of positrons to conversion electrons is 17/1. The L photoelectron spectrum has been observed with a thorium converter.

The Compton spectrum produced in a copper radiator has inflection points corresponding to gamma-rays of 3.30±0.14 Mev and 2.13±0.12 Mev.

It seems plausible to assume that Cl^{34} decays to S^{34} alternatively by three beta-gamma transitions. This is summarized in the following table. The figures involving the low energy gamma-ray are in parentheses because its position in the disintegration scheme is uncertain.

Beta-energy	Branching ratio	ft	Gamma-energy	Total energy
4.45±0.11 Mev	0.46	1.2×10^7	(0.145±0.003 Mev)	(4.6±0.11 Mev)
2.58±0.26 Mev	0.28	1.7×10^6	2.13±0.12 Mev	4.7±0.3 Mev
1.3 ±0.2 Mev	0.26	8.9×10^4	3.30±0.14 Mev	4.6±0.2 Mev

The portion of the Kurie plot due exclusively to the high energy positron component has been examined more extensively in this laboratory.[9] However, using the allowed Fermi function, no evidence of any divergence from linearity was found in the resulting plot.

[9] David Green (private communication).

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Neutron-Deuteron Scattering Amplitudes

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The diffraction of neutrons from deuterium containing compounds gives the coherent scattering section of deuterium which can be used to determine the spin dependent scattering amplitudes. Studies have been made on a number of deuterium containing compounds but the results reported here are concerned primarily with recent measurements on NaD. X-ray measurements have been used in interpreting the thermal vibration effects in the crystal and use has been made of Na scattering cross-section data obtained from various sodium containing compounds. This experiment gives $\sigma_{coh}=5.2\pm0.3$ barns. This value can be combined with the results of other deuterium cross-section measurements to give pairs of values for the individual quartet and doublet spin amplitudes for n-d scattering. Various experimental results bearing on this point are summarized. Recent theoretical work seems to have eliminated the ambiguity in the experiments with regard to which of two sets of values of the spin amplitudes is to be chosen.

THE first measurements of the coherent scattering cross section for neutrons on deuterium were made by studying the powder diffraction pattern of NaD.[1] These measurements showed the coherent scattering amplitude to have a positive sign and although the accuracy of the cross-section measurements suffered from uncertainty in the sample purity, they definitely indicated that the coherent scattering cross section is smaller than the total scattering cross section and hence that the scattering is spin dependent.

Since the time of these early experiments a number of other measurements involving the deuterium scattering amplitudes have been made. Powder diffraction patterns have been obtained at this Laboratory on several deuterium containing compounds; Fermi and Marshall[2] have studied the transmission of neutrons by deuterium gas, and Hurst and Alcock[3] have measured the angular dependence of the scattering by the gas.

Compounds studied by the powder technique include heavy ice,[4] ThD_2 and UD_3,[5] and more recent measurements on NaD and LiD, and in addition, a study by Levy and Peterson[6] has been made on ND_4Br. Most of these compounds have been studied for the primary purpose of determining the hydrogen positions in the respective crystal lattices. Pure compounds have, however, been used in every case and each experiment can be considered as a separate determination of the coherent scattering cross section of deuterium. The recent measurements with NaD and LiD* have, however, been made for this express purpose and these compounds have the advantage over the more complex compounds in that the determination of the cross section is not tied in with a simultaneous crystallographic study of the hydrogen atom locations. Only the recent measurements on NaD will be discussed in detail; the large absorption by Li makes the LiD results less significant. Samples were carefully prepared and analyzed for us by Mr. D. Lavalle of this Laboratory. A sufficient number of powder diffraction patterns were obtained by the automatic recording technique to obtain good statistics. The crystal structure scattering amplitudes calculated from the pattern by the powder formula are represented by the points in Fig. 1 plotted on semilog scale against $\sin^2\theta/\lambda^2$. The ordinate scale of

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[2] E. Fermi and L. Marshall, Phys. Rev. **75**, 578 (1949).
[3] D. G. Hurst and N. Z. Alcock, Can. J. Phys. **29**, 36 (1951).
[4] Wollan, Davidson, and Shull, Phys. Rev. **75**, 1348 (1949).
[5] Unpublished data by R. E. Rundle and the present authors.
[6] H. Levy and S. W. Peterson, private communication.
* The sample of LiD was kindly supplied by Metal Hydrides, Inc., Beverly, Massachusetts.

scattering amplitudes has been put on an absolute basis by our usual method of comparing the powder peak intensities in the pattern with the peaks from a standard scatterer. The standardization was based primarily on measurements with powdered samples of Ni^{58}, Th, and $Ni^{58}O$ which involve only zero spin nuclei.

