Explicit γ-γ Angular Correlations

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The theoretical γ-γ directional correlation function is given as a Legendre series: $W(\theta)=1+\Sigma A_L P_L(\cos\theta)$, for those cases where the γ-rays are emitted with the lowest angular momentum allowed by selection rules. The coefficients A_L are given algebraically and are also tabulated as four-place decimals for all cascades up to 2^4-pole—2^6-pole in which the largest nuclear spin does not exceed nine. Rules are given for obtaining from the tables: (a) the lowest order multipole mixture interference terms, (b) certain α-γ and α-α directional correlations.

1. GUIDE TO THE TABLES

THE general γ-γ directional correlation function[1] has been obtained by the author,[2] and independently by Racah,[3] in the form of a Legendre series in the cosine of the angle θ between the successive γ-rays:

$$W(\theta)=1+\sum A_L P_L(\cos\theta), \tag{1}$$

normalized to unit average. The sum is over even values of L: $L=2, 4, 6, \cdots, L_m$, where L_m will be given presently. In general, the coefficients A_L are functions of the initial, intermediate, and final nuclear angular momenta j_1, j, and j_2, respectively, and of the multipole orders and relative amplitudes of the various multipole fields describing the emitted γ-rays. We simplify the formula by assuming that the γ-rays are emitted with the lowest angular momentum allowed by angular momentum selection rules, so that if l_1 and l_2 are the multipole orders of the γ-rays emitted in the first and second transitions (respectively) of the cascade $j_1\rightarrow j\rightarrow j_2$, then $l_1=1$ if $j_1=j$, $l_1=|j_1-j|$ if $j_1\neq j$, and $l_2=1$ if $j_2=j_2$, $l_2=|j_2-j|$ if $j_2\neq j$. We denote this cascade by $j_1(l_1)j(l_2)j_2$, or by $j_1(\gamma, l_1)j(\gamma, l_2)j_2$ if necessary. The maximum value L_m of L in the sum, Eq. (1), is then the largest even integer satisfying each of $L_m\leq 2l_1$, $L_m\leq 2l_2$ and $L_m\leq 2j$.[4] There is no correlation if $j=\frac{1}{2}$. In Sec. II we give rules for obtaining the lowest order multipole mixture interference terms from the tabulated A_L.

We reproduce the formula used to obtain the tables,[5] in case points beyond the range of the tables are needed, or in case more accurate values of the coefficients are desired. The coefficients A_L are the same for both of the 2^{l_1}-pole–2^{l_2}-pole cascades $j-l_1(l_1)j(l_2)j+l_2$ and $j+l_1(l_1)j(l_2)j-l_2$; furthermore, the coefficients A_L for these particular cascades are independent of the value of j. We will refer to this correlation as the "basic" 2^{l_1}-pole–2^{l_2}-pole γ-γ correlation, and will denote the coefficients by $A_L^{(b)}$. Explicitly:

$$W(\theta)\doteq 1+\sum A_L^{(b)}P_L(\cos\theta) \quad \text{for} \quad j\pm l_1(l_1)j(l_2)j\mp l_2,$$

where

$$A_L^{(b)}=(2L+1)b_L(l_1)b_L(l_2),$$

with

$$b_L(l)=\left[1-\frac{L(L+1)}{2l(l+1)}\right]\frac{(2l+1)!L!(l+(L/2))!}{(2l+L+1)!((L/2)!)^2(l-(L/2))!}.$$

The other 2^{l_1}-pole–2^{l_2}-pole correlations are given by

$$W(\theta)=1+\sum w_L A_L^{(b)}P_L(\cos\theta),$$

where

$$w_L=\frac{(2j-L)!(2j+L+1)!}{(2j)!(2j+1)!} \quad \text{for} \quad j-l_1(l_1)j(l_2)j-l_2,$$

$$w_L=\frac{(2j)!(2j+1)!}{(2j-L)!(2j+L+1)!} \quad \text{for} \quad j+l_1(l_1)j(l_2)j+l_2,$$

$$w_2=-(2j+3)/j \quad \text{for} \quad j(1)j(l_2)j-l_2,$$

$$w_2=-(2j-1)/(j+1) \quad \text{for} \quad j(1)j(l_2)j+l_2,$$

$$w_2=(2j-1)(2j+3)/(j(j+1)) \quad \text{for} \quad j(1)j(1)j.$$

These "pure multipole" correlations do not depend on the parities of the γ-rays, so that "2^l-pole" means either an electric 2^l-pole or a magnetic 2^l-pole γ-ray.[6]

