Zenith Angle and Momentum Spectrum of μ-Mesons at Sea Level*

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The zenith angle dependence of the μ-meson intensity at sea level has been measured for several values of momentum between 280 and 1080 Mev/c. Results were obtained by three methods: the coincidence, anticoincidence, and delayed coincidence methods, and it has been found that the shape of the zenith spectrum depended on which technique was used. The delayed coincidence method was capable of determining meson intensities in the presence of other types of penetrating particles. It suggests the existence of a considerable number of nonmesonic particles in the low energy region of the hard component.

The lifetime of the meson was computed from its anomalous absorption in the atmosphere. It was found to be 2.2 ± 0.3 microseconds.

I. INTRODUCTION

THIS paper reports some measurements of the intensity of sea level mesons in several zenith directions for several momentum intervals. Similar measurements were made in the past, but in the present work a relatively new procedure has been used, making the identification of mesons much more certain. Formerly, it had been assumed that at sea level the number of particles in the hard component besides mesons was negligibly small. In recent years doubt has been cast on this. It has been found or suggested that:

a. A considerable number of protons are in the low energy part of the hard component. For instance, Conversi found an X component (presumably protons) making up sixteen percent of the hard component stopping per g/cm² of lead.[1] This is also supported by the cloud chamber experiments of Merkle, Goldwasser, and Brode.[2]

b. A significant number of electrons has been found by Reynolds to penetrate 10 cm of lead.[3]

c. An overestimation of the number of mesons that stop in the apparatus may result from scattering. Kraushaar and Rusk have analyzed this source of error.[4,5]

Two principal types of discrepancies may be found in the preceding work; (1) Coincidence and anticoincidence experiments gave different zenith angle variations of the meson intensity. (2) The lifetime of the meson deduced from its instability in the atmosphere has varied considerably in different experiments. It was usually found to be longer than the directly determined value of 2.2 microseconds. In order to see whether these discrepancies may have arisen from the complete identification of the hard component with mesons, the zenith and momentum spectrum was investigated by the method of delayed coincidences. Counters and electronic circuits were employed to determine the time intervals between the stopping of particles in a graphite absorber and the emission of decay electrons. It was found that the distribution of time intervals agreed with the known mean life of the meson. Hence, there was little doubt that what was measured was the absorption of μ-mesons only and their subsequent decay. This method was first used by Rossi, Sands, and Sard in their measurement of the slow meson intensity as a function of altitude.[6] Shortly afterward Shamos and Levy found the positive excess of mesons in this way.[7] Recently it has been employed by Kraushaar and by others.

Rossi normalized and combined the results of various observers and obtained a single curve representing the best contemporaneous estimate of the differential spectrum of vertical mesons at sea level.[8] This curve is here reproduced in Fig. 6 as the solid line labeled 0°. The momentum spectrum of mesons in inclined zenith directions is not so well known. The variation in intensity is usually expressed by:

$$I(\theta)=I_0\cos^n\theta; \quad (1)$$

where I_0 is the vertical intensity, θ is the zenith angle, and n is a constant determined from the experiment.

A precise determination of the zenith spectrum was made by Greisen at several altitudes, including near sea level, and with several thicknesses of lead.[9] For the total radiation he found n to be 2.3; for the hard component with momentum greater than about three hundred Mev/c (computed for mesons), n was 2.1; finally for the soft component, n was 3.4.

The most recent determination of n was by Kraushaar who used three techniques: coincidence, anticoincidence, and delayed coincidence.[4] Differences in the zenith variation were found, depending on the type of measurement. With the coincidence technique, n was found to be 2.09 for all particles whose range in lead exceeded that of two hundred Mev/c mesons. In the anticoincidence experiment the momentum range between 200

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[1] M. Conversi, Phys. Rev. **76**, 444, 849, 851 (1949).
[2] Merkle, Goldwasser, and Brode, Phys. Rev. **79**, 926 (1950).
[3] G. T. Reynolds, Revs. Modern Phys. **21**, 122 (1949).
[4] W. L. Kraushaar, Phys. Rev. **76**, 1045 (1949).
[5] R. D. Rusk and A. Rosenbaum, Phys. Rev. **76**, 1166 (1949).
[6] Rossi, Sands, and Sard, Phys. Rev. **72**, 120 (1947).
[7] M. H. Shamos and M. G. Levy, Phys. Rev. **73**, 1396 (1948).
[8] B. Rossi, Revs. Modern Phys. **20**, 537 (1948).
[9] K. Greisen, Phys. Rev. **61**, 212 (1942).

