On the Spin of the μ-Meson

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The behavior of a μ-meson of spin $\frac{3}{2}$ is investigated for the processes $\mu \rightarrow e+2\nu$, $\pi \rightarrow \mu+\nu$, $\mu^- +$ "p" $\rightarrow$ "n" $+\nu$. These processes have been chosen because they give direct information on the nature of the μ-meson and do not require second quantization of the μ-meson field, which is not feasible for spin values greater than 1 when electromagnetic interactions are present. The theory used to describe the μ-meson is due to Rarita and Schwinger.

The spectra for the $\mu \rightarrow e+2\nu$ decay are obtained for the same modes of decay considered by Tiomno and Wheeler for μ-mesons of spin $\frac{1}{2}$. Their most relevant feature is that none of them goes to zero at maximum electron energy. It is to be hoped that more careful measurements of this spectrum will permit a decision on the spin value of the μ-meson.

The μ-capture is studied by assuming the process to happen via an intermediate scalar or pseudoscalar π-meson. Comparison with the lifetime for $\pi \rightarrow \mu$-decay and use of experimental data allow an evaluation of the constant coupling the π-meson to nucleons. The plausibility of this value is taken as a test for the plausibility of the assumed spin value of the μ-meson. The results do not appear to contradict present-day evidence, due to the uncertainty of the nuclear matrix elements. The formulas obtained apply of course to any spin $\frac{3}{2}$ particle undergoing similar processes.

I. INTRODUCTION

THE question of the spin of the μ-meson does not have as yet a certain answer; experimental evidence requires the assumption of at least a three-particle decay process:[1]

$$\text{I.}\quad \mu \rightarrow \mu_0 + e + \nu,$$

where e and ν denote electron and neutrino respectively and μ_0 denotes the second neutral particle. More information can be obtained from the other known processes:

$$\text{II.}\quad \pi \rightarrow \mu + \mu_0.$$

$$\text{III.}\quad \mu^- + \text{“}p\text{”} \rightarrow \text{“}n\text{”} + \mu_0.$$

(The quotes denote nucleons bound in a nucleus.) With certainty, we can only conclude that μ and the neutral μ_0 are both bosons or fermions. Tiomno[2] has shown that μ and μ_0 can be assumed to be of spin zero, thus obtaining agreement with experiment for all the processes considered above, if the coupling is properly chosen. A spin 1 for μ is excluded by Christy and Kusaka's study of bursts.[3]

On the other hand, it is well known that a satisfactory agreement is reached by assuming both μ and μ_0 to be Dirac particles. Klein[4] and Tiomno and Wheeler[1] have emphasized the analogy between μ-decay and the β-decay of nucleons, so that it is tempting to identify μ_0 with a neutrino, thereby avoiding the introduction of a new particle. Even if μ_0 is a neutrino, it is possible that the spin of the μ-meson has a value larger than $\frac{1}{2}$, namely $\frac{3}{2}$. The object of this investigation is to determine the behavior of a μ-meson of spin $\frac{3}{2}$ in the processes I, II, III. The calculations will be presented in some detail, since, apart from their interest for the μ-meson, it is possible that a heavier meson may be found possessing spin $\frac{3}{2}$.

The first question which must be decided, when considering particles of spin higher than 1, is that of the theory to use for their description. There is indeed quite a variety of (physically equivalent) formalisms, because the requirements of relativistic covariance do not fix uniquely the form of the equations of motion;[5] these equations must be furthermore accompanied by auxiliary conditions, if we wish to restrict the state of the particle to one of definite mass and spin.

The theory here followed is that of Rarita and Schwinger.[6] It has the advantages of appearing to be the natural extension of the Dirac and Proca theories without the introduction of auxiliary potentials, and of eliminating formal complications as far as possible; it shares with other theories the disadvantage, pointed out by Kusaka and Weinberg,[7] that, due to the auxiliary conditions, second quantization is not feasible in the presence of an electromagnetic field without the introduction of finite distance operators in the expressions for the relativistic commutators. (This last feature prevents its use in a calculation of the type of Christy and Kusaka's, at least without a further theoretical investigation; a study of bremsstrahlung according to the pattern set by Dirac[8] and Bhabha[9] can be carried out in principle, but is prohibitive numerically.)

The essentials of the Rarita-Schwinger theory will be recalled here, with a view to establishing the notation used in this work. For further details, the reader is

[1] Tiomno, Wheeler, and Rau, Revs. Modern Phys. **21**, 144 (1949).
[2] J. Tiomno, Phys. Rev. **76**, 856 (1949).
[3] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 414 (1941).
[4] O. Klein, Nature **161**, 897 (1948).
[5] See for example H. J. Bhabha, Revs. Modern Phys. **21**, 451 (1949).
[6] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1940).
[7] S. Kusaka and J. W. Weinberg, thesis (University of California, June, 1943).
[8] P. A. M. Dirac, Proc. Cambridge Phil. Soc. **26**, 361 (1930).
[9] H. J. Bhabha, Proc. Roy. Soc. (London) **152**, 559 (1935).

referred to the original paper. Natural ($\hbar=c=1$) Heaviside units are used throughout the calculations; final results are restated in ordinary cgs units.

