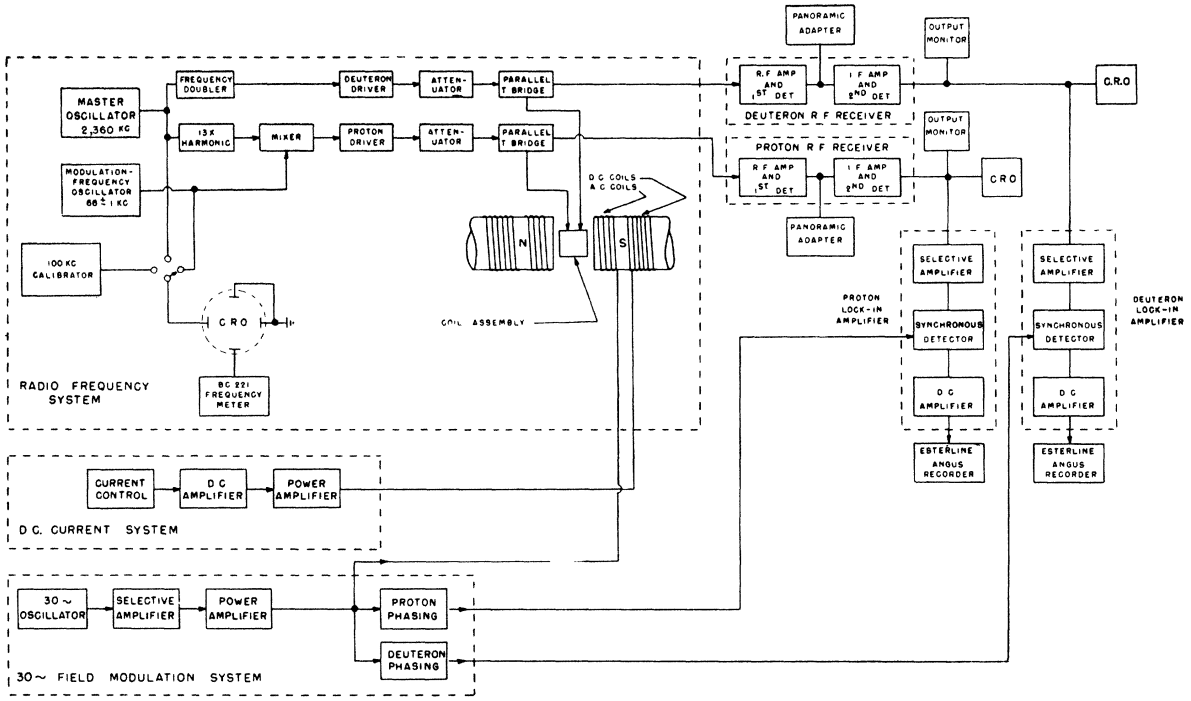


FIG. 1. Block diagram of nuclear magnetic resonance apparatus.

suitable conditions, the transitions which occur each time the resonance value of the field is traversed produced a change in the impedance of the coil and an unbalance of the bridge. Thus, the magnetic resonance effect produces a 25-cycle modulation in the radio-frequency signal which feeds the bridge, and this may be detected by means of an ordinary radio receiver. The extreme sharpness of the resonance which may be obtained makes possible the great accuracy of the method.

It is seen that a measurement of γ requires a measurement of both the frequency f and the magnetic field H. However, the ratio of the quantities γ for two substances can be obtained from a measurement of the ratio of two frequencies provided the resonances are observed in the same magnetic field at the same time. This was the technique of the present experiments.

The object of the measurement was to determine the ratio of the γ's for hydrogen and deuterium to a precision of a few parts in 10^7. The limit to the precision was set by the breadth of the resonance, which in turn was determined by the inhomogeneity of the magnetic field over the sample, and by the constancy of the magnetic field in time. By carefully lapping the pole faces of the magnet and by using a fairly small sample which could be moved about between the poles, it was possible to find a place where the half-width of the resonance at 7000 gauss amounted to 0.018 gauss. The center of the resonance curve was estimated to ± 0.0005 gauss. The magnet was the same as was used in **I**, but the constancy of its field with time was considerably improved.

It is important to recognize that the quantity measured in this experiment may be close to, but is not necessarily equal to, the ratio of the *nuclear* gyromagnetic ratios. Equation (4) holds for the *nuclear* gyromagnetic ratio only if H is the magnetic field at the nucleus. Thus, even though the external magnetic field is the same for the two substances, local differences in magnetic field might occur because of electronic and nuclear motions in the neighborhood of the nucleus under observation. In the previous measurements, this comparison was made using samples of water. A precision comparison using a mixture of heavy and light water was also made in the present work. These molecules are, indeed, similar to a high degree; but differences in their rotational and vibrational properties could be expected to introduce differences in the magnetic field at the nucleus amounting to several parts in 10^7. For this reason, the comparison was also made using hydrogen and deuterium gas mixtures. For these molecules, fairly accurate experimental data on the rotational and vibrational behavior are available, together with reasonably manageable electronic wave functions, so that at least the order of magnitude of the corrections can be estimated with some confidence.