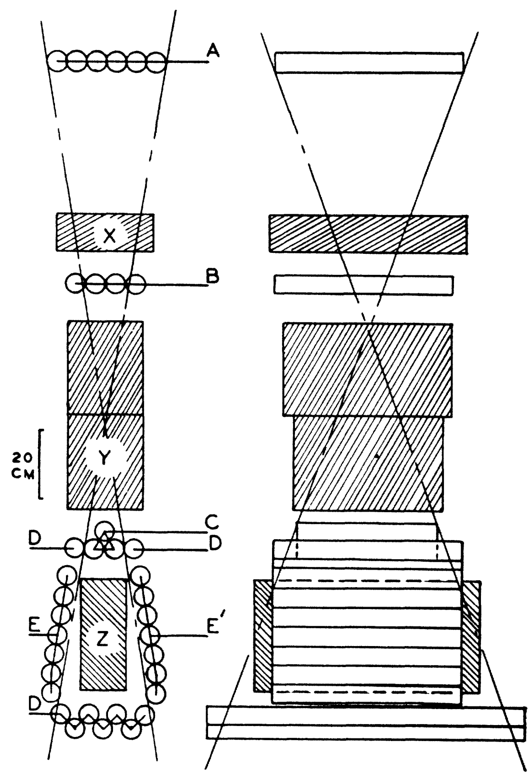

FIG. 1. Telescope for identification of mesons. A, B, and C form a threefold coincidence set with D in anticoincidence. E and E' detect decay electrons. X is lead, Y is iron, and Z is graphite. The whole apparatus may be tilted.

and 330 Mev/c was investigated. The delayed coincidence experiment was on mesons only, with momenta between 25 and 170 Mev/c. In the last two measurements the values obtained for n were both 3.3. The experiments that are here described were, in part, an extension of the work of Kraushaar to higher momentum.

When the intensity of the hard component was compared at different altitudes with allowance for the mass of the intervening column of air, it was found that the intensity lost in the atmosphere greatly exceeded the amount predicted by collision theory. This anomaly resulted from the spontaneous decay of mesons. If a meson is not absorbed, its mean range before decay is:

$$R = vt_0/(1-\beta^2)^{\frac{1}{2}} = pt_0/u_0. \tag{2}$$

Here v, t_0, u_0, and p are, respectively, the velocity, proper lifetime, rest mass, and mean momentum. There are two principal ways by which R may be measured and the lifetime deduced: (a) from the dependence of meson intensity on altitude, (b) from the intensity of mesons at one location but in different zenith directions.

Many observations of the "anomalous" absorption of mesons in the atmosphere have been made and lifetime values ranging from about 1.5 to 7 microseconds have been found. The most recent work in this field was by Cocconi and Tongiorgi.[10] They investigated the relation between the meson lifetime and the assumed mean free path of primaries, and showed that the calculated lifetime did not vary by more than plus or minus ten percent when the origin of the mesons was assumed to be 50, 100, or 200 g/cm² below the top of the atmosphere. Using 100 g/cm² for the mean free path of the primaries and 7.4×10^5 cm for z_0 (see Eq. (5)), they found lifetimes of 2.7 ± 0.4 and 3.1 ± 0.5 microseconds in two series of measurements. It will be shown that a better choice for z_0 would have been 7.0×10^5 cm. With this constant the values become 2.5 ± 0.4 and 2.9 ± 0.5 microseconds.

The delayed coincidence investigations of the present experiment afford an opportunity for measuring the anomalous absorption of mesons without requiring assumptions as to the composition of the hard component. Also, the observations were made sufficiently low in the atmosphere so as to preclude considerable local meson production. Therefore, as a part of this investigation, the estimation of the mean lifetime of the meson was undertaken to see whether an accurate value could be found by this technique.

II. APPARATUS

The apparatus was assembled in a thin-walled penthouse on the roof of the Washington Square College. Figure 1 shows a scale drawing of the essential apparatus. The telescope was mounted in a rigid, angle-iron frame and could be tilited to various zenith directions. Counter trays A, B, and C formed a threefold coincidence set for detecting incoming radiation. The anticoincidence section, D, determined the number of stopped particles. There were two guard counters in this section also for protection against cascade and side showers. Decay electrons were detected by two side trays, E and E'. The soft component was removed by a 116 g/cm² lead filter, X, which was always present. The removable absorbing material was in the form of iron bars, Y, of magnitude 0, 202, and 404 g/cm². These selected the specific region of the momentum spectrum where measurements were to be made. Lastly, a 53-g/cm² block of graphite, Z, was used to absorb mesons and provide a source of decay electrons.