One important symmetry property of the pure multipole correlation function is that the correlation for $j_2(l_2)j(l_1)j_1$ is the same as the one for $j_1(l_1)j(l_2)j_2$; this was shown by Hamilton in his original paper on γ-γ correlations,[7] and cuts the size of the tables roughly in half. In Tables I–IX the numerically smallest multipole in the cascade is supposed to occur in the first transition, so that, e.g., the $9/2\rightarrow 3/2\rightarrow 3/2$ octupole-dipole correlation is to be found as the $3/2\rightarrow 3/2\rightarrow 9/2$ dipole-octupole entry in the tables. The second symmetry is the equality of the basic correlation for the two cascades $j\pm l_1(l_1)j(l_2)j\mp l_2$, and the nondependence of this corre-

[1] D. L. Falkoff and G. E. Uhlenbeck, Phys. Rev. **79**, 323 (1950).
[2] S. P. Lloyd, Phys. Rev. **81**, 307 (1951).
[3] G. Racah, Phys. Rev. **82**, 309 (1951).
[4] These become Yang's rules when the Legendre polynomials are written out as polynomials in $\cos\theta$. C. N. Yang, Phys. Rev. **74**, 764 (1948).
[5] The derivation of a general two-step cascade angular correlation formula is part of the material to be submitted by the author as his inaugural dissertation (University of Illinois, 1951), under the sponsorship of Professor S. M. Dancoff. The results stated without proof in this article are special cases from the general formula. Complete publication will appear subsequently; it was felt that this interim presentation of some of the results would be of use to the experimentalist.
[6] D. S. Ling, Jr., and D. L. Falkoff, Phys. Rev. **76**, 1639 (1949).
[7] D. R. Hamilton, Phys. Rev. **58**, 122 (1940).

lation on the value of j. The coefficients $A_L^{(b)}$ of the basic correlation are to be found at the top of the columns in the tables. Thus in Tables II–III in the columns marked "2^1-2^3," one finds in the row marked "Basic" the entry: $A_2=-0.1250$, so that the correlation for all of the transitions: 0(1)1(3)4, 1/2(1)3/2(3)9/2, 1(1)2(3)5, $\cdots$, 4(1)3(3)0, 9/2(1)7/2(3)1/2, 5(1)4(3)1, $\cdots$, and also, from the first symmetry property, 4(3)1(1)0, 9/2(3)3/2(1)1/2, $\cdots$, 0(3)3(1)4, 1/2(3) 7/2(1)9/2, $\cdots$, is $W(\theta)=1-0.1250P_2(\cos\theta)$.

When the sequence of nuclear angular momenta is not definitely monotonic, the coefficients A are functions of the nuclear angular momenta, and are tabulated as functions of the intermediate nuclear angular momentum j. The initial and final nuclear angular momenta are to be obtained from the column markings; e.g., in Table III, the column "2^1-2^4," in the bottom half of the column under "$j-1\rightarrow j\rightarrow j-4$," one finds $A_2=-0.2833$ at $j=9/2$, so that the 7/2(1)9/2(4)1/2 correlation is $W(\theta)=1-0.2833P_2(\cos\theta)$. In the top half of the same column, under "$j+1\rightarrow j\rightarrow j+4$," one finds $A_2=-0.0843$ at $j=9/2$, so that the 11/2(1)9/2(4)17/2 correlation is $W(\theta)=1-0.0843P_2(\cos\theta)$.

2. MULTIPOLE MIXTURE INTERFERENCE TERMS[6]

Recent theoretical estimates of Weisskopf of the ratios of matrix elements[8] indicate that in parity forbidden γ-ray transitions (where magnetic 2^l-pole and electric 2^{l+1}-pole multipoles are the lowest and next lowest multipoles, respectively, allowed by both angular momentum and parity selection rules), the intensity of the electric 2^{l+1}-pole is a small fraction of the intensity of the magnetic 2^l-pole. Most of the γ-rays are emitted as the lowest allowed magnetic 2^l-pole, so that correlations from the tables are applicable as they stand. Reinterpretation of experimental data as well as new data support this conclusion. Nevertheless, we give the rules for obtaining the lowest order "multipole mixture" interference terms in the correlation from the tabulated "pure multipole" correlation.