The equations of motion of a spin $\frac{3}{2}$ μ-meson can be derived from the lagrangian:

$$L=\bar{\Psi}_\mu(\gamma_\tau\partial_\tau+m)\Psi_\mu-\tfrac{1}{3}\bar{\Psi}_\mu(\gamma_\mu\partial_\nu+\gamma_\nu\partial_\mu)\Psi_\nu+\tfrac{1}{3}\bar{\Psi}_\mu\gamma_\mu(\gamma_\tau\partial_\tau-m)\gamma_\nu\Psi_\nu, \quad (1)$$

where: $\bar{\Psi}_\mu=\Psi_\mu{}^*\gamma_4$, $\partial_\mu=\partial/\partial x_\mu-ie\varphi_\mu$, $m=$mass of μ-meson, $\varphi_\mu=$4-vector electromagnetic potential, $\{\Psi_\mu\}$ is the 4-vector (each component of which is a spinor) which describes the meson.

From the equations of motion:

$$(\gamma_\tau\partial_\tau+m)\Psi_\mu-\tfrac{1}{3}(\gamma_\mu\partial_\nu+\gamma_\nu\partial_\mu)\Psi_\nu+\tfrac{1}{3}\gamma_\mu(\gamma_\tau\partial_\tau-m)\gamma_\nu\Psi_\nu=0, \quad (2)$$

one obtains immediately

$$2\partial_\mu\Psi_\mu=m\gamma_\mu\Psi_\mu, \quad (3)$$

and from (2) and (3):

$$[m^2+(ie/3)\gamma_\mu\gamma_\tau F_{\tau\mu}]\gamma_\nu\Psi_\nu+2ie\gamma_\tau F_{\tau\mu}\Psi_\mu=0, \quad (4)$$

where $F_{\mu\nu}=\partial\varphi_\nu/\partial x_\mu-\partial\varphi_\mu/\partial x_\nu$. If no electromagnetic field is present, each component Ψ_μ obeys the Dirac equation:

$$(\gamma_\tau\partial_\tau+m)\Psi_\mu=0 \quad (5)$$

with the conditions

$$\gamma_\mu\Psi_\mu=0 \quad (6)$$

$$\partial\Psi_\mu/\partial x_\mu=0. \quad (7)$$

A set of four orthonormal positive energy solutions to (5), (6), (7) has been conveniently determined by Kusaka[10] in the form:

$$\begin{aligned}
&\{\Psi_\mu{}^{(1)}\}\equiv\{\mathbf{e}_1\psi_{+1},0\}\\
&\{\Psi_\mu{}^{(2)}\}\equiv\{\mathbf{e}_2\psi_{-1},0\}\\
&\{\Psi_\mu{}^{(3)}\}\equiv\{1/\sqrt{3}\mathbf{e}_2\psi_{+1}-\sqrt{2}/\sqrt{3}E/m\mathbf{e}_3\psi_{-1},\ -i\sqrt{2}/\sqrt{3}k/m\psi_{-1}\}\\
&\{\Psi_\mu{}^{(4)}\}\equiv\{1/\sqrt{3}\mathbf{e}_1\psi_{-1}+\sqrt{2}/\sqrt{3}E/m\mathbf{e}_3\psi_{+1},\ i\sqrt{2}/\sqrt{3}k/m\psi_{+1}\},
\end{aligned} \quad (8)$$

where

$$\mathbf{e}_1\equiv\{1/\sqrt{2},i/\sqrt{2},0\};\quad \mathbf{e}_2\equiv\{1/\sqrt{2},-i/\sqrt{2},0\};\quad \mathbf{e}_3=\mathbf{k}/k\equiv\{0,0,1\},$$

and ψ_{+1}, ψ_{-1} are the positive energy Dirac wave functions for spin parallel and antiparallel to the momentum $\mathbf{k}$ of the μ-meson.

In the general case, in so far as the external field can be considered as a perturbation, one obtains from (4):

$$\gamma_\nu\Psi_\nu\cong-(2ie/m^2)\gamma_\tau F_{\tau\mu}\Psi_\mu, \quad (9)$$

[10] S. Kusaka, Phys. Rev. **60**, 61 (1940).

where the right-hand term is extremely small (in ordinary units, for a coulomb field, $\sim(e^2/\hbar c)(\hbar/mc)^2$). Equation (6) is therefore a very good approximation to Eq. (9): this will considerably simplify the study of the μ-capture.

