the experiments were carried out (to about 90 percent of the total precipitation).

An interesting outcome is that the work affords a simple experimental method of measuring dislocation densities, although in a limited range and only in body-centered cubic alloys exhibiting this type of internal friction. It must be realized also that the theory leading to this estimated value is only approximate, as the solute atoms are treated as substitutional and hence only the relief of hydrostatic stress around the dislocation is considered. With the nonsymmetrical distortion produced by interstitial atoms, relaxation of shear stresses can also occur (Nabarro[12]), and thus there will be a tendency for such atoms to migrate to the Burgers type component of the dislocations.

Nevertheless the method may be of use in studying problems involving the amount of internal strain in a material.

Finally, by studying the case of carbon and nitrogen separately it has been possible to show that the temperature dependence of the process is identical with that of the diffusion of the elements in the mother metal.

The author wishes to thank Professor C. Zener for many helpful discussions during the course of this work.

PHYSICAL REVIEW VOLUME 83, NUMBER 4 AUGUST 15, 1951

The Contribution of the Pauli Moment to π-Meson Production by Photons*

M. F. KAPLON
University of Rochester, Rochester, New York

(Received February 26, 1951)

The photon-nucleon production of π-mesons has been calculated incorporating in the interaction a Pauli-type term representing the interaction of the photon with the anomalous magnetic moments of the nucleons. The effect of the added interaction for the scalar theory is to increase somewhat the charged meson production while leaving the neutral production virtually unchanged. For the pseudoscalar theory the charged meson production is relatively unaffected while the neutral meson production is greatly enhanced. Comparison with the neutral meson experiments in hydrogen therefore favors the pseudoscalar theory.

I. INTRODUCTION

WEAK coupling calculations of the production of neutral π-mesons by photons[1] yield cross sections which are much smaller than the charged π-meson production cross sections for both the scalar and pseudoscalar theories.[2] That is, it can be shown that if the anomalous magnetic moments of the nucleons are neglected,

$$d\sigma(\pi^0)/d\sigma(\pi^+) = (\boldsymbol{q}\cdot\boldsymbol{K}_0/\boldsymbol{P}\cdot\boldsymbol{K}_0)^2,$$

where ($K_{0\mu}$, q_μ, and P_μ are the four-momenta for the photon, meson, and recoil nucleon respectively) which, for instance, decreases from $\sim 1/5$ at 180° to $\sim 1/500$ in the forward direction at a photon energy of 250 Mev. This small ratio contradicts recent experimental results in hydrogen[3] which indicate that the neutral and charged cross sections are comparable.

One may ask the question whether it is possible within the context of weak coupling theory to eliminate the disagreement between theory and experiment by including a Pauli-type term in the hamiltonian to represent the interaction of the electromagnetic field with the anomalous magnetic moments of the nucleons. It turns out that such a phenomenological approach markedly improves the agreement for the pseudoscalar theory, whereas it actually increases the discrepancy for the scalar theory. At the same time it must be shown that the good qualitative agreement between the pseudoscalar theory[4] and experiment for charged π-meson production is not affected by taking into account the anomalous magnetic moments. We here report the results of calculations on the effect of the anomalous magnetic moments on meson production by photons, treating them in the Pauli fashion and assuming that the static values can be used at photon energies of several hundred Mev.

The calculations have been performed with the Feynman[5] techniques. The notation is that of Feynman with the exception $(\gamma^5)^2 = -1$. The interaction of the electromagnetic field with the anomalous magnetic moment[6] of the nucleon $(1/2)F_{\mu\nu}\gamma^\mu\gamma^\nu$, when transformed to momentum space, is $(\mu/2M)(\mathfrak{K}_0\mathfrak{A})$, where[7] $\mathfrak{A} = \epsilon_\mu\gamma^\mu$ (ϵ_μ is the 4-polarization of the photon), $\mathfrak{K}_0 = K_{0\mu}\gamma^\mu$ ($K_{0\mu}$

* Assisted by the AEC.