A. Radiofrequency System

The measurements were carried out at a magnetic field of 7223 gauss. The corresponding resonance frequencies are near 30,750 kilocycles per second for the proton and near 4720 kilocycles per second for the deuteron. To obtain the resonances simultaneously at

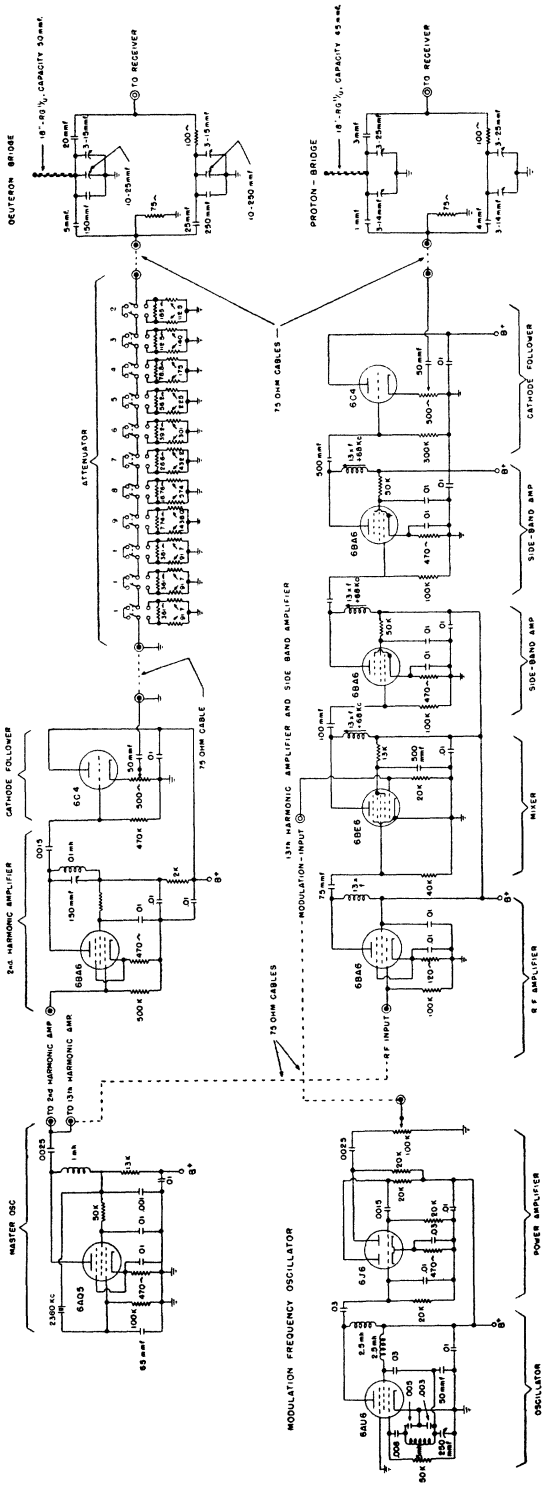


Fig. 2. Radiofrequency system.

the same magnetic field, at least one of these frequencies must be variable. Instead of the two oscillator arrangements used in **I**, advantage was taken of the fact that the required frequency ratio is close to 13/2. A master crystal was used to generate a fundamental frequency, f_0, near 2360 kilocycles per second. The second harmonic of this was amplified and used to drive the deuteron coil. The proton frequency was obtained by using the upper side band derived from a variable modulation oscillator operating at frequency f_m near 68 kilocycles per second and mixed with the thirteenth harmonic of the master oscillator. A block diagram of the scheme may be seen in Fig. 1. The actual circuit is given in Fig. 2.

The ratio of the proton to deuteron frequencies is given by

$$R=f_H/f_D=(13/2)+f_m/2f_0. \quad (5)$$

The relative error in the determination of R is only 1/450 of the relative error in the determination of either f_m or f_0. For an accuracy of 1 part in 10^7 in R, the accuracy with which f_0 and f_m need be determined is 3×10^{-5}. A standard Signal Corps BC221 frequency meter was used for these measurements. This instrument has a dual range from 125 kc sec^{-1} to 250 kc sec^{-1} and from 2000 kc sec^{-1} to 4000 kc sec^{-1} and uses the same vernier capacitor drum dial for both ranges. Through the use of harmonics, a continuous range is obtained from 125 kc sec^{-1} to 20 mc sec^{-1}. It may be read with a precision of about 2×10^{-5}. Instead of using the crystal oscillator of the instrument to provide check calibration points, a more accurate method was used. A General Radio 1101A–100 kc sec^{-1} frequency standard was used to provide check points by utilizing the optimum combination of harmonics to provide a Lissajou figure. For the modulating frequency, f_m, a 15:11 frequency ratio was used to establish a check point at $15/11\times100=136.364$ kc sec^{-1}. For the master frequency, f_0, a 47:2 frequency ratio was used to provide a check point at 2350.000 kc sec^{-1}. The 100-kc sec^{-1} standard was checked against WWV. The drifts to which it is subject can produce only quite negligible errors.