The G-M counters were of the metal type with a diameter of two inches and lengths of fifteen, twenty, and thirty-four inches, as may be seen in Fig. 1. Anodes were four-mil tungsten wire and cathodes were $\frac{1}{32}$-inch brass. A self-quenching filling of 9 cm argon and 1.3 cm ethyl acetate pressure was used. Pulses and counting rates were examined daily. The various counters in a tray were connected to a single preamplifier mounted close by. This contained a cathode follower and a one-shot multivibrator.[11] With this, the pulse rise was so rapid that the number of counts delayed by more than one microsecond (because of the natural time lags of the counters) was quite negligible. Trays A, B, and C

[10] G. Cocconi and V. Tongiorgi, Phys. Rev. **70**, 855 (1946).

[11] W. Y. Chang and J. R. Winckler, Rev. Sci. Instr. **20**, 276 (1949).

were connected to a Rossi type coincidence circuit with a resolving time of about thirty-five microseconds. Its counting rate gave the integral spectrum, that is, the number of particles able to penetrate absorbers X and Y. The anticoincidence circuit received pulses from both the D counters and the coincidence unit, and recorded fourfold coincidences (A, B, C, D). The number of stopped particles was therefore found by subtracting the readings of the anticoincidence from the coincidence registers. The delay circuit counted the decay electrons detected by the E or E' counters in from one to nine microseconds after a particle had stopped in the graphite absorber. It received pulses from the coincidence unit, the anticoincidence unit and the E counters. An output pulse was generated whose height was proportional to the time between the coincidence and the delayed event, provided there was no anticoincidence pulse at that time. The output pulses from the delay circuit were sorted and counted in an eight channel pulse-height discriminator. Calibration was accomplished by introducing artificial pulses with known amounts of delay into the preamplifiers.

The telescope admitted cosmic radiation with a maximum angular divergence in the direction of the zenith of ± 9.2 degrees, and in the direction of the azimuth of ± 19.7 degrees. Not all the rays penetrated the same thickness of absorber, however. In fact, the extreme rays just grazed it. After correction for this, there resulted a mean angular resolution in the zenith direction of ± 8.0 degrees, and in the azimuthal direction of ± 16.1 degrees.

III. EXPERIMENTAL PROCEDURE

The telescope was aligned with its axis of rotation pointing east and west. Observations were taken at zenith angles of thirty-two and fifty-five degrees tilted toward the south, and also vertically upwards. The south was chosen as the direction for measurement because it is in this quarter that the primaries are least disturbed by the magnetic field of the earth.

Before the momenta could be computed from the absorber thicknesses, several corrections were necessary. (1) The thicknesses were multiplied by 1.01 because a portion of the incident beam passed diagonally through the absorbers. (2) The roof above the equipment, the counter walls and the supports contained about 14 g/cm² of material of medium atomic number. This was equivalent to 19 g/cm² of lead in absorbing power, and was included in the computation by adding this amount to the lead. (3) The path of the radiation through the absorbers was increased by multiple scattering. To allow for this, the lead thickness was increased by 9.5 percent, and the iron thickness by 2.5 percent, according to the investigation of Koenig.[12] Multiple scattering in lighter materials was negligible. After including the above three corrections, the equivalent absorber thicknesses that were used in the momentum calculations for the experiment were: (a) 146 g/cm² lead in position X; (b) 146 g/cm² lead in position X and 207 g/cm² iron in position Y; (c) 146 g/cm² lead in position X and 414 g/cm² iron in position Y. The amount of graphite in position Z was 53.3 g/cm². The momentum values that correspond to these ranges were obtained from the curves by Gross.[13] These were slightly corrected to be in accord with a meson mass of 110 Mev/c^2. Since the Gross curves are based on the calculations of Wick, they include some allowance for the polarization of the absorbing medium.[14] The momentum values used in this experiment are given in Table I.

TABLE I. Meson momenta corresponding to specified ranges of absorbing material. Corrections have been made for multiple scattering and for divergence of the beam.