[1] G. Araki, Prog. Theor. Phys. **5**, 507 (1951). K. Brueckner, Phys. Rev. **79**, 641 (1950).

[2] The neutral π-meson is known to possess spin 0 and we shall assume that the charged π-meson also possesses spin 0 although this must still be demonstrated.

[3] W. Panofsky, private communication; preliminary results are $(d\sigma_{\pi^0}/d\Omega)(90°) \approx 3\times10^{-30}$ cm²/sterad compared to $(d\sigma_{\pi^+}/d\Omega)(90°) \approx 8\times10^{-30}$ cm²/sterad. Panofsky also finds $d\sigma_{\pi^0}(45°) \approx 2d\sigma_{\pi^0}(90°)$.

[4] Bishop, Steinberger, and Cook, Phys. Rev. **80**, 291 (1950); see Brueckner, reference 1.

[5] R. P. Feynman, Phys. Rev. **76**, 749, 769 (1949).

[6] W. Pauli, Handbuch d. Physik **24**, I (233).

[7] We use $\mathfrak{A} = A_\mu\gamma^\mu$; $\boldsymbol{A}\cdot\boldsymbol{B} = A_4B_4 - \mathbf{A}\cdot\mathbf{B}$; and A is the length of the three vector $\mathbf{A}$; thus $\mathfrak{A}$ is a 4-vector associated with a 4-matrix.

is the 4-momentum of the photon), γ_μ are the Dirac matrices defined by Feynman and $\mu_p=1.789$, $\mu_n=-1.91$ nuclear magnetons. We shall consider the scalar theory with scalar coupling (S, S), the pseudoscalar theory with direct coupling (PS, PS) and with derivative coupling (PS, PV).

For these theories with the inclusion of the Pauli term there are the 6 Feynman diagrams of Fig. 1, where $P_{0\mu}(E_0;\mathbf{P}_0)$, $P_\mu(E;\mathbf{P})$ are the initial and recoil nucleon 4-momenta, $K_{0\mu}(K_0;\mathbf{K}_0)$ is the photon 4-momentum and $q_\mu(\omega;\mathbf{q})$ is the meson 4-momentum. Diagrams 4 and 5 represent the anomalous moment interaction and diagram 6 represents the triple term required for gauge invariance in the (PS, PV) theory. The conservation laws require $(\boldsymbol{K}_0+\boldsymbol{P}_0=\boldsymbol{q}+\boldsymbol{P})$. The matrix elements are:

$$\mathbf{M}_1=-iC_1(\boldsymbol{P}|I(2\boldsymbol{A}\cdot\boldsymbol{P}_0+\mathfrak{K}_0\mathfrak{A})|\boldsymbol{P}_0)\varphi/2\boldsymbol{P}_0\cdot\boldsymbol{K}_0$$

$$\mathbf{M}_2=iC_2(\boldsymbol{P}|(2\boldsymbol{A}\cdot\boldsymbol{P}+\mathfrak{K}_0\mathfrak{A})I|\boldsymbol{P}_0)\varphi/2\boldsymbol{P}\cdot\boldsymbol{K}_0$$

$$\mathbf{M}_3=iC_3\boldsymbol{A}\cdot\boldsymbol{q}(\boldsymbol{P}|I|\boldsymbol{P}_0)\varphi/\boldsymbol{q}\cdot\boldsymbol{K}_0$$

$$\mathbf{M}_4=-iC_4(\boldsymbol{P}|I\{M\mathfrak{K}_0\mathfrak{A}-(\boldsymbol{A}\cdot\boldsymbol{P}_0)\mathfrak{K}_0+(\boldsymbol{P}_0\cdot\boldsymbol{K}_0)\mathfrak{A}\}|\boldsymbol{P}_0)\varphi/2M\boldsymbol{P}_0\cdot\boldsymbol{K}_0$$