II. DECAY OF THE μ-MESON

We have studied the same modes of decay of the μ-meson which Tiomno and Wheeler[1] have considered in the case of spin $\frac{1}{2}$, with a view to an eventual comparison with experiment. The possible modes of decay reduce to:

SCE (Simple Charge Exchange)

$$\begin{pmatrix}\mu^-\\ \bar{\nu}''\end{pmatrix}\rightarrow\begin{pmatrix}\nu'\\ e\end{pmatrix}$$

CR (Charge Retention)

$$\begin{pmatrix}\mu^-\\ \bar{\nu}''\end{pmatrix}\rightarrow\begin{pmatrix}e\\ \nu'\end{pmatrix}$$

ACE (Antisymmetric Charge Exchange)

$$\begin{pmatrix}\mu^+\\ \bar{e}\end{pmatrix}\rightarrow\begin{pmatrix}\nu'\\ \nu''\end{pmatrix}.$$

The last one requires, of course, antisymmetrization of the wave functions of the two neutrinos. The Dirac theory is used for electrons and neutrinos. If we consider in general a process $\begin{pmatrix}\mu\\ c\end{pmatrix}\rightarrow\begin{pmatrix}a\\ b\end{pmatrix}$ we readily find that the only possible interaction hamiltonians are:

V (Vector)

$$H_V=ig_V\{\bar{\psi}_b(\mathbf{x}_1)\gamma_\mu\psi_c(\mathbf{x}_1)\}\{\bar{\psi}_a(\mathbf{x}_2)\Psi_\mu(\mathbf{x}_2)\}\delta(\mathbf{x}_1-\mathbf{x}_2)$$

PV (Pseudovector)

$$H_{PV}=g_{PV}\{\bar{\psi}_b(\mathbf{x}_1)\gamma_5\gamma_\mu\psi_c(\mathbf{x}_1)\}\{\bar{\psi}_a(\mathbf{x}_2)\gamma_5\Psi_\mu(\mathbf{x}_2)\}\delta(\mathbf{x}_1-\mathbf{x}_2)$$

T (Tensor)

$$\begin{aligned}H_T&=g_T\{\bar{\psi}_b(\mathbf{x}_1)\gamma_\mu\gamma_\nu\psi_c(\mathbf{x}_1)\}\{\bar{\psi}_a(\mathbf{x}_2)\gamma_\mu\Psi_\nu(\mathbf{x}_2)\}\delta(\mathbf{x}_1-\mathbf{x}_2)\\
&\equiv g_T\{\bar{\psi}_b(\mathbf{x}_1)\gamma_\mu\gamma_\nu\psi_c(\mathbf{x}_1)\}\{\bar{\psi}_a(\mathbf{x}_2)\tfrac{1}{2}[\gamma_\mu\Psi_\nu(\mathbf{x}_2)-\gamma_\nu\Psi_\mu(\mathbf{x}_2)]\delta(\mathbf{x}_1-\mathbf{x}_2)\}.\end{aligned}$$

The scalar and pseudoscalar couplings vanish because of (6). From (6), it also follows that $\Psi_4=0$ for a μ-meson at rest, as considered here. The summation over spins for the Dirac particles is performed in the customary fashion; the average over-spin for the μ-meson should be done by actual use of the positive energy solutions (8) of the wave equation. It turns out, however, that when all other summations have been performed, the relations ($\sigma_1 = i\alpha_2\alpha_3$; $\cdots$):

$$(\boldsymbol{\alpha}\cdot\mathbf{u})(\boldsymbol{\alpha}\cdot\mathbf{v}) = \mathbf{u}\cdot\mathbf{v} - i\boldsymbol{\sigma}\cdot(\mathbf{u}\times\mathbf{v})$$

$$(\boldsymbol{\sigma}\cdot\mathbf{u})(\boldsymbol{\alpha}\cdot\mathbf{v}) = \gamma_5\mathbf{u}\cdot\mathbf{v} - i\boldsymbol{\alpha}\cdot(\mathbf{u}\times\mathbf{v})$$

allow all the terms bracketed between two $\{\Psi_\mu\}$'s to be expressed as a linear combination of 1, β, γ_5, $\boldsymbol{\alpha}\cdot\mathbf{u}$, $i\boldsymbol{\sigma}\cdot\mathbf{v}$. The last three terms are easily seen to give zero over-all contribution, and β, when operating directly on a $\{\Psi_\mu\}$, can be replaced by 1. The final expressions are therefore given in terms of

$$\sum_{\mu\ \mathrm{spin}} \langle \boldsymbol{\Psi}^* \cdot \boldsymbol{\Psi} \rangle = 4$$

$$\sum_{\mu\ \mathrm{spin}} \langle \mathbf{u}\cdot\boldsymbol{\Psi}^* \rangle \langle \mathbf{v}\cdot\boldsymbol{\Psi} \rangle + \text{compl. conj.} = (8/3)\mathbf{u}\cdot\mathbf{v}$$

and are evaluated on simple inspection.