Absorber	Momentum
115 g/cm² Pb	283 Mev/c
above+53 g/cm² C	391
115 g/cm² Pb+200 g/cm² Fe	615
above+53 g/cm² C	730
115 g/cm² Pb+400 g/cm² Fe	960
above+53 g/cm² C	1080

Frequent changes were made in the zenith angle and momentum region under observation. With each thickness of lead and iron, the telescope was operated for a day or two, first with the graphite absorber, then without it to obtain the background. On subtracting the background readings from those found with absorber, there were eliminated those disturbing effects which occurred with equal probability in the two cases. These effects included scattering in the metal absorbers, shower or accidental coincidences, the stopping of mesons in counter walls and supports, false delays caused by time lags in the counters, errors caused by dead-time of the counters or anticoincidence circuit, etc. When all the absorber thicknesses had been used, the telescope was tilted to another angle and the series of absorbers again run through.

Although rotation of the experimental conditions tended to smooth out the effects of atmospheric fluctuation, one could not be sure that all measurements represented identical average conditions unless some additional means of correction was employed. Consequently, a second threefold coincidence telescope was operated vertically under ten cm of lead. This continuously monitored the intensity of the hard component. Its readings were used to correct the data of the main zenith telescope in accordance with the day-to-day fluctuations of intensity of the hard component.

IV. MEASUREMENTS OF ZENITH ANGLE SPECTRA

A. Integral Spectrum

The integral spectrum of the hard component is shown as a function of zenith angle in Fig. 2. The ex-

[12] H. P. Koenig, Phys. Rev. **69**, 590 (1946).

[13] E. P. Gross, *Range, Energy, Ionization Curves*. Distributed by Princeton University (1947).

[14] G. C. Wick, Nuovo cimento, Sec. 9 1, 310 (1943).

TABLE II. Data and corrections for the differential spectrum of penetrating particles as a function of momentum and zenith angle.

1	2	3	4	5	6	7	8	9	10
			Data with absorber			Data for background		Difference	
Zen. angle deg.	Mean momentum Mev/c	Hours	Counts per hour	Corr. daily fluc.	Hours	Counts per hour	Corr. daily fluc.	Counts per hour	Corr. for first microsec.
0	337	178	50.4±0.5	51.2±0.5	92	30.4±0.6	30.6±0.6	20.6±0.8	22.2±0.8
0	672	212	30.3±0.4	30.5±0.4	113	15.7±0.4	15.9±0.4	14.6±0.5	16.2±0.5
0	1020	334	27.4±0.3	27.5±0.3	198	14.4±0.3	14.4±0.3	13.1±0.4	14.7±0.4
32	337	232	35.1±0.4	35.0±0.4	125	23.4±0.4	23.2±0.4	11.8±0.6	12.7±0.6
32	672	224	19.4±0.3	19.2±0.3	129	11.0±0.3	10.8±0.3	9.4±0.4	10.3±0.4
32	1020	186	18.2±0.3	18.3±0.3	84	10.7±0.4	10.8±0.3	7.6±0.5	8.5±0.5
55	337	452	16.0±0.2	16.0±0.2	305	11.6±0.2	11.6±0.2	4.4±0.3	4.7±0.3
55	672	243	7.8±0.2	7.8±0.2	137	5.1±0.2	5.1±0.2	2.7±0.3	3.0±0.3
55	1020	223	7.0±0.2	7.0±0.2	155	4.7±0.2	4.7±0.2	2.4±0.3	2.7±0.3

ponent, n, was found to be 2.04, 1.93, and 1.85 for p greater than 283, 615, and 960 Mev/c. This result is in harmony with previous work where exponents between 2.0 and 2.2 were found for the least penetrating portion of the hard component.

B. Differential Spectrum for Penetrating Particles

The number of particles between two momentum limits was found from the readings of the first channel of the delay recorder. This gave the number of particles that failed to set off either the anticoincidence or the delay counters within the first microsecond after a coincidence. Most of the particles that were widely scattered in the graphite were not recorded in this way. On the other hand, the mesons that stopped and decayed within the first microsecond, sending an electron through the E or E' trays of counters, were also not recorded. However, the number of these was found by extrapolation from the measured numbers decaying after one microsecond.