B. Sample and Coil Arrangement

For the measurements with the gases, a brass cell was used which could be filled with a suitable mixture of hydrogen and deuterium under pressure. Two coils were used having a rectangular cross section and placed at right angles to one another and to the magnetic field. The proton coil nested snugly inside the deuteron coil. The space in the cell exterior to the coils was filled with non-hydrogen-containing Teflon plastic, thereby confinding the gas to the region inside the coils. The construction is illustrated in Fig. 3. The volume enclosed by the coils was 0.613 cm^3, small enough to bring the inhomogeneity of the field over the sample down to less than 3×10^{-6}. A typical coil assembly used for the water samples is shown in Fig. 4. Here the deuteron coil was wound on the outside of a small glass tube. The proton coil was placed inside the tube. Care was taken to fill the tube with the water sample so that very little of the sample was outside the coils. The volume of the sample was about 0.23 cm^3.

This arrangement is not ideal, since the proton coil measures a different average over the sample volume than does the deuteron coil. The error was estimated to be less than 2×10^{-7}. The obvious procedure for eliminating this source of error by interchanging the roles of the two coils was not practical in the present experiment because of the large difference in inductance of the two coils. Evidence that it did not amount to more than this is had from the general consistency of the data, which were taken with a variety of coils and coil positions over a period of more than one year. In particular, a measurement made with the coils rotated through 90° gave a result which was not outside the experimental uncertainty.

C. Detection

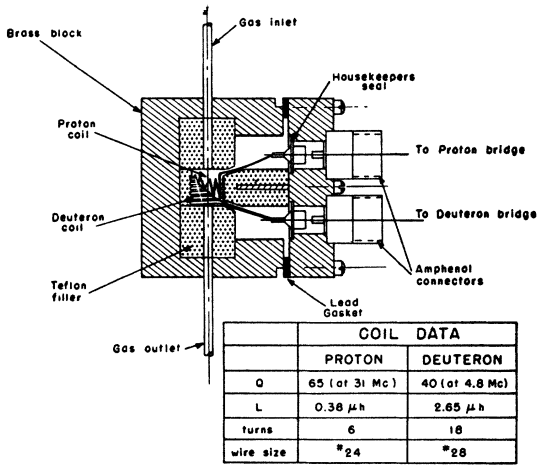
The output of the bridge was connected to a National HRO-5A radio receiver. The balance of the bridge was judged with a panoramic adapter at the output of the first detector, and it could also be read directly on a microammeter at the output of the second detector.

In a typical experiment using 880 lb/in^2 of D_2 and 320 lb/in^2 of H_2, the input into the deuteron bridge was set to 100 millivolts and the rf voltage across the deuteron coil, as calculated from the constants of the bridge circuit, was 175 millivolts. The bridge was adjusted to resistive balance and reactive unbalance in the amount of about 10 microvolts at the input of the receiver. The gain of the receiver was set to give a carrier output of 2.5 volts at the second detector. The amplitude of the signal obtained was about one-fourth of the maximum obtainable by increasing the radio-frequency input power to the bridge while maintaining the carrier level constant at the second detector. The effects of saturation were kept small thereby. The proton bridge and receiver were operated under identical conditions, except for the input voltage being reduced to about 20 millivolts.

The property of the "Twin T" bridge of balancing separately the reactive and resistive components of the signal is an important feature of the present measurement. The presence of an appreciable amount of resistive unbalance in the signal results in a systematic shift in the center of symmetry of the pattern. This could easily be mistaken for an apparent difference in the value of the resonant field. In the present measurements, the resistive unbalance was always less than 1/100 the reactive unbalance. A special experiment showed that, with a resistive unbalance ten times greater than that usually used, the shift in the center of the pattern was not detectable.

D. Magnetic Field Control

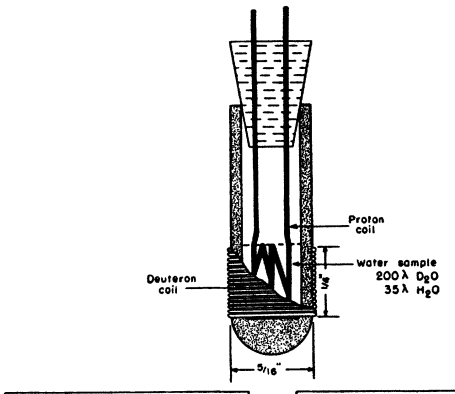
The magnet used in this experiment was the same as that used in I. It had a 5-inch-diameter pole with a 1-inch gap and was provided with a high voltage, low current, field winding so that precise electronic control of the magnet current could be arranged. The dc voltage



	COIL DATA	
	PROTON	DEUTERON
Q	65 (at 31 Mc)	40 (at 4.8 Mc)
L	0.38 μh	2.65 μh
turns	6	18
wire size	#24	#28

FIG. 3. Gas chamber assembly for nuclear resonance measurement. Axes of proton and deuteron coil are perpendicular to each other and to magnetic field, H_0 (directed into the figure).

amplifier used in the current regulator was improved to provide a loop gain of the feed-back circuit of 6000. The dc field sweep rate was controlled by a motor-driven Helipot and was usually set to change the field at the rate of 3.0 milligauss per second. Ample time was afforded thereby for the response of the lock-in amplifiers. The deuteron and proton patterns were usually observed with a time difference less than three seconds. Thus, the requirement on the regulator was to keep the field constant over times of only about this duration. Occasionally, a shift in magnetic field of as much as 3×10^{-6} was detected during the traversal of a resonance pattern. Such patterns were not excluded from the data, and they contributed their share to the lack of precision in the final result. Inconsistency of the field, together



	COIL DATA	
	PROTON	DEUTERON
Q	65 (at 31 Mc)	57 (at 4.8 Mc)
L	0.36 μh	2.4 μh
turns	3	18
wire size	#24	#28

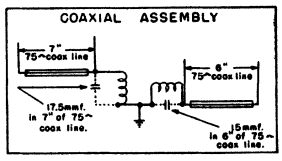


FIG. 4. Water chamber assembly for nuclear resonance measurement. Axes of proton and deuteron coil are perpendicular to each other and to magnetic field, H_0 (directed into the figure).