The squares of the matrix elements, averaged over the spin of the μ-meson and summed over all other spins, result finally given by the following expressions (nat. units):

SCE

$$I_V = \frac{g_V^2}{g_{PV}^2} I_{PV} = \frac{1}{2} g_V^2 \left(1 - \frac{1}{3}\frac{\mathbf{k}_e\cdot\mathbf{k}_{\nu''}}{E_e E_{\nu''}}\right)$$

$$I_T = g_T^2 \left\{1 + \frac{1}{3}\left[\frac{\mathbf{k}_e\cdot\mathbf{k}_{\nu''}}{E_e E_{\nu''}} - \frac{\mathbf{k}_e\cdot\mathbf{k}_{\nu'}}{E_e E_{\nu'}} - \frac{\mathbf{k}_{\nu'}\cdot\mathbf{k}_{\nu''}}{E_{\nu'} E_{\nu''}}\right]\right\}$$

CR

$$I_V = \frac{1}{2} g_V^2 \left(1 + \frac{m_e}{E_e}\right)\left(1 - \frac{1}{3}\frac{\mathbf{k}_{\nu'}\cdot\mathbf{k}_{\nu''}}{E_{\nu'} E_{\nu''}}\right)$$

$$I_{PV} = \frac{1}{2} g_{PV}^2 \left(1 - \frac{m_e}{E_e}\right)\left(1 - \frac{1}{3}\frac{\mathbf{k}_{\nu'}\cdot\mathbf{k}_{\nu''}}{E_{\nu'} E_{\nu''}}\right)$$

$$I_T = g_T^2 \left\{1 + \frac{1}{3}\left[\frac{\mathbf{k}_{\nu'}\cdot\mathbf{k}_{\nu''}}{E_{\nu'} E_{\nu''}} - \frac{\mathbf{k}_e\cdot\mathbf{k}_{\nu'}}{E_e E_{\nu'}} - \frac{\mathbf{k}_e\cdot\mathbf{k}_{\nu''}}{E_e E_{\nu''}}\right]\right\}$$

ACE

$$I_V = (g_V^2/g_{PV}^2) I_{PV} = \tfrac{1}{2} I_V \quad \text{for CR}$$

$$I_T = g_T^2 \left\{1 + \frac{m_e}{E_e} - \left(\frac{1}{3} + \frac{m_e}{E_e}\right)\frac{\mathbf{k}_{\nu'}\cdot\mathbf{k}_{\nu''}}{E_{\nu'} E_{\nu''}}\right\}.$$

The μ-decay spectra for electron momentum between p_e and $p_e + dp_e$ (using now ordinary units) are obtained by standard methods (see reference 1, appendix by Rau). They are conveniently expressed in terms of the maximum electron energy

$$E_0 = \tfrac{1}{2} m_\mu c^2 + (m_e/m_\mu)\cdot\tfrac{1}{2} m_e c^2.$$

Introducing the dimensionless quantities

$$\epsilon = E_e/m_e c^2; \quad \epsilon_0 = E_0/m_e c^2; \quad k = p_e/m_e c; \quad r = m_e/m_\mu$$

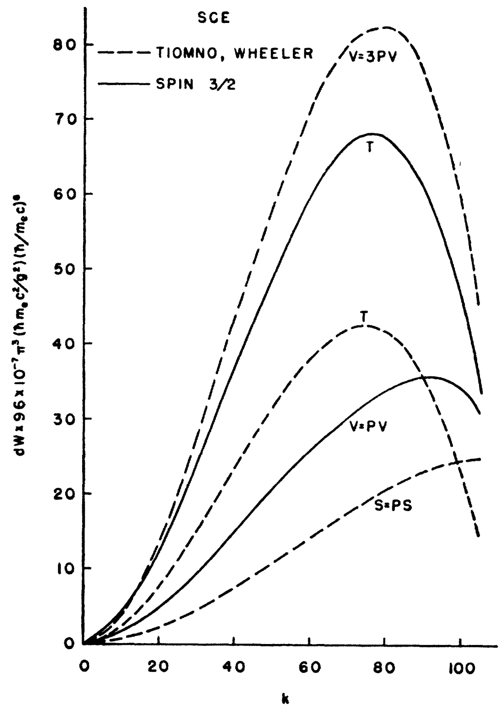


FIG. 1. Decay spectra for simple charge exchange theory. Vector and pseudovector couplings yield the same spectrum. (The ordinate, in this and in the other figures, is actually $1.41\times K$ as given in the text.)

the probabilities for the aforesaid decays become (g in erg cm^3):

$$dW = g^2 \frac{m_\mu (m_e c)^4}{144\pi^3\hbar^7} K dk$$

where the dimensionless factor K is defined as:

SCE

$$K_V = K_{PV} = 9k^2(\epsilon_0 - \epsilon) + k^4(2r + 1/\epsilon)$$

$$K_T = 4[6k^2(\epsilon_0 - \epsilon) + rk^4]$$

CR

$$K_V = 2(1 + 1/\epsilon)[6k^2(\epsilon_0 - \epsilon) + rk^4]$$

$$K_{PV} = 2(1 - 1/\epsilon)[6k^2(\epsilon_0 - \epsilon) + rk^4]$$

$$K_T = 4[3k^2(\epsilon_0 - \epsilon) + k^4(r + 1/\epsilon)]$$

ACE

$$K_V = K_{PV} = \tfrac{1}{2} K_V \quad \text{for CR}$$

$$K_T = 4[9k^2(\tfrac{2}{3} + 1/\epsilon)(\epsilon_0 - \epsilon) + rk^4].$$

A common feature of these spectra is that none of them goes to zero when $\epsilon \to \epsilon_0$; this is perhaps the most significant result obtained in this section. Figures 1–3 show how these spectra compare with those given by Tiomno and Wheeler for the case of spin $\frac{1}{2}$ (some of the curves for spin $\frac{1}{2}$ might have been affected by slight errors in numerical computation). The ordinate scale has been chosen so as to coincide with the one used by Tiomno and Wheeler; this simply amounts to plotting