The peak intensities were corrected for a small amount of second-order ($\lambda/2$) scattering, the amount of which has been determined from second-order reflections from the (200) planes of powdered crystals having strong (200) peaks. The total amount of second-order scattering is less than two percent of the corresponding first-order scattering. Correction was also made for the presence of about 3 percent by weight of NaOH. Patterns for NaOH samples have been obtained and the small effect of NaOH contaminant on the NaD pattern and also on the effective weight of sample have been taken into account. The data in Fig. 1 have been corrected for both these effects.

To obtain the deuterium scattering amplitude from the measured points it is necessary to take into account the effects of thermal vibration of the atoms in the crystal. The Debye-Waller temperature factor, which was derived for the x-ray case, has been shown by Weinstock[7] to apply also to the case of neutron scattering. This factor is strictly applicable only to a monatomic cubic crystal for which the true scattering amplitude, f_0, for an atom at rest is related to the value f_T measured in a crystal at a given temperature T and at a given $(\sin\theta)/\lambda$ by

$$f_T = f_0 \exp[-W(\Theta, m, T)(\sin^2\theta)/\lambda^2], \qquad (1)$$

where Θ is the characteristic temperature, and m is the mass of the atoms which make up the crystal. It has been found in practice[8] that the factor holds for diatomic crystalline compounds if the masses of the constituent atoms are not too different in which case a single factor involving the average mass in the function W applies to the amplitudes of both atoms.

If this condition could be assumed to hold also for a compound such as NaD where the ratio of the masses of the constituent atoms is large, then the best straight lines through the experimental points in curves A and D of Fig. 1 would represent the effect of lattice vibrations, and the intercept at $(\sin^2\theta)/\lambda^2=0$ would give the values of the sum and difference of the deuterium and sodium scattering amplitudes.

In related studies (unpublished) which we have made on the scattering of neutrons by NaH, this point has been investigated by a method which depends on the characteristic differences between x-ray and neutron diffraction for just such a case. With x-rays the diffraction by NaH arises almost wholly from the scattering by the Na atoms, and hence the lattice vibration effects in the pattern will also be due primarily to Na atoms. The slope of the dashed curve C in Fig. 1 represents

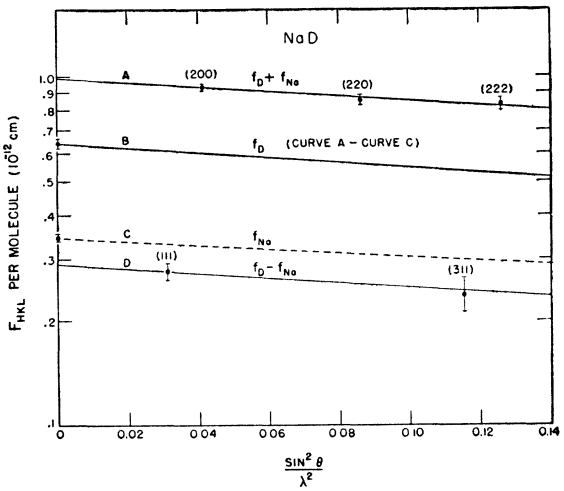

FIG. 1. Crystal structure scattering amplitudes as determined in NaD. Curves A and D represent the experimental data for a series of all-even and all-odd reflections respectively. Curve C corresponds to the scattering amplitude in this crystal for sodium only, making use of x-ray data on NaD and sodium scattering in other compounds. Curve B is obtained by subtracting C from A and corresponds to deuterium scattering only.

this lattice vibration effect on the sodium scattering amplitude as determined from x-ray intensity measurements on NaH. Although similar measurements were not made with NaD, it would be expected that the sodium amplitude would be very nearly the same for the two compounds.

It is to be noted also that the curve for Na atoms alone is almost parallel within experimental error to the lines through the sum and difference reflections for NaD, and hence one can conclude that the thermal motion effects for the Na and D atoms in this crystal are very nearly equal. The NaD data could then be used directly for determining the deuterium scattering amplitude. If one were to proceed in this way, however, the accuracy of the results would depend strongly on the relatively weak and hence less accurately determined odd index reflections represented by the points on curve D. The deuterium amplitude can, however, be more accurately determined by making use of data on the sodium scattering amplitude obtained with compounds better suited to give accurate results. In line with this procedure, the intercept at $(\sin^2\theta)/\lambda^2=0$ of the dashed curve whose slope has been taken from x-ray measurements has been normalized to the average value of the sodium cross section ($\sigma = 1.50 \pm 0.04$ barns) obtained from several sodium-containing compounds.[9] Subtracting this curve C from the experimental data in curve A gives curve B, which then represents the case for the scattering by deuterium atoms alone. With this procedure the accuracy of the scattering amplitude for deuterium determined from curve B depends primarily on the accuracy of the present measurements of the strong (200) reflection and on the accuracy with which

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[8] Kathleen Lonsdale, Acta Cryst. **1**, 142 (1948).
[9] C. G. Shull and E. O. Wollan, Phys. Rev. **81**, 527 (1951).