Let the real[9] quantity δ be such that its square is the ratio of the electric 2^{l+1}-pole intensity to the intensity of the (lowest allowed) magnetic 2^l-pole in a given parity forbidden transition; the sign of δ is the relative sign of the reduced (magnetic quantum number independent) nuclear matrix elements for the emission of the two multipoles, i.e., δ is the ratio of these matrix elements. If both transitions in the cascade $j_1(l_1)j(l_2)j_2$ are parity forbidden, and δ_1 and δ_2 are the matrix element ratios for the first and second transitions, respectively, then, neglecting terms in δ_1^2, δ_2^2 and $\delta_1\delta_2$, the directional correlation of the two γ-rays is

$$W(\theta)=1+\sum A_LP_L(\cos\theta)+x_1\delta_1\sum y_L(l_1)A_LP_L(\cos\theta)-x_2\delta_2\sum y_L(l_2)A_LP_L(\cos\theta). \quad (2)$$

The coefficients A_L are those tabulated; the coefficients $y_L(l)$ are independent of nuclear angular momenta and are explicitly

$$y_L(l)=2\left(\frac{l(2l+3)}{l+2}\right)^{\frac{1}{2}}\cdot\frac{L(L+1)}{2l(l+1)-L(L+1)}. \quad (3)$$

The coefficients x_1 and x_2 depend only on the quantities describing the first and second transitions, respectively; for either transition:

$$x=((j-l)/(j+1))^{\frac{1}{2}} \quad \text{if} \quad j'=j-l,$$
$$x=-((j+l+1)/j)^{\frac{1}{2}} \quad \text{if} \quad j'=j+l,$$
$$x=-((2j-1)(2j+3)/3)^{-\frac{1}{2}} \quad \text{if} \quad j'=j \text{ (dipole only)}, \quad (4)$$

where $(j', l)=(j_1, l_1)$ or (j_2, l_2).

These rules are quite general and give the multipole mixture terms in any correlation involving a γ-ray. Suppose that

$$W(\theta)=1+\sum B_LP_L(\cos\theta)$$

is the directional correlation between a γ-ray emitted in the first transition of the cascade $j_1(l_1)j(X)j_2$ and some other nuclear radiation X (such as an α-particle, β-ray, etc.) emitted in the second transition. The (first) γ-ray is supposed to be emitted as the lowest allowed 2^{l_1}-pole multipole. If the first transition is parity forbidden and the matrix element ratio δ_1 is not negligible (assume that δ_1^2 *is* negligible), then the correlation is modified by multipole mixture to

$$W(\theta)=1+\sum B_LP_L(\cos\theta)+x_1\delta_1\sum y_L(l_1)B_LP_L(\cos\theta),$$

where the quantities x_1, $y_L(l_1)$ are those defined in Eqs. (3)–(4). If the γ-ray transition is second, the X–γ correlation for $j_1(X)j(l_2)j_2$,

$$W(\theta)=1+\sum B_L'P_L(\cos\theta),$$

becomes

$$W(\theta)=1+\sum B_L'P_L(\cos\theta)-x_2\delta_2\sum y_L(l_2)B_L'P_L(\cos\theta)$$

when multipole mixture effects in the second transition are taken into account to order δ_2.

3. CORRELATIONS INVOLVING α-PARTICLES

We describe here a method whereby certain correlations involving α-particles can be obtained from the γ-γ tables given. Suppose that an α-particle is emitted in the first transition of $j_1(\alpha)j(X)j_2$, and that the nuclear parity changes in the first transition is $(-1)^{j_1-j}$ if $j_1\neq j$, and (-1) if $j_1=j$. Suppose also that the corresponding γ-X correlation for $j_1(\gamma, l_1)j(X)j_2$ is available and has the value

$$W(\theta)=1+\sum C_LP_L(\cos\theta),$$

[8] Unpublished; the formulas are given by R. D. Hill, Phys. Rev. **81**, 470 (1951). See also A. W. Sunyar and M. Goldhaber, Phys. Rev. **83**, 216(A) (1951).