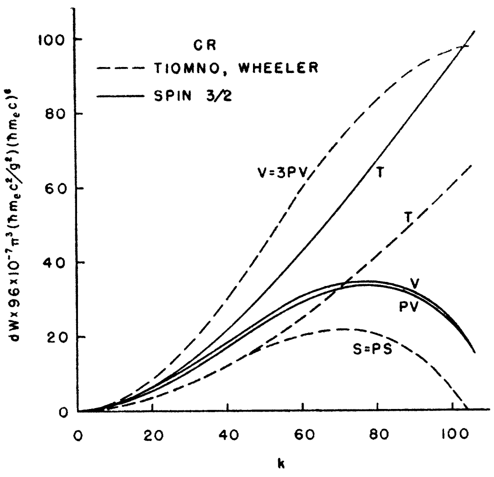
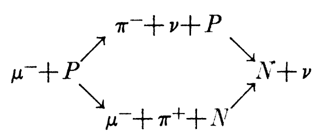


FIG. 2. Decay spectra for charge retention theory.

as ordinate 1.41K instead of K (we use $m_\mu=212m_e$, Tiomno and Wheeler $m_\mu=210m_e$: the difference is not significant)

III. $\pi\rightarrow\mu+\nu$ DECAY AND μ^--CAPTURE

(a) It is known[11] that the μ^--capture data can be accounted for by a scheme



under the assumptions: π-scalar or pseudoscalar, μ of spin $\frac{1}{2}$. This fact is very satisfactory, and leads us to consider the same scheme for a μ^--meson of spin $\frac{3}{2}$. Comparison with the lifetime for the $\pi\rightarrow\mu_{\frac{3}{2}}+\nu$ decay and use of experimental data for the μ^- capture allow indeed an indirect determination of the coupling constant g for the μ-meson with the nucleon: the consistency of the assumption of a spin $\frac{3}{2}$ for the μ-meson will be judged from the plausibility of the values found for g.

Simplifying assumptions required to make the above calculation feasible are: use of free waves for the π-meson, use of the equivalence theorems for the couplings of the π-meson with the nucleon, neglect of the energy imparted to the nucleus with respect to the rest energy of the virtual π-meson (this simplifies the energy denominators), use of Eq. (6) instead of Eq. (9) in forming the interaction lagrangians and neglect of the electromagnetic interaction in the derivative couplings. These assumptions are easily justified (see reference 11) to a good approximation. Further approximations will be specifically stated when needed.

Let φ, ψ_P, ψ_N, χ be, respectively, the π-meson, proton, neutron, neutrino-wave functions. Let g, f/m_π refer to the (pseudo) scalar and (pseudo) vector couplings (specified, when needed, by a superscript ^{S} or ^{PS}). The possible interaction lagrangians between the π-meson and the (P, N), (μ, ν) fields reduce, as a consequence of our assumptions, to

$$L'=-g\varphi^*U-(f/m_\pi)\partial_\mu\varphi V_\mu+\text{compl. conj.},$$

where:

$$U=\psi_N^*O\psi_P;\quad V_\mu=\chi^*O\Psi_\mu$$

and $O=\gamma_4$ for scalar π-meson, $=i\gamma_4\gamma_5$ for pseudoscalar π-meson. The compl. conj. part does not contribute to the capture process. Quantization of the π-meson field in the customary manner:

$$\varphi=\sum(2\omega_k)^{-\frac{1}{2}}(a_k+b_k^*)\exp(i\mathbf{k}\cdot\mathbf{x})$$

$$\pi=\sum i(\omega_k/2)^{\frac{1}{2}}(a_k^*-b_k)\exp(-i\mathbf{k}\cdot\mathbf{x})$$

($\omega_k=(m_\pi^2+k_\pi^2)^{\frac{1}{2}}$, $k=\pm k_{\pi\pm}$; normalization per unit volume) gives the interaction hamiltonian in the form ($V_4=iV_0$)

$$H'=\sum\Big\{g(2\omega_k)^{-\frac{1}{2}}(a_k^*+b_k)\int\exp(-i\mathbf{k}\cdot\mathbf{x})Ud\mathbf{x} + (if/m_\pi)(2\omega_k)^{-\frac{1}{2}}\Big[\omega_k(-a_k+b_k^*)\int\exp(i\mathbf{k}\cdot\mathbf{x})V_0d\mathbf{x} + (a_k+b_k^*)\int\exp(i\mathbf{k}\cdot\mathbf{x})\mathbf{k}\cdot\mathbf{V}d\mathbf{x}\Big]\Big\}. \quad (10)$$

(b) *Decay of π-meson at rest.* Consideration of the equivalent process $\pi+\bar{\nu}\rightarrow\mu$ and use of (10) yield immediately for the lifetime τ_μ the expression:

$$\frac{1}{\tau_\mu}=\frac{1}{12}\frac{f_{\text{cgs}}^2}{\hbar c}\frac{m_\pi^3}{m_\mu^2}\frac{c^2}{\hbar}\left[1-\left(\frac{m_\mu}{m_\pi}\right)^2\right]^4 \quad (11)$$

(in ordinary cgs units). Equation (11) is valid for both scalar and pseudoscalar π-mesons, as a consequence of the assumption $m_\nu=0$.