$$\mathbf{M}_5=iC_5(\boldsymbol{P}|\{M\mathfrak{K}_0\mathfrak{A}+(\boldsymbol{A}\cdot\boldsymbol{P})\mathfrak{K}_0-(\boldsymbol{P}\cdot\boldsymbol{K}_0)\mathfrak{A}\}I|\boldsymbol{P}_0)\varphi/2M\boldsymbol{P}\cdot\boldsymbol{K}_0$$

$$\mathbf{M}_6=-iC_6(\boldsymbol{P}|\gamma^5\mathfrak{A}|\boldsymbol{P}_0)\varphi$$

with

$$I=1 \qquad (S, S)$$
$$=\gamma^5 \qquad (PS, PS)$$
$$=\gamma^5\begin{cases}\mathfrak{q} & \text{Diagrams 1, 2, 4, 5}\\ (\mathfrak{P}_0-\mathfrak{P}) & \text{Diagram 3}\end{cases} \qquad (PS, PV)$$

and the normalizations $\varphi^*\varphi=2\pi/\omega$, $(\boldsymbol{P}/\boldsymbol{P})=M/E$ (M is the nucleon mass taken to be equal for neutron and proton).

Table I gives the coefficients C_1 to C_6 corresponding to the Feynman diagrams 1 to 6 for the various production possibilities, where e is the charge of the proton and C_i is to be multiplied by g', g, f/μ; g_P', g_P, f_P/μ; g_N', g_N, f_N/μ (μ is the meson mass taken to be equal for π^+ and π^0), for processes (I, II), (III), and (IV) respectively where g', g, f are the coupling constants for the (S, S), (PS, PS), and (PS, PV) theories, respectively.

One can demonstrate a partial equivalence between the direct and derivative couplings of the pseudoscalar theory:

$$\sum_i \mathbf{M}_i^{(PS,PV)}=\sum_i[2M\mathbf{M}_i^{(PS,PS)}+a_i(\boldsymbol{P}|\gamma^5\mathfrak{K}_0\mathfrak{A}|\boldsymbol{P}_0)\varphi]$$

$$\sum a_i=\mu_P+\mu_N \qquad \text{for I, II}$$
$$=2\mu_P,\ 2\mu_N \qquad \text{for III, IV, respectively.}$$

$\mathfrak{A}\mathfrak{B}=(A_\mu\gamma^\mu)(B_\nu\gamma^\nu)$ is the product of $\mathfrak{A}$ and $\mathfrak{B}$, while $\boldsymbol{A}\cdot\boldsymbol{B}$ is a scalar.

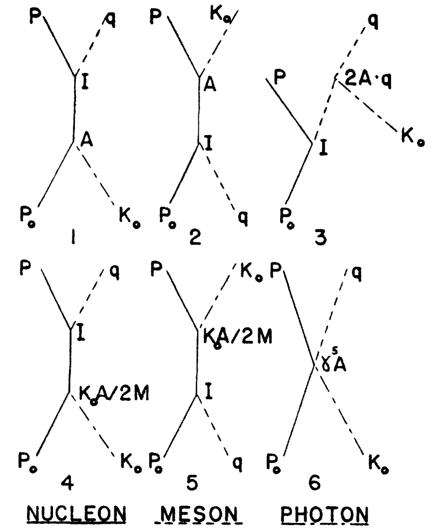

FIG. 1. Feynman diagrams for photonucleon production of mesons.

That is to say, the equivalence theorem holds if the Pauli term is omitted.