Table II contains the data and corrections for the differential spectrum of penetrating particles. The first correction applied, that given in columns 5 and 8, was for the daily variations of the hard component. The second correction, applied in column 10, was the estimated number of mesons that decayed during the first microsecond. The results are plotted in Fig. 3, where it may be seen that one value of n, 2.97, fits all data obtained for different meson momenta. (The errors shown in the table and in the figure are the standard statistical deviations of the numerical data.) This result is consistent with the measurements by Kraushaar.[4] With a similar method of determining the differential spectrum, he reported a value for n of 3.3 for low momentum mesons. Greisen, who measured the differential zenith spectrum for the soft component, found n to be 3.4.[15]

C. Differential Spectrum for Mesons

Mesons stopping in the graphite absorber were identified by detection of their decay electrons as already

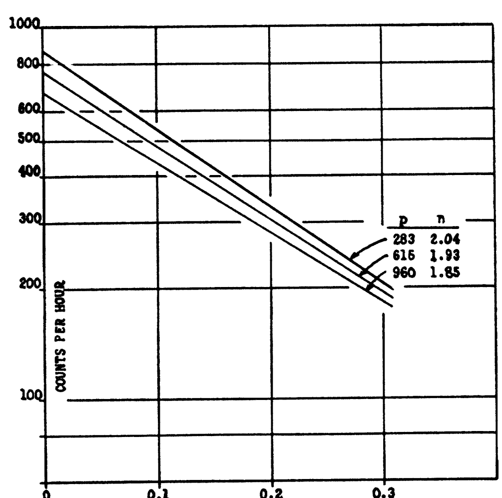

FIG. 2. Integral spectrum of the hard component as a function of zenith angle. This is expressed as a $\cos^n\theta$ law. The values of n decrease with increasing momentum.

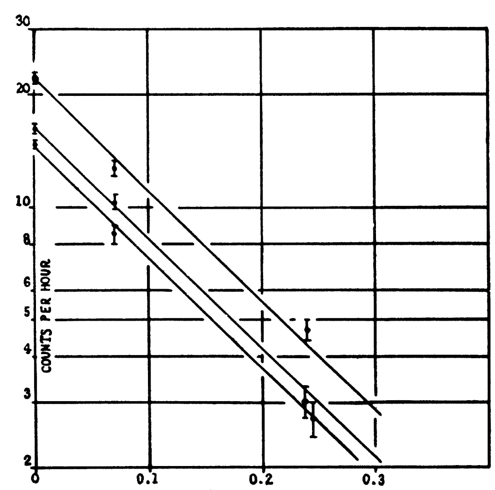

FIG. 3. The differential spectrum of the hard component as a function of zenith angle. The results were obtained with an anticoincidence counter telescope. The exponent n is 2.97 for all momenta.

[15] K. Greisen, Phys. Rev. 61, 212 (1942).

TABLE III. Data and corrections for the differential spectrum of mesons as a function of momentum and zenith angle.

		Data with absorber			Data for background			Difference
Zen. angle deg.	Mean momentum Mev/c	Hours	Counts per hour	Corr. for daily fluc.	Hours	Counts per hour	Corr. for daily fluc.	Counts per hour
0	337	178	3.48±0.14	3.54±0.14	92	1.54±0.13	1.56±0.13	1.98±0.19
0	672	212	3.79±0.13	3.82±0.13	113	1.55±0.12	1.57±0.12	2.25±0.18
0	1020	334	3.62±0.10	3.64±0.10	198	1.42±0.08	1.42±0.08	2.22±0.13
32	337	232	2.31±0.10	2.30±0.10	125	1.07±0.09	1.06±0.09	1.24±0.14
32	672	224	2.30±0.10	2.28±0.10	129	0.98±0.09	0.96±0.09	1.32±0.14
32	1020	186	2.52±0.12	2.53±0.12	84	1.00±0.11	1.01±0.11	1.52±0.16
55	337	452	0.91±0.05	0.91±0.05	305	0.60±0.04	0.60±0.04	0.31±0.06
55	672	243	0.90±0.06	0.90±0.06	137	0.52±0.06	0.52±0.06	0.38±0.09
55	1020	223	0.93±0.06	0.93±0.06	155	0.45±0.05	0.45±0.05	0.48±0.08

explained. The data were corrected for the number of delayed events that would have been observed after 9.0 microseconds had the recorder been sensitive for all time. The corrected integral decay curves for each zenith angle are shown in Fig. 4, where they may be seen to be consistent with the known lifetime of mesons in carbon, namely, 2.15 microseconds. There is one point lying off the curves by about twice the standard deviation. This is expected to occur occasionally as a result of the normal distribution of errors. In Table III are shown the observed intensities of mesons with the background subtracted from the data with absorber. The results are plotted in Fig. 5. The error introduced into the zenith distribution by the finite angular resolution of the telescope was found to be about one percent or less, and was ignored by comparison with the much larger statistical errors.