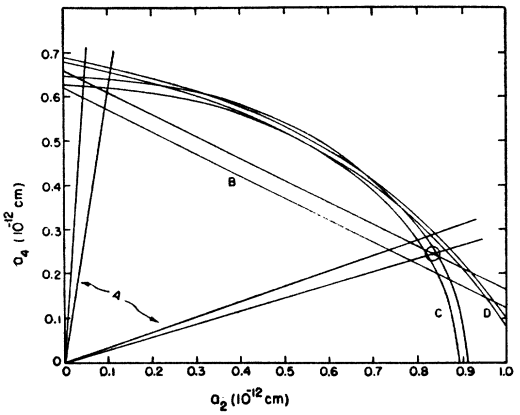

FIG. 2. Summary graph of the various experimental results for the quartet and doublet scattering amplitudes of neutrons on deuterons. The experiment designation is as follows: (A) Hurst and Alcock angular distribution of scattering by D_2 gas, (B) coherent scattering amplitude evaluation from crystal diffraction, (C) free scattering cross section by transmission at epi-thermal neutron energy and (D) Fermi-Marshall transmission at very low neutron energy. Theory suggests that the amplitude assignment should follow $a_2>a_4$, and the circled point corresponds to mean values of the amplitudes as suggested by all of the experiment data.

the sodium cross section was determined from the study of other sodium-containing compounds. The results obtained from curves A and D or from curve B lead to the same value for the deuterium scattering amplitude, the latter, however, being the more accurate. The deuterium scattering amplitude obtained from this experiment is

$$f_D=+(0.64\pm0.02)\times10^{-12}\text{ cm}$$

and the corresponding cross section

$$\sigma_{\text{coh}}=5.2\pm0.3\text{ barns.}$$

The experiments with heavy ice, ThD_2, and LiD are less accurate than those with NaD, but they are all consistent with the above value of σ_{coh}.

On the basis of measurements on ND_4Br, Levy and Peterson give the same value ($\sigma=5.2\pm0.3$ barns) as that obtained with NaD. This value has then been taken as representing the crystal diffraction data.

SPIN SCATTERING AMPLITUDES

The coherent scattering cross section can be expressed in terms of the individual scattering amplitudes for the two spin states, $i+\frac{1}{2}$ and $i-\frac{1}{2}$, of the compound nucleus

$$\sigma_{\text{coh}}=4\pi f_{\text{coh}}{}^2=\frac{4\pi}{(2i+1)^2}[(i+1)f_{i+\frac{1}{2}}+if_{i-\frac{1}{2}}]^2, \quad (2)$$

where the f's refer to bound scattering amplitudes, which are related to the free amplitudes a for a nucleus of mass number A by $f=[(A+1)/A]a$. For the case of n-d scattering, Eq. (2) can be written as:

$$f_{\text{coh}}=\pm\tfrac{1}{2}(2a_4+a_2), \quad (3)$$

where a_4 and a_2 are the free amplitudes for scattering in the quartet and doublet states of the compound nucleus, the triton.

The total free scattering cross section is given by

$$\sigma_s{}^f=\tfrac{4}{3}\pi(2a_4{}^2+a_2{}^2). \quad (4)$$

These two equations give solutions for the scattering amplitudes a_4 and a_2. Since f_{coh} has already been determined to be positive there will be only two sets of values for these scattering amplitudes.

Hurst and Alcock have made measurements of the angular distribution of the scattering by deuterium gas and these experiments can be expressed in terms of the ratio a_2/a_4, but again the quadratic character of the cross-section relation leads to two roots for this ratio. Fermi and Marshall have measured the transmission by deuterium gas with long wavelength neutrons and this leads to still another relation between a_4 and a_2.

These results are all represented graphically in Fig. 2. The total scattering cross section of deuterium is taken from transmission measurements[2,10] on D_2O for epi-thermal neutrons with the contribution from the oxygen cross section subtracted. A value of $\sigma_{\text{total}}=3.4\pm0.1$ barns seems consistent with these measurements. The errors in the other data are those given by the experimenters.

As seen in Fig. 1, there exist two regions of common intersection among the four independent sets of amplitudes. The experiments are not capable of resolving the selection ambiguity, but recent theoretical work[11–13] on neutron-deuteron scattering indicates that the only acceptable pair of values for the scattering amplitudes is that for which $a_2>a_4$. This case is represented in Fig. 2 by the circle, which might be considered as a reasonable mean value for the over-all experimental results. The corresponding mean values for the various amplitudes and cross sections according to this selection are given as

$$a_2=+0.83\pm0.015\times10^{-12}\text{ cm}$$

$$a_4=+0.24\pm0.015\times10^{-12}\text{ cm}$$

$$\sigma_{\text{coh}}=5.4\pm0.4\text{ barns}$$

and

$$\sigma_s{}^f=3.40\pm0.15\text{ barns.}$$

The algebraic signs associated with these scattering amplitudes follow the convention of assigning a positive sign to potential scattering amplitudes.

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