[9] S. P. Lloyd, Phys. Rev. **81**, 161 (1951). With the author's choice of phase for the reduced matrix elements, the matrix δ is antihermitian. This leads to the difference in sign of the interference terms in Eq. (2), following.

where l_1 is the lowest allowed γ-ray multipole order in the first transition. If all the α-particles are emitted with angular momentum l_1, the α-X correlation is

$$W(\theta)=1+\sum z_L(l_1)C_LP_L(\cos\theta),$$

where

$$z_L(l)=\frac{2l(l+1)}{2l(l+1)-L(L+1)}.$$

For α-particles emitted in the second transition the formula is essentially the same. If the X-γ lowest γ-multipole correlation for $j_1(X)j(\gamma, l_2)j_2$ is

$$W(\theta)=1+\sum C_L'P_L(\cos\theta),$$

the corresponding X-α correlation for $j_1(X)j(\alpha, l_2)j_2$ is

$$W(\theta)=1+\sum z_L(l_2)C_L'P_L(\cos\theta).$$

Double application of the rule gives the $j_1(\alpha, l_1)j(\alpha, l_2)j_2$ correlation

$$W(\theta)=1+\sum z_L(l_1)z_L(l_2)A_LP_L(\cos\theta)$$

from the coefficients A_L of the $j_1(\gamma, l_1)j(\gamma, l_2)j_2$ correlation.

The results of this section apply to any scalar particle emitted in a nuclear transition; α-particles, for example, and also conversion electrons, if the electron is from an atomic s-shell (e.g., K or L_I shell) and the converted γ-ray is an electric multipole. In this case the angular momentum of the electron is to be taken as the multipole order of the γ-ray.[10,*]

4. POWER SERIES AND LEGENDRE SERIES

That the Legendre series is the theoretically natural form for the directional correlation function is clearly indicated in Secs. II–III, preceding. The rule for obtaining the multipole mixture terms from the pure multipole γ-γ correlation, for instance, would become quite unwieldy if both were to be presented in the familiar form as a power series in $\cos^2\theta$. One might also argue that the orthogonality properties of the Legendre polynomials on the unit sphere makes them an intuitively more natural set with which to describe the non-isotropy of the relative angular distribution of the emitted particles. Angular correlations have been previously computed and reported as power series in $\cos^2\theta$, however, so we reproduce the following reversion formulas, obtained from formulas in MacRobert's *Spherical Harmonics*.[11]

$$Q+R\cos^2\theta+S\cos^4\theta+T\cos^6\theta+U\cos^8\theta$$
$$=A_0+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)+A_6P_6(\cos\theta)+A_8P_8(\cos\theta),$$

then

$$A_0=Q+\tfrac{1}{3}R+\tfrac{1}{5}S+(1/7)T+(1/9)U,$$
$$A_2=\tfrac{2}{3}R+(4/7)S+(10/21)T+(40/99)U,$$
$$A_4=(8/35)S+(24/77)T+(48/143)U,$$
$$A_6(16/231)T+(64/495)U,$$
$$A_8+(128/5435)U,$$

and

$$Q=A_0-\tfrac{1}{2}A_2+\tfrac{3}{8}A_4-\tfrac{5}{16}A_6+(35/128)A_8,$$
$$R=\tfrac{3}{2}A_2-(30/8)A_4+(105/16)A_6-(1260/128)A_8,$$
$$S=(35/8)A_4-(315/16)A_6+(6930/128)A_8,$$
$$T=(231/16)A_6-(12012/128)A_8,$$
$$U=(6435/128)A_8.$$

5. TABLES[12]

TABLE I. Dipole-dipole correlations: $W(\theta)=1+A_2P_2(\cos\theta)$. Basic: $A_2=0.05$.