One may suppose that the exponent for the zenith angle distribution decreases as the meson energy increases. This is suggested by the observation that n for the integral spectrum is approximately two, whereas for the differential spectrum, it is approximately three. This variation of n with energy is consistent with the data in Fig. 5. It should be noted, however, that the meson counting rate was quite low at inclined zenith angles, and so the final numerical results are not known with great accuracy. The spectrum obtained by anticoincidence, as shown in Fig. 3, fails to show this expected variation of n with energy. Two possible explanations may be suggested.

(1) Protons near the end of their range have a greater specific ionization than mesons; and so an absorber in an anticoincidence experiment has a greater stopping power for protons than for mesons. Consequently, if protons of low energy were present in the penetrating component they would have a disproportionate influence on the shape of the anticoincidence differential spectrum.

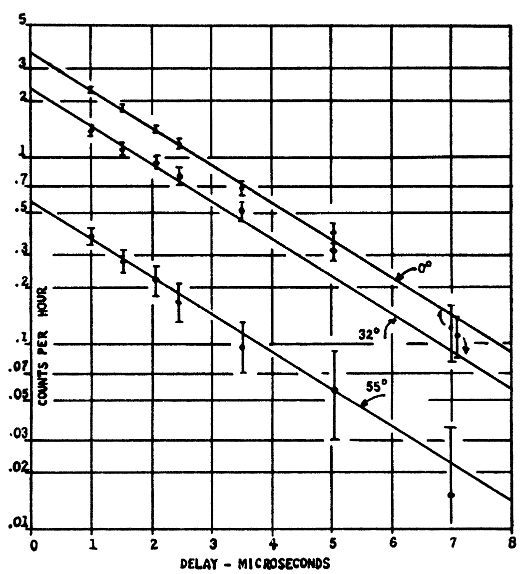

FIG. 4. Meson decay curves. Values are readings of delay discriminator for absorber minus background. They have been corrected to include delays to infinite time. Curves are drawn for a mean life of 2.15 microseconds.

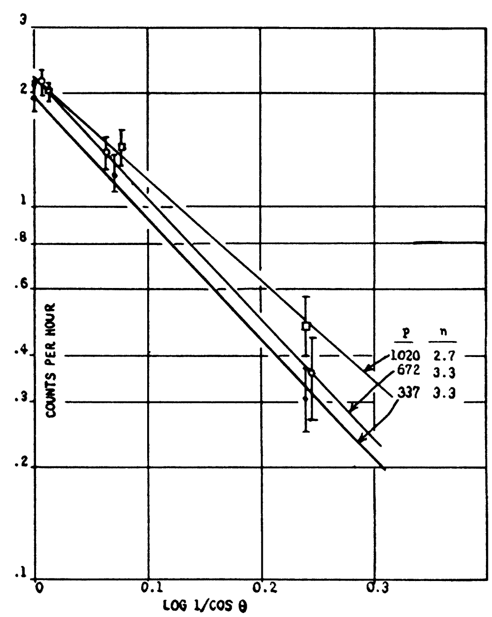

FIG. 5. The zenith angle dependence of mesons at sea level obtained by the method of delayed coincidences. Background intensities have been subtracted from measurements taken with a graphite absorber.

Since there is no *a priori* reason to suppose that the zenith angle distribution for protons is the same as for mesons at sea level, the lack of variation of n in Fig. 3 may in part be attributed to the presence of protons in the low energy region of the hard component.

(2) Kraushaar has pointed out how carefully an anticoincidence telescope must be designed so that errors produced by the scattering of particles in the heavy metal absorbers may be eliminated.[4] The difference between his value of 3.3 and the value obtained here for n may have resulted from different telescope designs.

D. Differential Range Spectrum

A comparison of the zenith and momentum spectra that have been obtained thus far by a number of investigators is shown in Fig. 6. The abscissa is given in terms of the range in air, and the conversion from momentum values to range units was made with the aid of the Gross curves.[12] The solid line graph labeled 0° is the vertical differential range spectrum as given by Rossi.[8] For comparison, the measurements of Kraushaar (K) and of Shamos and Levy (S) are shown plotted along with the delayed coincidence results of the present experiment (circles).[4,7] In order to obtain absolute intensities, the present data were normalized to Rossi's vertical curve at the 280 g/cm² point. The points for the curves at zenith angles of thirty and sixty degrees have been interpolated from Fig. 5 without further normalization.