The following identities hold and are convenient for computational purposes:

$$\sum_{i=\mathrm{II}}\mathbf{M}_i(\mu_{P,N}=0)=-(\boldsymbol{P}_0\cdot\boldsymbol{K}_0/\boldsymbol{P}\cdot\boldsymbol{K}_0)\sum_{i=\mathrm{I}}\mathbf{M}_i(\mu_{P,N}=0)$$

$$\sum_{i=\mathrm{III}}\mathbf{M}_i(\mu_{P,N}=0)=-(\boldsymbol{K}_0\cdot\boldsymbol{q}/\boldsymbol{P}\cdot\boldsymbol{K}_0)\sum_{i=\mathrm{I}}\mathbf{M}_i(\mu_{P,N}=0).$$

One demonstrates gauge invariance for any process by the condition:

$$\sum_i \mathbf{M}_i(\mathfrak{A}\rightarrow\mathfrak{K}_0)=0.$$

The cross section in the laboratory system is

$$d\sigma=(2\pi)^2\langle|\sum_i\mathbf{M}_i|^2\rangle_{\mathrm{Av}}\rho_f/K_0,$$

where the symbol $\langle\ \rangle_{\mathrm{Av}}$ signifies an average over photon polarizations and initial nucleon spin and a sum over

TABLE I. Coefficients for matrix elements.

	C_1	C_2	C_3	C_4	C_5	$C_6(PS, PV)$ only
I. $\gamma+p\rightarrow\pi^++n$	e	0	e	μ_Pe	μ_Ne	$-e$
II. $\gamma+n\rightarrow\pi^-+p$	0	e	$-e$	μ_Ne	μ_Pe	e
III. $\gamma+p\rightarrow\pi^0+p$	e	e	0	μ_Pe	μ_Pe	0
IV. $\gamma+n\rightarrow\pi^0+n$	0	0	0	μ_Ne	μ_Ne	0

TABLE II. Coefficients for Eqs. (1) and (2).

	G	A	μ_1	μ_2	α	β
I. (S, S); (PS, PS); (PS, PV)	g'; g; f/μ	1	μ_P	μ_N	1; $2M$	0; 1
II. (S, S); (PS, PS); (PS, PV)	g'; g; f/μ	$-(\boldsymbol{P}_0\cdot\boldsymbol{K}_0/\boldsymbol{P}\cdot\boldsymbol{K}_0)$	μ_N	μ_P	1; $2M$	0; 1
III. (S, S); (PS, PS); (PS, PV)	g'_P; g_P; f_P/μ	$-(\boldsymbol{K}_0\cdot\boldsymbol{q}/\boldsymbol{P}\cdot\boldsymbol{K}_0)$	μ_P	μ_P	1; $2M$	0; 1
IV. (S, S); (PS, PS); (PS, PV)	g'_N; g_N; f_N/μ	0	μ_N	μ_N	1; $2M$	0; 1

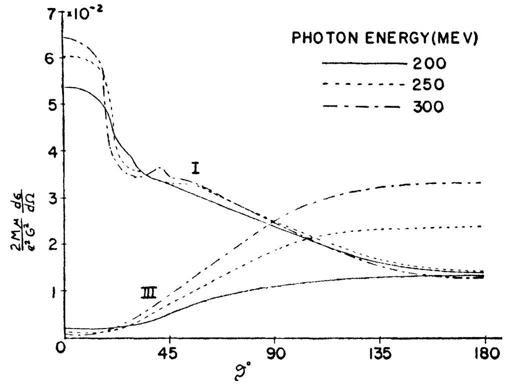

FIG. 2. Differential cross section in the laboratory system for production of π^+ mesons (curves I, anomalous magnetic moment not included) and π^0 mesons (curves III) in photon-proton collisions for the (PS, PS) theory.

final nucleon spin, ρ_f is the density of final states per unit energy, and the factor $2\pi/K_0$ arises from the expansion of the photon field.