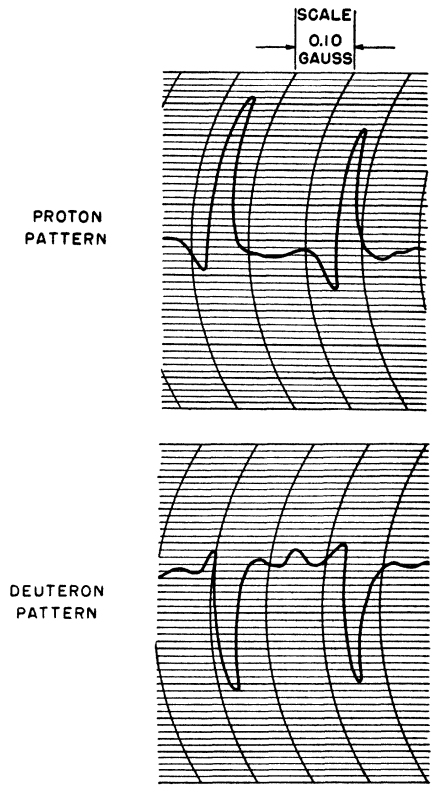


FIG. 5. Proton and deuteron resonance patterns in H_2-D_2. Dispersion signal patterns as they appear at output of lock-in amplifiers; ac sweep set to 0.024 gauss.

with inhomogeneity of the field, constituted the major source of inaccuracy in these measurements. Typical patterns are shown in Fig. 5 for the H_2-D_2 mixture and in Fig. 6 for the H_2O-D_2O mixture.

E. ac Field Control

The small sinusoidally varying field was applied through a separate set of coils wound about the pole pieces. The frequency was determined by a fixed low frequency oscillator and was coupled to an appropriate power amplifier permitting control of the amplitude of the ac field from zero to 2.5 gauss. Measurements were usually made with an amplitude of 24 milligauss. Since the selective amplifier had a band width of 0.5 cycle, it was found necessary to stabilize the frequency to less than 0.1 cycle to insure a stable gain from the amplifier. Moreover, the phase of the resonance signal influenced the size and shape of the recorded pattern; and, hence, it was necessary to avoid variable unknown phase shifts through the selective stage of the lock-in amplifiers. Thus, for the D_2O-H_2O measurements, the 30-cycle oscillator was of multivibrator design locked into the 60-cycle power line frequency. For the gas measurements, it had become necessary to avoid interference effects from adjacent motor generators, and so a 25-cycle oscillator was used, locking into the 100-cycle subharmonic of the 100-kc frequency standard. For studies of frequency modulation effects, the 100-cycle signal was used directly. The phasing arrangement was of the variable RC type driven from the output of a center-tapped transformer which permitted phase shifts from zero to almost 180 degrees.

F. Line Shape

In the initial experiments, we were puzzled by an apparent splitting of the deuteron line. Following a suggestion by Pound, the cause was sought and found to be a frequency modulation effect. The frequency modulation comes because of the use of the 25-cycle magnetic field modulation. In the absence of this modulation, the nuclei precess with frequencies in a narrow band about the central frequency given by Eq. (3). When the modulation is applied, however, side bands appear in the spectrum of the precessional frequency spaced by almost integral multiples of the modulation frequency on either side of the central frequency. The amplitude of the modulation determines the amplitude of the side bands. Where the modulation amplitude is large, the distant side bands contribute and the pattern obtained is broadened. A series of maxima will appear if the width of the resonance is small compared with the modulation frequency. In the present case, the width of the resonance was determined by the inhomogeneity in the magnetic field. The effect was particularly marked when the water samples were used, since then this inhomogeneity amounted to about 18 milligauss for the proton and 36 milligauss for the deuteron. The corresponding line widths are 23 cycles/sec for the deuteron and 77 cycles/sec for the proton. Thus, with a modulation frequency of 30 cycles/sec, a splitting in the line could be expected for the deuteron but not for the proton, and this was as observed. Splitting in the proton pattern was observed when the modulation frequency was increased to 100 cycles/sec.

A closer understanding of the effects of frequency modulation was obtained by applying the analysis of Karplus[9] to the present case. When saturation effects are unimportant, the equations of motion of the magnetic moment of a collection of nuclei in a homogeneous magnetic field $H(t)$ in the z direction, acted upon by a rotating magnetic field of amplitude H_1 and angular frequency ω may be written,[10] in terms of the complex function $F=v+iu$,

$$dF/dt=[\omega_r+i\Delta\omega(t)]F=-|\gamma|H_1M. \qquad (6)$$

Here u and v are the components of the magnetic moment in- and out-of-phase, respectively, with the rotating field H_1; $|\gamma|$ is the absolute value of the gyromagnetic ratio; $\Delta\omega=|\gamma|H_0(t)-\omega$; ω_r is the inverse of the relaxation time; and $M=\chi H_0(t)$ is the equilibrium magnetic moment.