The anticoincidence data of Fig. 4 were also plotted in Fig. 6 as crosses. For the lowest range, the crosses occur considerably above the meson curves. The anticoincidence measurements were made by subtracting background intensities, and so those sources of systematic error which depended on the momentum of the incident meson were largely eliminated. It is therefore concluded that ionizing particles besides mesons occur

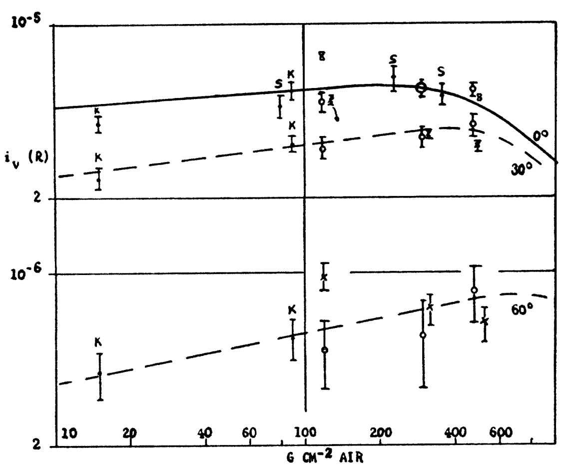

FIG. 6. Differential range spectrum of mesons at sea level. Ordinates are particles per cm² steradian second. Circles are delayed coincidence data; crosses are anticoincidence measurements. Points marked K and S are by Kraushaar and by Shamos and Levy.

with ranges between 100 and 400 g/cm² of air in all zenith directions at sea level. These particles are believed to be protons because: (1) it is unlikely that electrons could penetrate the ten cm of lead absorber in appreciable numbers; (2) an absorber in an anticoincidence experiment has a greater stopping power for low energy heavy particles than for lighter ones of similar range.

The present experimental results extend our knowledge of the meson zenith spectrum. Whenever comparisons can be made with previous work, the agreement appears satisfactory.

V. LIFETIME OF THE μ-MESON

The method of delayed coincidences permits the determination of the lifetime of the μ-meson from its anomalous absorption in the atmosphere. This technique was not affected by the presence of protons in the hard component, and systematic errors were largely eliminated by subtracting background readings. The calculations follow the method given by Janossy.[16]

In a group of $N(t)$ mesons, the number decaying is:

$$-dN=N(z)(\mu_0/t_0)dz/p(z). \tag{3}$$

If $N(0)$ mesons reach sea level with momentum p_1, the number that must have existed at an altitude, z, with momentum $p(z)$ is obtained by integration.

The atmospheric pressure may be approximated by:

$$\theta(z)=\theta_0\exp(-z/z_0), \tag{4}$$

where θ_0 and z_0 are constants.

Let p_A denote the momentum loss of a specified group of mesons in traversing the entire atmosphere.

$$p(z)=p_1+p_A[1-\exp(-z/z_0)]. \tag{5}$$

Equation (3) may now be integrated to give:

$$\frac{N(0)}{N(z)}=\left[\frac{p_A}{p_1}\frac{p(z)}{p_A+p_1-p(z)}\right]\exp\left[\frac{-z_0}{ct_0}\frac{\mu_0 c}{p_1+p_A}\right]. \tag{6}$$

Mesons, however, are not produced at the top of the atmosphere but throughout its depth. Take an altitude z_p, where the density is θ_p, representing the average height at which mesons are produced. The probability that they do not decay on their way down to sea level is then:

$$\frac{N(0)}{N(z_p)}=\left[\frac{\theta_0}{\theta_p}\left(1+\frac{p_A}{p_1}\right)-\frac{p_A}{p_1}\right]\exp\left[\frac{-z_0}{ct_0}\frac{\mu_0 c}{p_1+p_A}\right]. \tag{7}$$

The accuracy of the preceding is not significantly altered by the fact that mesons are created as π's which then decay into μ's. This is because in the π–μ-decay of

[16] L. Janossy, *Cosmic Rays* (Oxford University Press, London, 1950).

energetic particles the momentum of the π is almost entirely transferred to the μ; and, since the lifetime of the π-meson is only about one percent of the μ, the effective value of t_0 that would apply to the whole process is very nearly the lifetime of the μ-meson alone.