			A_2		
j	$j\to j\to j$	$j\to j\to j-1$	$j\to j\to j+1$	$j-1\to j\to j-1$	$j+1\to j\to j+1$
1	0.1250	−0.2500	−0.0250	0.5000	0.0050
3/2	0.1600	−0.2000	−0.0400	0.2500	0.0100
2	0.1750	−0.1750	−0.0500	0.1750	0.0143
5/2	0.1829	−0.1600	−0.0571	0.1400	0.0179
3	0.1875	−0.1500	−0.0625	0.1200	0.0208
7/2	0.1905	−0.1429	−0.0667	0.1071	0.0233
4	0.1925	−0.1375	−0.0700	0.0982	0.0255
9/2	0.1939	−0.1333	−0.0727	0.0917	0.0273
5	0.1950	−0.1300	−0.0750	0.0867	0.0288
11/2	0.1958	−0.1273	−0.0769	0.0827	0.0302
6	0.1964	−0.1250	−0.0786	0.0795	0.0314
13/2	0.1969	−0.1231	−0.0800	0.0769	0.0325
7	0.1973	−0.1214	−0.0813	0.0747	0.0335
15/2	0.1976	−0.1200	−0.0824	0.0729	0.0343
8	0.1979	−0.1188	−0.0833	0.0713	0.0351
17/2	0.1981	−0.1176		0.0699	
9	0.1983	−0.1167		0.0686	

TABLE II. Certain dipole–2^l-pole correlations: $W(\theta)=1+A_2P_2(\cos\theta)$.

			$j\to j\to j\pm l$		
	2^1-2^2	2^1-2^3	2^1-2^4	2^1-2^5	2^1-2^6
			A_2		
Basic	−0.0714	−0.1250	−0.1545	−0.1731	−0.1857
j	$j\to j\to j+2$	$j\to j\to j+3$	$j\to j\to j+4$	$j\to j\to j+5$	$j\to j\to j+6$
1	0.0357	0.0625	0.0773	0.0865	0.0929
3/2	0.0571	0.1000	0.1236	0.1385	0.1486
2	0.0714	0.1250	0.1545	0.1731	0.1857
5/2	0.0816	0.1429	0.1766	0.1978	0.2122
3	0.0893	0.1563	0.1932	0.2163	0.2321
7/2	0.0952	0.1667	0.2061	0.2308	
4	0.1000	0.1750	0.2164	0.2423	
9/2	0.1039	0.1818	0.2248		
5	0.1071	0.1875	0.2318		
11/2	0.1099	0.1923			
6	0.1122	0.1964			
13/2	0.1143				
7	0.1161				

[10] J. W. Gardner, Proc. Phys. Soc. (London) **62A**, 763 (1949).
* *Note added in proof:*—This holds only when the electrons are treated nonrelativistically.
[11] T. MacRobert, *Spherical Harmonics* (Methuen and Company, Ltd., London, 1928), first edition.

[12] These tables are a corrected version of a set circulated privately.

TABLE II.—*Continued*

	$j\to j \to j-2$	$j\to j \to j-3$	$j\to j \to j-4$	$j\to j \to j-5$	$j\to j \to j-6$
2	0.2500				
5/2	0.2286				
3	0.2143	0.3750			
7/2	0.2041	0.3571			
4	0.1964	0.3438	0.4250		
9/2	0.1905	0.3333	0.4121		
5	0.1857	0.3250	0.4018	0.4500	
11/2	0.1818	0.3182	0.3934	0.4406	
6	0.1786	0.3125	0.3864	0.4327	0.4643
13/2	0.1758	0.3077	0.3804	0.4260	0.4571
7	0.1735	0.3036	0.3753	0.4203	0.4510
15/2	0.1714	0.3000	0.3709	0.4154	0.4457
8	0.1696	0.2969	0.3670	0.4111	0.4411
17/2	0.1681	0.2941	0.3636	0.4072	0.4370
9	0.1667	0.2917	0.3606	0.4038	0.4333

TABLE III. Certain dipole–2^l-pole correlations: $W(\theta)=1+A_2P_2(\cos\theta)$.