[9] R. Karplus, Phys. Rev. **73**, 1029 (1948).
[10] B. A. Jacobsohn and R. K. Wangsness, Phys. Rev. **73**, 942 (1948).

For sinusoidal modulation of the magnetic field

$$\Delta\omega(t)=\bar{\omega}+\omega_m\cos\omega_s t. \tag{7}$$

The time-independent solution of Eq. (6) is

$$F=|\gamma|H_1M_0\int_0^\infty dT \times\exp\left[-iT\bar{\omega}-T\omega_r-i\omega_m\int_{t-T}^{t}\cos\omega_s t'dt'\right]. \tag{8}$$

The exponential in the integrand is expanded in a fourier series in terms of the ordinary bessel function $J_k(x)$,

$$\exp[-i\beta\sin x]=\sum_{k=-\infty}^{\infty}J_k(\beta)\exp[-ikx]. \tag{9}$$

On integrating, Eq. (8) becomes

$$F=-i|\gamma|H_1M_0\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}J_k\left(\frac{\omega_m}{\omega_s}\right)J_l\left(\frac{\omega_m}{\omega_s}\right) \times\exp[i(k-l)\omega_s t]\frac{k\omega_m+\bar{\omega}+i\omega_r}{(k\omega_m+\bar{\omega})^2+\omega_r^2}. \tag{10}$$

In the experiments, the bridge was usually unbalanced to give the dispersion signal, which is the imaginary part of F. Moreover, the detector system responded only to the fundamental of the sweep frequency ω_s, so that only terms with $k-l=\pm1$ contribute. The signal is proportional to

$$u=A\cos\omega_s t+B\sin\omega_s t,$$

where

$$A=\sum_{k=0}^{\infty}\frac{2k\omega_s}{\omega_m}J_k{}^2\frac{2k\omega_s(k^2\omega_s{}^2+\omega_r{}^2-\bar{\omega}^2)}{([k\omega_s+\bar{\omega}]^2+\omega_r{}^2)([k\omega_s-\bar{\omega}]^2+\omega_r{}^2)}, \tag{11}$$

$$B=\sum_{k=0}^{\infty}\left(\frac{2k\omega_s}{\omega_m}J_k{}^2-2J_kJ_{k+1}\right) \times\frac{2\omega_r(k^2\omega_s{}^2+\omega_r{}^2+\bar{\omega}^2)}{([k\omega_s+\bar{\omega}]+\omega_r{}^2)([k\omega_s-\bar{\omega}]^2+\omega_r{}^2)},$$

$$J_k=J_k(\omega_m/\omega_s).$$

In the synchronous detector, this signal is mixed with a reference signal $V=\cos(\omega_s t-\theta)$, and the product of the two is integrated to give the dc output which drives the Esterline Angus recorder:

$$V_0\propto\oint uVdT=A\cos\theta+B\sin\theta. \tag{12}$$

The size and shape of the pattern may be altered markedly by the setting of the phase angle θ. For purposes of frequency comparison, the important feature of (10) and (11) is that the pattern is symmetric with respect to $\bar{\omega}$ and this dictates the use of the center of symmetry of the pattern in the comparisons.

There is an optimum phase setting which gives maximum response at $\bar{\omega}=0$.

$$\theta_{\text{opt}}=\tan^{-1}[B(\bar{\omega}=0)/A(\bar{\omega}=0)]. \tag{13}$$

At phase angles 90° removed from this, the response at $\bar{\omega}=0$ becomes zero. When $\omega_s>\omega_r$, frequency modulation effects occur which cause a splitting in the pattern where θ is not close to θ_{opt}. A comparison of patterns as calculated with $\omega_s/\omega_r=1.22$ and $\omega_m/\omega_s=0.64$ with those observed is given in Figs. 7 and 8 for $\theta=\theta_{\text{opt}}$ and $\theta=\theta_{\text{opt}}+\pi/2$, respectively. Such curves confirm the general behavior outlined here and form the basis for the estimates of field inhomogeneity which have been given.

The width of the pattern is minimized by lowering the modulation amplitude until only the terms involving $k=0, 1$ contribute appreciably. Beyond this, the amplitude of the signal is reduced without further reduction in width.