The cross sections in the laboratory system become

$$(S, S)\quad d\sigma/d\Omega = \frac{q^3e^2G^2}{2MK_0[\omega(K_0M+\mu^2/2)-\mu^2(M+K_0)]}$$
$$\times\left\{A^2\left[\boldsymbol{P}\cdot\boldsymbol{K}_0/2\boldsymbol{P}_0\cdot\boldsymbol{K}_0+\frac{M^2q^2\sin^2\vartheta}{(\boldsymbol{K}_0\cdot\boldsymbol{q})^2}(1-\mu^2/4M^2)\right]\right.$$
$$+A(\mu_1-\mu_2)\boldsymbol{K}_0\cdot\boldsymbol{q}/4\boldsymbol{P}_0\cdot\boldsymbol{K}_0$$
$$-(\mu_1\mu_2/4M^2)(q^2\sin^2\vartheta\boldsymbol{P}_0\cdot\boldsymbol{K}_0/\boldsymbol{P}\cdot\boldsymbol{K}_0)$$
$$+1/2[(\mu_1-\mu_2)^2-\mu_1\mu_2(\boldsymbol{K}_0\cdot\boldsymbol{q})^2/((\boldsymbol{P}_0\cdot\boldsymbol{K}_0)(\boldsymbol{P}\cdot\boldsymbol{K}_0))]$$
$$\left.+(\mu_1+\mu_2)^2(\boldsymbol{K}_0\cdot\boldsymbol{q}-\mu^2/2)/4M^2\right\}\quad(1)$$

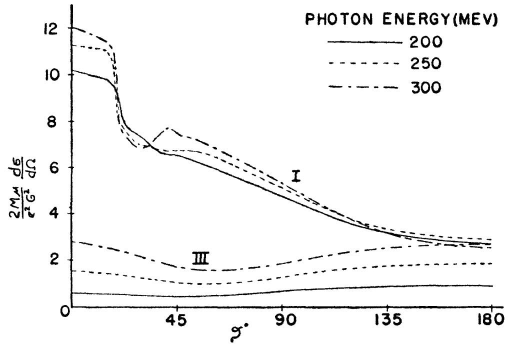

FIG. 3. Differential cross section in the laboratory system for production of π^+ mesons (curves I) and π^0 mesons (curves III) in photon-proton collisions for the (PS, PV) theory.

$$\left.\begin{matrix}(PS, PS)\\(PS, PV)\end{matrix}\right\}\ d\sigma/d\Omega$$
$$=\frac{q^3e^2G^2}{2MK_0[\omega(K_0M+\mu^2/2)-\mu^2(M+K_0)]}$$
$$\times\{\alpha^2[A^2(\boldsymbol{P}\cdot\boldsymbol{K}_0/2\boldsymbol{P}_0\cdot\boldsymbol{K}_0-\mu^2q^2\sin^2\vartheta/4(\boldsymbol{q}\cdot\boldsymbol{K}_0)^2)$$
$$-A(\mu_1+\mu_2)\boldsymbol{K}_0\cdot\boldsymbol{q}/2\boldsymbol{P}_0\cdot\boldsymbol{K}_0$$
$$+(\mu_1{}^2+\mu_2{}^2)(\boldsymbol{K}_0\cdot\boldsymbol{q}-\mu^2/2)/4M^2$$
$$+\mu_1\mu_2(\boldsymbol{K}_0\cdot\boldsymbol{q})^2/(4(\boldsymbol{P}_0\cdot\boldsymbol{K}_0)(\boldsymbol{P}\cdot\boldsymbol{K}_0))]$$
$$+\beta[((\mu_1+\mu_2)^2/2)((\boldsymbol{P}\cdot\boldsymbol{K}_0)(\boldsymbol{P}_0\cdot\boldsymbol{K}_0)/M^2-2\boldsymbol{K}_0\cdot\boldsymbol{q})$$
$$+A(\mu_1+\mu_2)(2\boldsymbol{P}\cdot\boldsymbol{K}_0-q^2\sin^2\vartheta\boldsymbol{P}_0\cdot\boldsymbol{K}_0/\boldsymbol{q}\cdot\boldsymbol{K}_0)]\},\quad(2)$$

where ϑ is the angle between $\mathbf{q}$ and $\mathbf{K}_0$ and the coefficients are given in Table II.