	$j\pm1\to j\to j\pm l$				
	2^1-2^2	2^1-2^3	2^1-2^4	2^1-2^5	2^1-2^6
			A_2		
Basic	−0.0714	−0.1250	−0.1545	−0.1731	−0.1857
j	$j+1\to j \to j+2$	$j+1\to j \to j+3$	$j+1\to j \to j+4$	$j+1\to j \to j+5$	$j+1\to j \to j+6$
1	−0.0071	−0.0125	−0.0155	−0.0173	−0.0186
3/2	−0.0143	−0.0250	−0.0309	−0.0346	−0.0371
2	−0.0204	−0.0357	−0.0442	−0.0495	−0.0531
5/2	−0.0255	−0.0446	−0.0552	−0.0618	−0.0663
3	−0.0298	−0.0521	−0.0644	−0.0721	−0.0774
7/2	−0.0333	−0.0583	−0.0721	−0.0808	
4	−0.0364	−0.0636	−0.0787	−0.0881	
9/2	−0.0390	−0.0682	−0.0843		
5	−0.0412	−0.0721	−0.0892		
11/2	−0.0432	−0.0755			
6	−0.0449	−0.0786			
13/2	−0.0464				
7	−0.0478				
	$j-1\to j \to j-2$	$j-1\to j \to j-3$	$j-1\to j \to j-4$	$j-1\to j \to j-5$	$j-1\to j \to j-6$
2	−0.2500				
5/2	−0.2000				
3	−0.1714	−0.3000			
7/2	−0.1531	−0.2679			
4	−0.1403	−0.2455	−0.3036		
9/2	−0.1310	−0.2292	−0.2833		
5	−0.1238	−0.2167	−0.2679	−0.3000	
11/2	−0.1182	−0.2068	−0.2557	−0.2864	
6	−0.1136	−0.1989	−0.2459	−0.2753	−0.2955
13/2	−0.1099	−0.1923	−0.2378	−0.2663	−0.2857
7	−0.1068	−0.1868	−0.2310	−0.2587	−0.2776
15/2	−0.1041	−0.1821	−0.2252	−0.2522	−0.2706
8	−0.1018	−0.1781	−0.2202	−0.2466	−0.2646
17/2	−0.0998	−0.1746	−0.2159	−0.2418	−0.2595
9	−0.0980	−0.1716	−0.2121	−0.2376	−0.2549

TABLE IV. Quadrupole–2^2-, 2^3-, and 2^4-pole correlations: $W(\theta)=1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)$.

	2^2-2^2		2^2-2^3		2^2-2^4	
	A_2	A_4	A_2	A_4	A_2	A_4
Basic	0.1020	0.0091	0.1786	−0.0043	0.2208	−0.0018
j	$j+2\to j\to j+2$		$j+2\to j\to j+3$		$j+2\to j\to j+4$	
1	0.0102	0	0.0179	0	0.0221	0
3/2	0.0204	0	0.0357	0	0.0442	0
2	0.0292	0.0001	0.0510	−0.0000	0.0631	−0.0000
5/2	0.0364	0.0002	0.0638	−0.0001	0.0788	−0.0000
3	0.0425	0.0004	0.0744	−0.0002	0.0920	−0.0001
7/2	0.0476	0.0006	0.0833	−0.0003	0.1030	−0.0001
4	0.0519	0.0009	0.0909	−0.0004	0.1124	−0.0002
9/2	0.0557	0.0011	0.0974	−0.0005	0.1204	−0.0002
5	0.0589	0.0014	0.1030	−0.0007	0.1274	−0.0003
11/2	0.0617	0.0016	0.1079	−0.0008		
6	0.0641	0.0019	0.1122	−0.0009		
13/2	0.0663	0.0021				
7	0.0683	0.0023				
	$j-2\to j\to j-2$		$j+2\to j\to j-3$		$j-2\to j\to j-4$	
2	0.3571	1.1429				
5/2	0.2857	0.3810				
3	0.2449	0.1995	0.4286	−0.0952		
7/2	0.2187	0.1283	0.3827	−0.0612		
4	0.2004	0.0926	0.3508	−0.0442	0.4337	−0.0184
9/2	0.1871	0.0721	0.3274	−0.0344	0.4048	−0.0143
5	0.1769	0.0590	0.3095	−0.0281	0.3827	−0.0117
11/2	0.1688	0.0500	0.2955	−0.0239	0.3653	−0.0099
6	0.1623	0.0436	0.2841	−0.0208	0.3512	−0.0086
13/2	0.1570	0.0388	0.2747	−0.0185	0.3397	−0.0077
7	0.1525	0.0351	0.2669	−0.0168	0.3300	−0.0070
15/2	0.1487	0.0322	0.2602	−0.0154	0.3217	−0.0064
8	0.1454	0.0298	0.2545	−0.0142	0.3146	−0.0059
17/2	0.1426	0.0279	0.2495	−0.0133	0.3084	−0.0055
9	0.1401	0.0262	0.2451	−0.0125	0.3030	−0.0052

TABLE V. Quadrupole–2^5- and 2^6-pole correlations: $W(\theta)=1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)$.