III. RATIO DETERMINATION

A. Procedure

It was found convenient to follow a set procedure for each separate determination. For the D_2O:H_2O system,

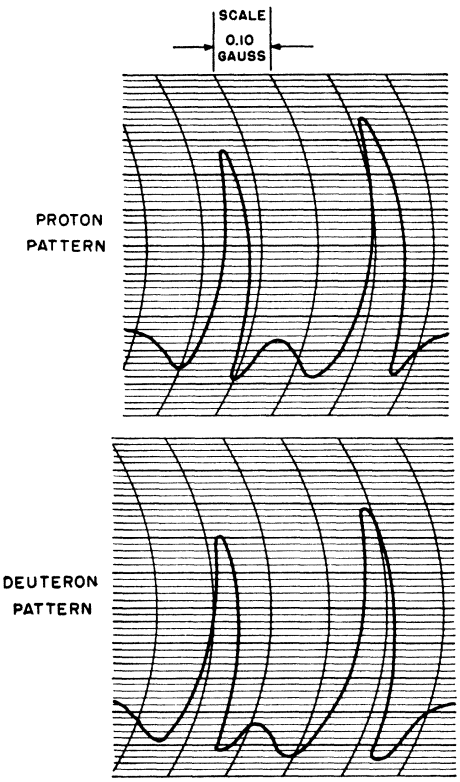


FIG. 6. Proton and deuteron resonance patterns in H_2O-D_2O. Dispersion signal patterns as they appear at output of lock-in amplifiers; ac sweep set to 0.024 gauss.

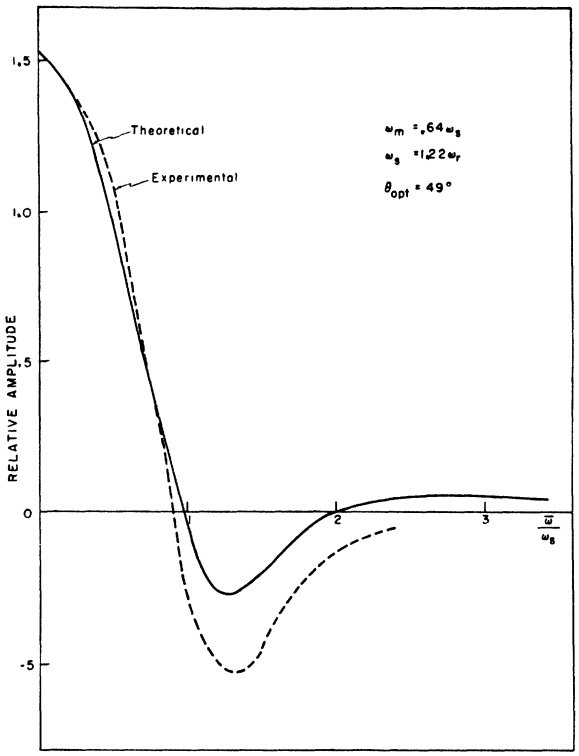


FIG. 7. Resonance line shape for $\theta=\theta_{opt}$. Line has symmetry about the $\tilde{\omega}=0$ axis. Curves are normalized at their maximum.

the sample was prepared in a small vial containing the desired ratio of D to H and Mn^{++} ion concentration. (The gas mixtures were made from a manifold system containing tanks of H_2 and D_2.) The radiofrequency power was set to a low value. The proton and deuteron detector phases were varied through 180 degrees; and the maximum amplitude of the dispersion pattern was determined, thus establishing the optimum phase. The signal amplitude and line width were then determined as a function of modulation amplitude, yielding a value for ω_r. With the modulation amplitude set for minimum line width and optimum phase, a saturation curve was run and the radiofrequency power was reduced to about 0.10 the value at which $\frac{1}{2}$ saturation had been obtained. (In the case of the gas measurements, the values of rf power to the deuteron coil were somewhat higher.)

Frequency checks and calibrations were made at the beginning and end of each run and at several intervals during its course. The modulation oscillator was set to a sequence of values, straddling the value where the patterns coincided in time. The steps were usually 13 cycles/sec apart and constantly monitored against the frequency meter by observation of the Lissajou figure on the oscilloscope. The bridges were balanced to a null and then adjusted to the proper reactive unbalance so that input level at the receivers was 10 microvolts. This adjustment was usually checked at each setting of the modulation oscillator. With the correct pure dispersion pattern attained of approximately equal height in both channels, a series of runs was taken of the line pattern in increasing and decreasing fields and the average displacement of the center of the patterns noted.

B. Results

The detailed measurements are given in Tables I and II for water and gas samples, respectively, where the displacement is given in milligauss. The value of modulation frequency for zero displacement between the center of the deuteron and proton pattern (i.e., identical magnetic field) was determined by the method of least squares. The results for the aqueous system are given in Table III, while those for the gas are shown in Table IV.

In any given run the relative probable error amounted to about 2×10^{-7}, owing primarily to the uncertainty in selection of the line center. The relative error in establishing the frequency ratio amounted to about 1×10^{-7}. In order to reveal the presence of other systematic errors, the experimental conditions were varied in the various runs. Thus, there was some question whether the central field for the two substances was at precisely the same position. Accordingly, in all measurements with water samples new coils were constructed for each determination. For the gas, measurements were taken with coils rotated 90° from their conventional position.