Examination of Eq. (1) shows that for the scalar theory the π^+ production in hydrogen is strongly enhanced relative to the π^0 production by the introduction of the Pauli moment so that the ratio of the π^+ to the π^0 cross sections is increased even more. The Pauli moments contribute an approximately constant term to $\langle|\mathbf{M}(\pi^+)|^2\rangle_{Av}$, yielding a net angular distribution that still has a dipole character though less exaggerated. The contribution of the Pauli term to the π^0 production is $\sim$1/10 that due to the charge of the proton.[8] This coupled with the fact that the π^0 angular distribution (which has its maximum at $\sim$90° and decreases in both the forward and backward directions) is in complete disagreement with experiment (see reference 3) makes it unlikely that the π^0 is scalar.

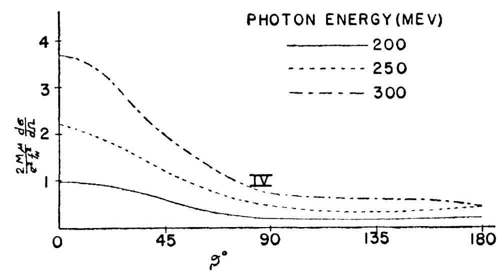

FIG. 4. Differential cross section in the laboratory system for production of π^0 mesons in photon-neutron collisions for the (PS, PV) theory.

Figure 2 gives the (PS, PS) angular distribution in the laboratory system at 3 photon energies including the Pauli term only in the π^0 case. The angular distributions of (PS, PS) π^0 mesons produced in photon-proton collisions without the inclusion of the Pauli term

[8] This can be seen by considering in the nonrelativistic approximation the matrix-elements for meson production by those diagrams representing the interaction of the photon with the anomalous moments of the nucleons: $(F|\boldsymbol{\sigma}\cdot\mathbf{H}|I)(\mu_1-\mu_2)/2MK_0$ which yields $|\mu_P|+|\mu_N|$ for charged meson production and 0 for neutral meson production.

TABLE III. Total cross sections$\times 10^{28}$ cm^2.

Photon energy (Mev)	200	250	300
I. (PS, PS)[a]	0.033	0.034	$0.034\times g^2$
I. (PS, PV)	6.6	6.9	$7.1 \times f^2$
III. (PS, PS)	0.013	0.023	$0.031\times g_P^2$
III. (PS, PV)	0.52	1.9	$3.0 \times f_P^2$
IV. (PS, PV)	0.16	0.85	$1.55 \times f_N^2$

[a] Does not include anomalous moment.

are similar to those in Fig. 2, though of much smaller magnitude. Figure 3 gives the (PS, PV) angular distributions for hydrogen including the anomalous moment interaction for both π^+ and π^0 production. To a good approximation ($\sim$3 percent),

$$d\sigma(+)^{(PS,PS)}/d\Omega = (\mu/2M)^2 d\sigma(+)^{(PS,PV)}/d\Omega$$

even when the Pauli term is included. Figure 4 gives the (PS, PV) angular distribution for π^0 meson production in photon-neutron collisions;[9] these curves will be useful for calculations of π^0 production in photon-deuteron collisions. Table III lists the total cross sections for the pseudoscalar theories at 3 different photon energies.

Since the angular distribution of the π^0 production for the direct coupling cases (with or without inclusion of the Pauli term) is in poor agreement with experiment (see reference 3), we consider further only the derivative coupling of the pseudoscalar theory. Figure 5 gives the excitation functions at 90° for π^+ and π^0 production, normalized so that $d\sigma(+)/d\Omega \approx d\sigma(0)/d\Omega$ when integrated over the experimental bremsstrahlung spectrum[10] (the experimental ratio of $\pi^0/\pi^+ \sim \frac{1}{3}$ at 90° requires $f_P \sim f$). The included experimental points[3,4] are normalized in the same way. The agreement is fair and indeed the qualitative features of the excitation functions at 90° agree well with the experimental points.