The treatment of the problem for travel in an inclined direction, making an angle ϕ with the vertical, is analogous to that which preceded, except that different constants are required, namely: $z_0'=z_0 \sec\phi$; $p_A'=p_A \sec\phi$; $\theta_0'=\theta_0 \sec\phi$; $z_p'=z_0 \ln(\theta_0/\theta_p \cos\phi)$. The probability that these mesons do not decay between the production region and sea level is found in the same way as Eq. (7).

$$\frac{N'(0)}{N'(z_p)}=\left[\frac{\theta_0}{\theta_p \cos\phi}\left(1+\frac{p_A}{p_1 \cos\phi}\right)-\frac{p_A}{p_1 \cos\phi}\right] \times\exp\left[\frac{-z_0}{ct_0}\,\frac{\mu_0 c}{p_1 \cos\phi+p_A}\right]. \quad (8)$$

From the experiments of Winckler and Stroud, it is known that the penetrating cosmic radiation near the top of the atmosphere is uniform in all directions.[17] Hence,

$$N(z_p)=N'(z_p). \quad (9)$$

Then

$$t_0=\frac{z_0\mu\left\{\ln\left[\frac{\theta_0}{\theta_p \cos\phi}\,\frac{p_0}{p_1'}-\frac{p_A}{p_1' \cos\phi}\right]-\cos\phi \ln\left[\frac{\theta_0}{\theta_p}\,\frac{p_0}{p_1}-\frac{p_A}{p_1}\right]\right\}}{(p_1+p_A)(\cos\phi)\ln[N(0)/N'(0)]}. \quad (10)$$

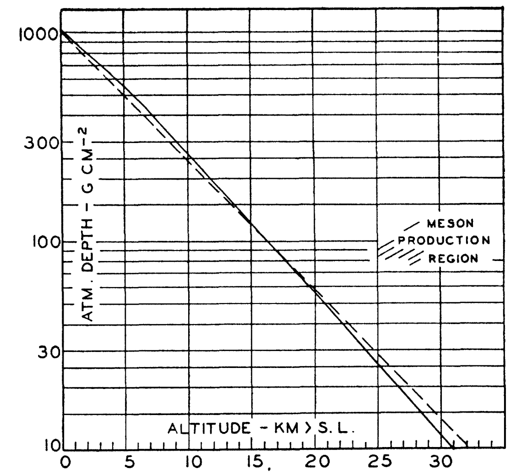

FIG. 7. The standard atmosphere. The solid curve is the atmospheric density variation as given by The National Advisory Committee on Aeronautics. The dashed curve is the exponential approximation used in deriving the probability for meson decay.

Former investigators have used values of z_0 of (7.0, 7.5, or even 8.0)$\times 10^5$ cm, and have justified their choice by assuming mean temperatures for the atmosphere that were consistent with their selection. Assumptions about the atmospheric temperature, however, are not necessary since all that is required is that the approximate function given in Eq. (4) should fit the atmosphere as well as possible. The "standard atmosphere" is shown in Fig. 7 as the solid curve.[18] The dashed curve is Eq. (4) plotted with $z_0=7.0\times 10^5$ cm. It may be seen that this approximation introduces the same path length and change in density as the exact atmospheric function between the region of meson production (approximately 100 g/cm^2) and sea level. This, therefore, gives the same probability for decay.

The mean free path of the primaries, θ_p, was taken as 150 g/cm^2, in keeping with the experimental findings of Conversi and others.[19] (Note that the meson lifetime is not strongly dependent on this quantity.) The values chosen for p_1, p_1', and (p_1+p_A) were 1020, 337, and 3430 Mev/c, respectively. The zenith angle appropriate to these was 39 degrees. For this angle, $N(0)/N'(0)$ was $2.22\pm0.13/0.88\pm0.12$ which was 2.52 ± 0.38. The meson lifetime was then computed to be 2.2 ± 0.3 microseconds. The agreement between the lifetime value found here and 2.15 microseconds is evidence that this experimental technique is effective for the determination of meson intensities in the presence of other particles.

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[17] J. R. Winckler and W. G. Stroud, Phys. Rev. **76**, 1012 (1949).
[18] W. G. Brombacher, National Committee of Aeronautics, Report No. 538 (1935).
[19] M. Conversi, Phys. Rev. **79**, 749 (1950).