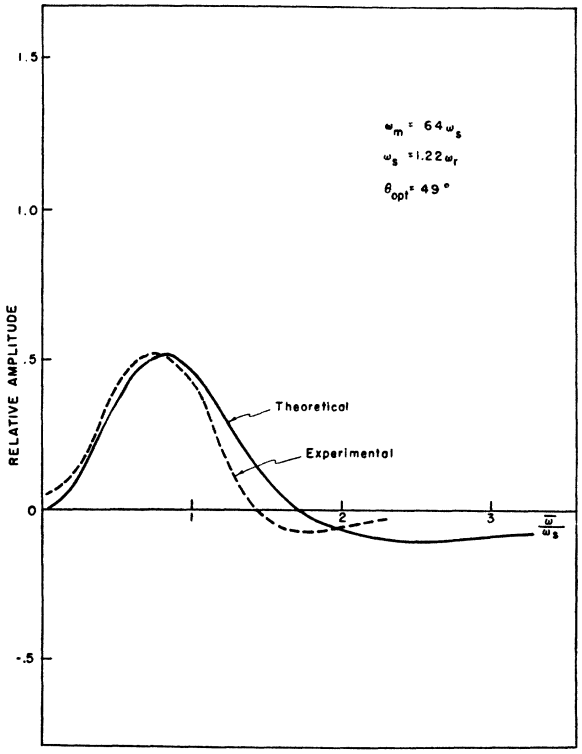


FIG. 8. Resonance line shape for $\theta=\theta_{opt}+\frac{1}{2}\pi$. Line has symmetry about $\tilde{\omega}=0$ axis, thus presenting a doublet appearance. Both curves are normalized at their maximum.

TABLE I. Difference in resonance values of the magnetic field for D+H for various modulation frequencies (using H_2O-D_2O mixture).

Run No.	$\frac{Mn^{++}}{D^+}$	f_0 Master oscillator frequency (cycles sec^{-1})	f_m Modulation oscillator frequency (cycles sec^{-1})	$\langle H(D)-H(H)\rangle_{Av}$ (milligauss)
31	0	2360290	68030.8	−12.7
			68017.6	−9.1
			68004.4	−5.2
			67991.2	−1.4
			67978.0	+1.1
			67964.8	+4.2
			67951.6	+5.6
			67938.4	+9.5
33	1×10^{-6}	2360250	68030.8	−10.5
			68017.6	−8.0
			68004.4	−6.3
			67991.2	−2.5
			67978.0	+0.5
			67964.8	+3.0
			67951.6	+6.9
			67938.4	+11.3
35	1.4×10^{-5}	2360270	68004.4	−8.8
			67991.2	−6.4
			67984.6	−3.6
			67978.0	−1.3
			67964.8	0
			67951.6	+2.7
			67938.4	+7.4
37	1×10^{-5}	2360290	68017.6	−13.1
			68004.4	−12.6
			67991.2	−1.9
			67984.6	−5.8
			67978.0	−1.2
			67971.4	−2.6
			67964.8	0
			67951.6	+2.6
36	1×10^{-4}	2360310	68017.6	−13.6
			68004.4	−6.2
			67991.2	−4.2
			67984.6	−2.5
			67978.0	+2.4
			67964.8	+4.0
			67951.6	+9.9
			67938.4	+9.4

TABLE II. Difference in resonance values of the magnetic field for D+H for various modulation frequencies (using H_2-D_2 mixture).

Run No.	f_0 Master oscillator frequency (cycles sec^{-1})	f_m Modulation oscillator frequency (cycles sec^{-1})	$\langle H(D)-H(H)\rangle_{Av}$ (milligauss)
47	2360250	68004.4	−8.4
		67978.0	−7.0
		67964.8	+3.1
		67951.6	+3.5
		67938.4	+3.7
		67912.0	+12.3
49	2360270	68004.4	−11.3
		67991.2	−8.0
		67978.0	−4.0
		67964.8	−1.7
		67951.6	0
		67938.4	+5.4
		67925.2	+7.3
56	2360330	68004.4	−13.2
		67991.2	−6.5
		67978.0	−2.1
		67964.8	+1.6
		67951.6	+3.4
		67938.4	+5.0
		67925.2	+6.0
57	2360330	67993.4	−2.0
		67959.2	+7.0
		67940.8	+9.8
58	2360200	68004.4	−4.6
		67991.2	−2.6
		67978.0	0
		67964.8	+4.1
		67951.6	+5.7
		67938.4	+9.3
59	2360170	68004.4	−3.4
		67991.2	−5.1
		67978.0	−1.1
		67964.8	−4.9
		67951.6	0
		67938.4	0
		67925.2	+5.9
60	2360250	68004.4	−4.3
		67991.2	−3.9
		67978.0	−2.7
		67964.8	−0.3
		67951.6	+0.5
		67938.4	+1.8
		67925.2	+5.3

Measurements were also made with the coil in different positions in the magnetic field. For some measurements the dc field sweeping rate was varied, the recording systems were interchanged, and the radiofrequency power level altered. In the water measurements different amounts of manganese ion were added, while for the gas the pressure was varied to some extent. The effect of all of these changes can be judged from the self-consistency of the several runs and appears to be of the order of 2×10^{-7}.

An additional uncertainty of about 1×10^{-7} should be included due to the magnetic shielding correction. This correction, when calculated using Ramsey's formula[11] and new values for the rotational field,[12,13] is

[11] N. F. Ramsey, Phys. Rev. **78**, 699 (1950).

[12] Kolsky, Phipps, Ramsey, and Silsbee, Phys. Rev. **79**, 883 (1950).

[13] Kolsky, Phipps, Ramsey, and Silsbee, Phys. Rev. **80**, 483 (1950).

TABLE III. Ratio of the magnetic moment of the proton to that of the deuteron in H_2O-D_2O.