(c) *μ^- capture by a nucleus.*

An alternative treatment of the μ^- capture by complex nuclei consists in considering first the capture by a free proton and then using some nuclear model for the study of the actual case.

Since results obtained in this way do not at present give any better accuracy than those above reported, we only state here for possible future reference the relevant formulas, which require some computational work. Taking into account the two possible intermediate states, one obtains for the capture of a free μ^--meson by a free proton (with the neglect of electromagnetic interaction).

$$I=\sum_{\text{spins}}\text{Av}|H''|^2=\frac{1}{12}\left(\frac{gf}{m_\pi m_\mu}\right)^2\frac{(E_\mu E_\nu-\mathbf{k}_\mu\cdot\mathbf{k}_\nu)^3}{E_\mu E_\nu[m_\pi^2-m_\mu^2+2(E_\mu E_\nu-\mathbf{k}_\mu\cdot\mathbf{k}_\nu)]^2}\times\left(1-\frac{\mathbf{k}_P\cdot\mathbf{k}_N}{E_PE_N}\pm\frac{M_PM_N}{E_PE_N}\right)$$

(strictly relativistic) where the + and − signs refer respectively to the cases of scalar and pseudoscalar π-mesons. The lifetime

[11] R. Latter, thesis (California Institute of Techology, 1949).

for μ^- capture from a K-orbit is then, in ordinary cgs units:

$$\frac{1}{\tau^S}=\frac{1}{24}\frac{(g_{\rm cgs}{}^S)^2}{\hbar c}\frac{(f_{\rm cgs}{}^S)^2}{\hbar c}\left(\frac{e^2}{\hbar c}\right)^3\frac{m_\mu c^2}{\hbar}\frac{m_\mu{}^6[m_\mu{}^2+4M(m_\mu+M)]}{m_\pi{}^2[m_\pi{}^2(M+m_\mu)+Mm_\mu{}^2]^2}\times\left(1+\frac{M}{M+m_\mu}\right)^4$$

$$\frac{1}{\tau^{PS}}=\left(\frac{g^{PS}f^{PS}}{g^Sf^S}\right)^2_{\rm cgs}\frac{1}{\tau^S}\frac{m_\mu{}^2}{m_\mu{}^2+4M(m_\mu+M)}.$$

If a Z^4 law is assumed, one can very roughly extrapolate, and obtain from the value $1/\tau$ capt.$=47$/sec for a free proton the values $(g_{\rm cgs}{}^S)^2/\hbar c=0.09$; $(g_{\rm cgs}{}^{PS})^2/\hbar c=30.43$. These values are a poorer approximation than those found above. The adoption of a free particle model for the nucleus[12] would increase them by a factor of about 4.

The nucleus can take up any amount of momentum. From (10) once obtains then easily, remembering the stated assumptions, that the expression for the matrix element of the capture process can be reduced to the form:

$$H''=(gf/m_\pi)\int d\mathbf{x}\int d\mathbf{x}'\times\left[\sum_{k_\pi}\frac{\sin[\mathbf{k}_\pi\ (\mathbf{x}'-\mathbf{x})]}{\omega_\pi{}^2}\mathbf{k}_\pi\cdot\mathbf{V}(\mathbf{x}')\right]U(\mathbf{x})$$

$$=-(gf/4\pi m_\pi)\int d\mathbf{x}\int d\mathbf{x}'U(\mathbf{x})V(\mathbf{x}')\nabla_R\times[\exp(-m_\pi R)/R]\quad(\mathbf{R}=\mathbf{x}'-\mathbf{x}).$$

The main contribution to this integral comes from distances of order $\hbar/mc$ from the nucleus; we approximate $\{\Psi_\mu(\mathbf{x}')\}$ over this range by the constant $\{\Psi_\mu(0)\}$, its value at the center of the nucleus. The gradient operator and the factor $\exp(-m_\pi R)/R$ are eliminated by two further integrations. We find:

$$H''=-\frac{igf}{m_\pi(m_\pi{}^2+k_\nu{}^2)}\langle v^*O\mathbf{k}_\nu\cdot\boldsymbol{\Psi}(0)\rangle\sum_{\substack{\text{all}\\ \text{protons}}}\int\exp(-i\mathbf{k}_\nu\cdot\mathbf{x})\langle\psi_N{}^*(\mathbf{x})O\psi_P(\mathbf{x})\rangle d\mathbf{x},\quad(12)$$

where v is the neutrino wave amplitude. The lifetime for μ^- capture is determined by the equation

$$1/\tau_{\rm capt}=2\pi/(2\pi)^3\int Ik_\nu{}^2d\Omega,$$

where I is the average over μ and P spins and the sum over the other spins of $|H''|^2$. The statistical factor is determined by the consideration of the neutrino states only. One obtains, treating the nucleons nonrelativistically:

[12] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 153 (1949).