	2^2-2^5		2^2-2^6	
	A_2	A_4	A_2	A_4
Basic	0.2473	−0.0293	0.2653	−0.0384
j	$j+2\to j\to j+5$		$j+2\to j\to j+6$	
1	0.0247	0	0.0265	0
3/2	0.0495	0	0.0531	0
2	0.0706	−0.0002	0.0758	−0.0003
5/2	0.0883	−0.0007	0.0948	−0.0009
3	0.1030	−0.0013	0.1105	−0.0017
7/2	0.1154	−0.0021		
4	0.1259	−0.0029		
	$j-2\to j\to j-5$		$j-2\to j\to j-6$	
5	0.4286	−0.1905		
11/2	0.4091	−0.1616		
6	0.3934	−0.1409	0.4221	−0.1847
13/2	0.3804	−0.1254	0.4082	−0.1644
7	0.3695	−0.1135	0.3965	−0.1487
15/2	0.3603	−0.1040	0.3866	−0.1364
8	0.3523	−0.0964	0.3781	−0.1263
17/2	0.3454	−0.0901	0.3706	−0.1181
9	0.3394	−0.0848	0.3641	−0.1112

TABLE VI. Octuple–2^3- and 2^4-pole correlations: $W(\theta)=1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)+A_6P_6(\cos\theta)$.

	2^3-2^3			2^3-2^4		
	A_2	A_4	A_6	A_2	A_4	A_6
Basic	0.3125	0.0021	0.0010	0.3864	0.0086	0.0002
j	$j+3\to j\to j+3$			$j+3\to j\to j+4$		
1	0.0313	0	0	0.0386	0	0
3/2	0.0625	0	0	0.0773	0	0
2	0.0893	0.0000	0	0.1104	0.0001	0
5/2	0.1116	0.0000	0	0.1380	0.0002	0
3	0.1302	0.0001	0.0000	0.1610	0.0004	0.0000
7/2	0.1458	0.0001	0.0000	0.1803	0.0006	0.0000
4	0.1591	0.0002	0.0000	0.1967	0.0008	0.0000
9/2	0.1705	0.0003	0.0000	0.2107	0.0011	0.0000
5	0.1803	0.0003	0.0000	0.2229	0.0013	0.0000
11/2	0.1889	0.0004	0.0000			
6	0.1964	0.0004	0.0000			
	$j-3\to j\to j-3$			$j-3\to j\to j-4$		
3	0.7500	0.0455	1.7045			
7/2	0.6696	0.0292	0.4261			
4	0.6138	0.0211	0.1776	0.7589	0.0877	0.0284
9/2	0.5729	0.0164	0.0947	0.7083	0.0682	0.0152
5	0.5417	0.0134	0.0585	0.6697	0.0558	0.0094
11/2	0.5170	0.0114	0.0399	0.6393	0.0473	0.0064
6	0.4972	0.0099	0.0292	0.6147	0.0413	0.0047
13/2	0.4808	0.0088	0.0224	0.5944	0.0367	0.0036
7	0.4670	0.0080	0.0179	0.5774	0.0332	0.0029
15/2	0.4554	0.0073	0.0148	0.5630	0.0305	0.0024
8	0.4453	0.0068	0.0125	0.5506	0.0282	0.0020
17/2	0.4366	0.0064	0.0108	0.5398	0.0264	0.0017
9	0.4289	0.0060	0.0095	0.5303	0.0248	0.0015

TABLE VII. Octuple–2^5- and 2^6-pole correlations: $W(\theta)=1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)+A_6P_6(\cos\theta)$.