Run No.	Mn^{++}/D^+	μ_H/μ_D	Probable error
31	0	3.25720041	±0.00000022
33	1.0×10^{-6}	3.25720065	0.00000022
35	1.4×10^{-5}	3.25719906	0.00000023
37	1.0×10^{-5}	3.25719906	0.00000079
36	1.0×10^{-4}	3.25720014	0.00000068
	Mean	3.25719986	±0.00000023

very closely the same for both H_2 and D_2. However, a difference in the magnetic shielding constant, σ, arises because of changes in the amplitude of the molecular vibration.[14] For the first term in Ramsey's formula, Newell[15] has calculated that the contribution to σ is greater for D_2 by $(1.1\pm0.2)\times10^{-7}$. He points out, however, that this change could be canceled by the effect of the molecular vibration in the second term.

The result for the ratio of the magnetic moments of the proton and the deuteron from the measurements with the gas is thus

$$\mu_H/\mu_D=3.2571990\pm0.0000010.$$

The correction in the case of the water measurements is less certain and may possibly be twice as large as for the gas. The result of the water measurements is, thus,

$$\mu_H/\mu_D=3.2571999\pm0.0000012.$$

TABLE IV. Ratio of the magnetic moment of the proton to that of the deuteron in H_2-D_2.

Run No.	Pressure (lb/in²) H_2	D_2	μ_H/μ_D	Probable error
47	300	700	3.25719876	±0.00000069
49	300	500	3.25719803	0.00000022
56	520	880	3.25719818	0.00000060
57	300	500	3.25720074	0.00000016
58	320	880	3.25720064	0.00000024
59	660	740	3.25719854	0.00000096
60	660	740	3.25719825	0.00000049
		Mean	3.25719902	±0.00000030

[14] G. F. Newell, Phys. Rev. **80**, 476 (1950).
[15] G. F. Newell (private communication).

A comparison of these results with previous determinations of the ratio μ_D/μ_H is given in Table V. The present result is in agreement with that of Levinthal, but with an accuracy of about fifteen times that of the latter. However, it lies outside the stated error of the values given by Lindstrom, as well as that of Bitter *et al.* and of Siegbahn.

Using the value of μ_H/μ_D as determined with the gas and the recent value of m_p/m_e given by Sommer,[16] the expected value of the hyperfine structure ratio ν_H/ν_D is obtained from Eq. (1). The deviation from that measured by Prodell and Kusch[5] is thus

$$\epsilon=\Delta\kappa/\kappa=1.702\pm0.007\times10^{-4}.$$

TABLE V. Comparison with previous values of μ_D/μ_H.

Author	Sample	μ_D/μ_H	Stated error
Bloch[a]	Water	0.3070126	±0.000002
Bitter[b]	Liquid H_2	0.307021	0.000005
Siegbahn[c]	Water	0.3070183	0.0000015
Wimett[d]	Water	0.3070117	0.0000017
Lindstrom[e]	Water	0.30701337	0.00000050
Levinthal[f]	Water	0.3070117	0.0000015
Present work	Water	0.30701217	0.00000012
Present work	Gas	0.30701225	0.00000010

[a] Bloch, Graves, Packard, and Spence, Phys. Rev. **71**, 551 (1947).
[b] Bitter, Alpert, Nagle, and Poss, Phys. Rev. **72**, 1271 (1947).
[c] K. Siegbahn and G. Lindstrom, Nature **163**, 211 (1949).
[d] T. F. Wimett, M.I.T. Research Laboratory of Electronics Report (July, 1949).
[e] G. Lindstrom, Phys. Rev. **78**, 817 (1950).
[f] E. C. Levinthal, Phys. Rev. **78**, 204 (1950).

This value is consistent with the theoretical estimate of this required correction, but possesses a considerably higher precision and thereby provides a critical test of the theory of the structure of the deuteron.

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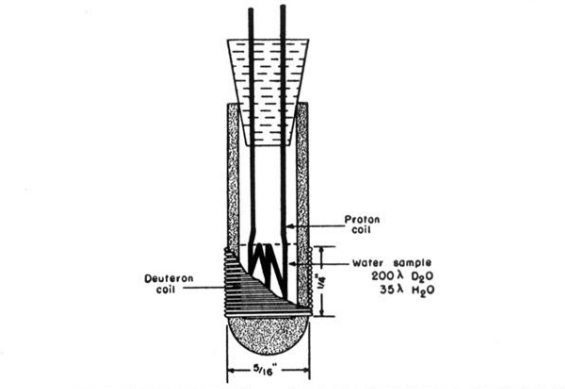
[16] Sommer, Thomas, and Hipple, Phys. Rev. **80**, 487 (1950).



	COIL DATA	
	PROTON	DEUTERON
Q	65 (at 31 Mc)	57 (at 4.8 Mc)
L	0.36 μh	2.4 μh
turns	3	18
wire size	#24	#28

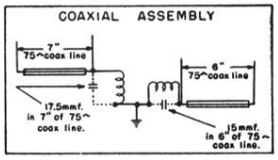


FIG. 4. Water chamber assembly for nuclear resonance measurement. Axes of proton and deuteron coil are perpendicular to each other and to magnetic field, H_0 (directed into the figure).