(1) *Scalar π-meson:* $O=\gamma_4$

$$\frac{1}{\tau^S}=\frac{1}{6\pi}\left(\frac{g^Sf^S}{m_\pi}\right)^2\frac{E_\nu{}^4}{[m_\pi{}^2+E_\nu{}^2]^2}|\boldsymbol{\Psi}(0)|^2\cdot\left|\sum\int\exp(-i\mathbf{k}_\nu\cdot\mathbf{x})\psi_N{}^*(\mathbf{x})\psi_P(\mathbf{x})d\mathbf{x}\right|^2.\quad(13)$$

(2) *Pseudoscalar π-meson*: $O=i\gamma_4\gamma_5$. With our approximation, we get:

$$\langle\psi_N{}^*O\psi_P\rangle\cong-\frac{1}{2M}\nabla\cdot\langle\psi_N{}^*\boldsymbol{\sigma}\psi_P\rangle.$$

($M=$ nucleon mass.) This gives:

$$\sum_{\rm spins}|H''^{PS}|^2=\frac{1}{2}\left[\frac{g^{PS}f^{PS}}{2m_\pi M(m_\pi{}^2+k_\nu{}^2)}\right]^2\times\langle[\mathbf{k}_\nu\cdot\boldsymbol{\Psi}^*(0)][\mathbf{k}_\nu\cdot\boldsymbol{\Psi}(0)]\rangle\times\left|\mathbf{k}_\nu\cdot\sum\int\exp(-i\mathbf{k}_\nu\cdot\mathbf{x})\langle\psi_N{}^*(\mathbf{x})\boldsymbol{\sigma}\psi_P(\mathbf{x})\rangle d\mathbf{x}\right|^2.$$

Taking, then, the z-axis in the direction of $\boldsymbol{\Psi}(0)$, we write $\boldsymbol{\sigma}=\boldsymbol{\sigma}_{||}+\boldsymbol{\sigma}_\perp$ (for each of the terms of the $\sum$ over all protons), where $\boldsymbol{\sigma}_{||}$ is parallel and $\boldsymbol{\sigma}_\perp$ is perpendicular to $\boldsymbol{\Psi}(0)$. Then;

$$\int d\Omega\sum_{N,\nu\ \rm spins}\underset{P\ \rm spin}{\rm Av.}|H''^{PS}|^2=\frac{2\pi}{5}\left[\frac{g^{PS}f^{PS}}{2m_\pi M(m_\pi{}^2+k_\nu{}^2)}\right]^2\cdot k_\nu{}^4|\boldsymbol{\Psi}(0)|^2\left\{\left|\sum\int\exp(-i\mathbf{k}_\nu\cdot\mathbf{x})\psi_N{}^*(\mathbf{x})\sigma_{||}\psi_P(\mathbf{x})d\mathbf{x}\right|^2+\frac{1}{3}\left|\sum\int\exp(-i\mathbf{k}_\nu\cdot\mathbf{x})\psi_N{}^*(\mathbf{x})\sigma_\perp\psi_P(\mathbf{x})d\mathbf{x}\right|^2\right\}.$$

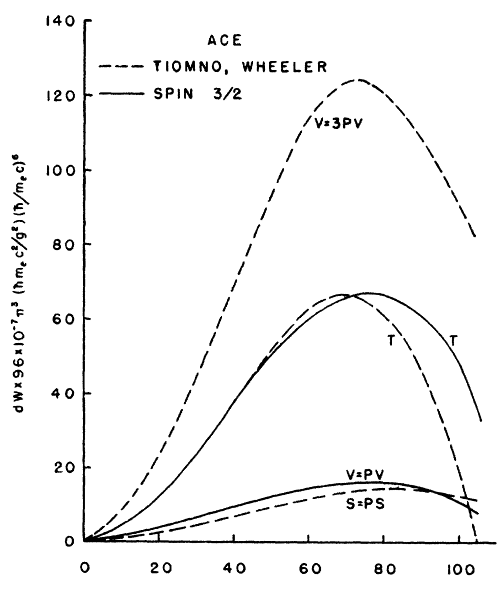


FIG. 3. Decay spectra for antisymmetric charge exchange theory. Vector and pseudovector couplings yield the same spectrum.