	2^3-2^5			2^3-2^6		
	A_2	A_4	A_6	A_2	A_4	A_6
Basic	0.4327	0.0140	−0.0015	0.4643	0.0183	−0.0035
j	$j+3\to j\to j+5$			$j+3\to j\to j+6$		
1	0.0433	0	0	0.0464	0	0
3/2	0.0865	0	0	0.0929	0	0
2	0.1236	0.0001	0	0.1327	0.0001	0
5/2	0.1545	0.0003	0	0.1658	0.0004	0
3	0.1803	0.0006	−0.0000	0.1935	0.0008	−0.0000
7/2	0.2019	0.0010	−0.0000			
4	0.2203	0.0014	−0.0000			
	$j-3\to j\to j-5$			$j-3\to j\to j-6$		
5	0.7500	0.0909	−0.0909			
11/2	0.7159	0.0771	−0.0620			
6	0.6884	0.0672	−0.0453	0.7386	0.0882	−0.1033
13/2	0.6657	0.0599	−0.0348	0.7143	0.0785	−0.0795
7	0.6467	0.0542	−0.0279	0.6939	0.0710	−0.0636
15/2	0.6305	0.0496	−0.0230	0.6765	0.0651	−0.0524
8	0.6166	0.0460	−0.0194	0.6616	0.0603	−0.0443
17/2	0.6045	0.0430	−0.0168	0.6486	0.0564	−0.0383
9	0.5939	0.0405	−0.0147	0.6373	0.0531	−0.0336

TABLE VIII. 2^4-pole–2^4-pole and 2^4-pole–2^5-pole correlations: $W(\theta)=1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)+A_6P_6(\cos\theta)+A_8P_8(\cos\theta)$.

	2^4-2^4				2^4-2^5			
	A_2	A_4	A_6	A_8	A_2	A_4	A_6	A_8
Basic	0.4777	0.0356	0.0000	0.0001	0.5350	0.0581	−0.0002	0.0001
j	$j+4\to j\to j+4$				$j+4\to j\to j+5$			
1	0.0478	0	0	0	0.0535	0	0	0
3/2	0.0955	0	0	0	0.1070	0	0	0
2	0.1365	0.0003	0	0	0.1528	0.0005	0	0
5/2	0.1706	0.0008	0	0	0.1911	0.0014	0	0
3	0.1990	0.0016	0.0000	0	0.2229	0.0026	−0.0000	0
7/2	0.2229	0.0025	0.0000	0	0.2497	0.0041	−0.0000	0
4	0.2432	0.0035	0.0000	0.0000	0.2723	0.0057	−0.0000	0.0000
9/2	0.2606	0.0045	0.0000	0.0000				
5	0.2756	0.0055	0.0000	0.0000				
	$j-4\to j\to j-4$				$j-4\to j\to j-5$			
4	0.9383	0.3641	0.0045	2.1930				
9/2	0.8758	0.2832	0.0024	0.4386				
5	0.8280	0.2317	0.0015	0.1515	0.9273	0.3776	−0.0145	0.1097
11/2	0.7904	0.1966	0.0010	0.0689	0.8851	0.3204	−0.0099	0.0498
6	0.7600	0.1714	0.0007	0.0371	0.8511	0.2793	−0.0072	0.0268
13/2	0.7349	0.1526	0.0006	0.0224	0.8230	0.2486	−0.0056	0.0162
7	0.7139	0.1380	0.0005	0.0147	0.7995	0.2250	−0.0045	0.0107
15/2	0.6961	0.1265	0.0004	0.0103	0.7795	0.2062	−0.0037	0.0075
8	0.6807	0.1172	0.0003	0.0076	0.7623	0.1910	−0.0031	0.0055
17/2	0.6674	0.1096	0.0003	0.0058	0.7474	0.1786	−0.0027	0.0042
9	0.6556	0.1032	0.0002	0.0046	0.7343	0.1681	−0.0024	0.0033

TABLE IX. 2^4-pole–2^6-pole correlations: $W(\theta)=1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)+A_6P_6(\cos\theta)+A_8P_8(\cos\theta)$.

	2^4-2^6			
	A_2	A_4	A_6	A_8
Basic	0.5740	0.0762	−0.0006	−0.0001
j	$j+4\to j\to j+6$			
1	0.0574	0	0	0
3/2	0.1148	0	0	0
2	0.1640	0.0006	0	0
5/2	0.2050	0.0018	0	0
3	0.2392	0.0035	−0.0000	0
	$j-4\to j\to j-6$			
6	0.9132	0.3662	−0.0165	−0.0356
13/2	0.8831	0.3259	−0.0127	−0.0215
7	0.8579	0.2949	−0.0102	−0.0141
15/2	0.8364	0.2703	−0.0084	−0.0099
8	0.8180	0.2504	−0.0071	−0.0073
17/2	0.8019	0.2341	−0.0061	−0.0056
9	0.7879	0.2204	−0.0054	−0.0044