It is interesting to note that the π^-/π^+ ratio, which is given for the spin 0 theories without the inclusion of the Pauli moment by

$$\pi^-/\pi^+ = (\boldsymbol{P}_0\cdot\boldsymbol{K}_0/\boldsymbol{P}\cdot\boldsymbol{K}_0)^2 = (K_0/(\omega-\mu^2/2M))^2,$$

is only slightly changed by the inclusion of the anomalous magnetic moment interaction. It is, however, somewhat energy sensitive; for example at 90° to the beam it varies from $\sim$5 to 12 percent less than the ratio given above for photon energies in the range 200 to 300 Mev.

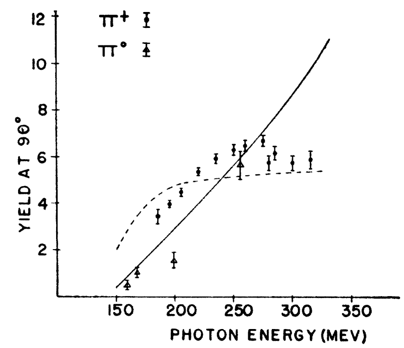

FIG. 5. Excitation functions at 90° to the photon beam for π^+ (dotted curve) and π^0 (solid curve) production in photon-proton collisions for the (PS, PV) theory.

Although the inclusion of the Pauli term can only be justified on an *ad hoc* basis,[11] it seems to us significant that in the pseudoscalar theory, the π^0 production can be raised by a large factor (for the energy region considered) without affecting the π^+ production to any appreciable extent. In addition, the fact that the same qualitatively correct excitation function and angular distribution are predicted by the weak coupling and classical (PS, PV) theories is evidence for a magnetic origin of the π^0 mesons in photon-proton collisions. Further evidence for the magnetic moment explanation of π^0 production can be obtained from experiments on π^0 production in deuterium since the neutron can interact only through its anomalous magnetic moment.[12]

I should like to thank Professor R. E. Marshak for suggesting this calculation and for many valuable discussions. I am indebted to Professor W. Panofsky for a discussion of the experimental data and to Professor R. P. Feynman and Dr. K. Brueckner for helpful discussion.

[9] For the (PS, PV) theory in nonrelativistic approximation, the matrix elements involving the interaction of the photon with the anomalous magnetic moments are

$$[\mu_1(F|(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{H})|I) - \mu_2(F|(\boldsymbol{\sigma}\cdot\mathbf{H})(\boldsymbol{\sigma}\cdot\mathbf{q})|I)]/2MK_0$$

which yield $d\sigma/d\Omega = (e^2f^2/K_0)(\mu_N^2/2M^2)(q^3/\mu^2)\sin^2\vartheta$ for the photoneutron production of charged π^- mesons ($|\mu_P| \approx |\mu_N|$) and $d\sigma/d\Omega = (e^2f_N^2/K_0)(\mu_N^2/2M^2)(q^3/\mu^2)(1+\cos^2\vartheta)$ for the photo-neutron production of neutral π^0 mesons. The latter result is identical with the classical cross section given by Serber (Summer lectures, University of Michigan, 1950) if one sets $\boldsymbol{\sigma}_0^2=3$ in the latter formula. The formula given in Serber's notes is too low by a factor of 4 due to an incorrect spin equation of motion; I am indebted to Dr. K. M. Watson for confirmation of this point.

[10] Hartsough, Hill, and Powell (to be published).

[11] It is fully realized that a consistent theory of the meson nucleon interaction should lead to an explanation of the anomalous magnetic moments of the nucleons in higher order; however, such a theory does not exist and this is the justification for attempting a phenomonological approach.

[12] The spin zero π^0 meson cannot possess an anomalous magnetic moment; however, it should be remembered that the π^0 meson may be directly coupled to the electromagnetic field plus the (bilinear) nucleon field so that a large photon-neutron cross section would not necessarily imply a large anomalous moment interaction.