We must finally average over the μ-spin. Keeping in mind that for the evaluation of $|\Psi(0)|^2$ we shall use the nonrelativistic approximation, i.e., the ground-state hydrogen-like wave functions, for which the vector character is lost, we recognize that such an average will simply reduce a term

$$\left|\sum\int \exp(-i\mathbf{k}_\nu\cdot\mathbf{x})\psi_N^*(\mathbf{x})\sigma_{||}^{(r)}\psi_P(\mathbf{x})d\mathbf{x}\right|^2,$$

r denoting the projection of $\boldsymbol{\sigma}$ onto the $\Psi^{(r)}(0)$ corresponding to the rth spin state, to a sum of such terms divided by their number. As we ultimately cannot evaluate them, but must resort to an approximation, nothing is practically changed by writing the final result in the form:

$$\frac{1}{\tau^{PS}}=\frac{1}{40\pi}\left(\frac{g^{PS}f^{PS}}{m_\pi}\right)^2\frac{E_\nu^4}{[m_\pi^2+E_\nu^2]^2}\frac{E_\nu^2}{M^2}|\Psi(0)|^2$$
$$\cdot\left\{\left|\sum\int \exp(-i\mathbf{k}_\nu\cdot\mathbf{x})\psi_N^*(\mathbf{x})\sigma_{||}\psi_P(\mathbf{x})d\mathbf{x}\right|^2\right.$$
$$\left.+\frac{1}{3}\left|\sum\int \exp(-i\mathbf{k}_\nu\cdot\mathbf{x})\psi_N^*(\mathbf{x})\sigma_{\perp}\psi_P(\mathbf{x})d\mathbf{x}\right|^2\right\}. \quad (14)$$

(d) We take now in (13) and (14) $|\Psi(0)|^2=(Zm_\mu e^2)^3/\pi$ and $Z/3$ for the sums squared of the nuclear matrix elements (see reference 11), and express the final results in ordinary cgs units. They are:

$$\frac{1}{\tau^S}=\frac{8}{9}\frac{(g_{\text{cgs}}^S)^2}{\hbar c}\frac{(f_{\text{cgs}}^S)^2}{\hbar c}\frac{m_\mu^3e^6}{m_\pi^2\hbar^4c}\frac{E_\nu^4}{[m_\pi^2c^4+E_\nu^2]^2}Z^4 \quad (15)$$

$$\frac{1}{\tau^{PS}}=\frac{8}{45}\frac{(g_{\text{cgs}}^{PS})^2}{\hbar c}\frac{(f_{\text{cgs}}^{PS})^2}{\hbar c}\frac{m_\mu^3e^6}{m_\pi^2\hbar^4c}\frac{E_\nu^4}{[m_\pi^2c^4+E_\nu^2]^2}\times\left(\frac{E_\nu}{Mc^2}\right)^2Z^4. \quad (16)$$

We are interested in the comparison of (15) and (16) with (11). Denoting by r_μ the ratio m_μ/m_π we obtain:

$$\left(\frac{\tau_\mu}{\tau_{\text{capt}}}\right)^S=\frac{32}{3}\frac{(g_{\text{cgs}}^S)^2}{\hbar c}\left(\frac{e^2}{\hbar c}\right)^3\frac{r_\mu^5}{(1-r_\mu^2)^4}\frac{E_\nu^4}{[m_\pi^2c^4+E_\nu^2]^2}Z^4$$

$$\left(\frac{\tau_\mu}{\tau_{\text{capt}}}\right)^{PS}=\frac{1}{5}\left(\frac{g_{\text{cgs}}^{PS}}{g_{\text{cgs}}^S}\right)^2\left(\frac{E_\nu}{Mc^2}\right)^2\left(\frac{\tau_\mu}{\tau_{\text{capt}}}\right)^S.$$

For a numerical determination of $g^2/\hbar c$, we use the values: $m_\pi=280m_e$; $m_\mu=212m_e$; $Z=10$; $\tau_\mu=1.97\times10^{-8}$ sec; $\tau_{\text{capt}}=2.15\times10^{-6}$ sec. We have then:

$$E_\nu=E_\mu-[E_{``n"}-E_{``p"}]=98 \text{ Mev},$$

taking the excitation energy of the nucleus to be 10 Mev (a reasonable estimate from the absence of stars accompanying the μ^--capture). We obtain finally:

$$(g_{\text{cgs}}^S)^2/\hbar c=0.29;\quad (g_{\text{cgs}}^{PS})^2/\hbar c=136.$$

These values are uncertain chiefly for two reasons: (1) the value assigned to the squares of the nuclear matrix elements, (2) the value of 10 Mev taken for the nuclear excitation energy. Only τ^{PS} is however, sensitive to the second uncertainty: an increased value for the excitation energy would sensibly increase $(g_{\text{cgs}}^{PS})^2/\hbar c$.

IV. CONCLUSION

We have investigated the processes

I $\mu\rightarrow e+2\nu$

II $\pi\rightarrow\mu+\nu$

III $\mu^-+``p"\rightarrow``n"+\nu$

assuming a spin $\frac{3}{2}$ for the μ-meson. Process III has been studied under assumption of an intermediate scalar or pseudoscalar π-meson. The results obtained do not disagree with the experimental information available at present. It may be possible to exclude spin $\frac{3}{2}$ for the μ-meson when the nuclear matrix elements and the nuclear excitation energy are known with greater accuracy. The spectra obtained from the study of process I may be even more decisive when the experimental electron spectrum is known very accurately.

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