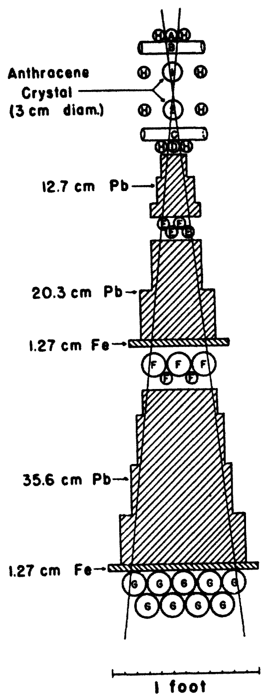

FIG. 1. Experimental arrangement.

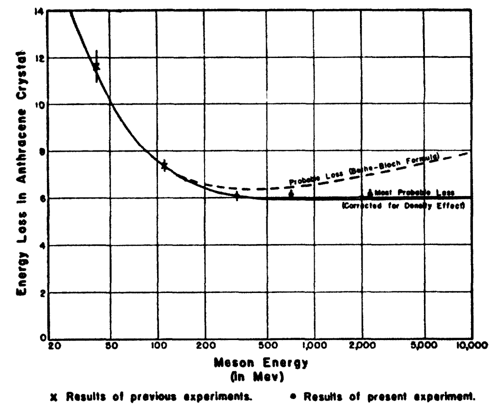

FIG. 2. Energy loss *vs* meson energy.

used, as these represent the three energy-ranges for μ-mesons. The data is recorded photographically when the master coincidence *ABCD* initiates the sweep of an oscilloscope. Pulses from scintillation counters (1) and (2), as well as from Geiger-Müller counters *E*, *F*, *G*, and *H*, are each delayed with respect to the master coincidence by different fixed amounts of time; hence they appear at characteristic positions on the oscilloscope trace. This system makes it possible to obtain the data for all three energy ranges during the same run of the equipment, so that slow time variations in the apparatus cannot affect the results in comparing the data for the various energy ranges. It was estimated that the multiple scattering of mesons out of the solid angle covered by trays *F* and *G* was of no consequence for the purpose of this experiment. The pulse-height scale was calibrated in units of energy loss (Mev) with the Compton electron distribution from the 2.62-Mev gamma-line from ThC″. This should yield, in addition to an accurate relative comparison with the theory, a slightly less precise absolute comparison.

The three curves representing frequency *vs* pulse height for each scintillation counter, corresponding to energy ranges (a), (b), and (c), were found to be in agreement within experimental error with the ionization energy-loss distribution calculated by Landau[5] for charged particles traversing thin absorbers. Table I gives the most probable energy loss for each of these distributions.

The averages from the table are also shown in Fig. 2. Two additional points at 40 and 110 Mev in Fig. 2 were taken from the results of a previous experiment.[4] A correction which was made for the change in light output with the specific ionization, as found by Frey *et al.*,[6] and others, was found to be significant only for these two additional points. The solid curve is the theoretically expected

TABLE I. Most probable energy loss for three energy ranges.

Average energy	Most probable energy loss (Mev)		
	Counter 1	Counter 2	Average of 1 and 2
(a) 325 Mev	6.07±0.15	6.14±0.15	6.11±0.10
(b) 710 Mev	6.19±0.12	6.19±0.12	6.18±0.08
(c) 2700 Mev	6.15±0.07	6.15±0.07	6.15±0.05

curve for the most probable energy loss, when corrected for the density effect in anthracene. The density effect correction for anthracene was estimated to be similar to those for light elements as found by Wick,[2] and Halpern and Hall.[3] The uncertainty of the ordinates of the theoretical curve are of the order of 2 or 3 percent. The experimental points are seen to be in good agreement with the theoretical curve, and show, as expected, that the most probable ionization loss in anthracene indicates a relativistic increase of less than 2 percent between 300 and 3000 Mev for μ-mesons. The curve for the most probable energy loss without the density effect correction is shown for comparison. The results of this experiment appear to establish definitely the existence of the reduction in ionization loss caused by the density effect.

We wish to thank Professor Marcel Schein for his continued interest in this work.

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† On leave from Universidad Catolica, Rio de Janeiro, Brazil.

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Meson Scattering

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EXPERIMENTS in this laboratory[1,2] have given the cross section σ_a for nuclear events (stars, large-angle scattering, apparent absorption) produced by mesons of about 45 Mev in carbon, as well as that for diffraction scattering, σ_d. The former cross section determines the opacity $O=\sigma_a/\pi R^2$ of the nucleus and the mean free path of the meson in nuclear matter, λ, which turns out to be about $(3.7\pm0.7)\times10^{-13}$ cm corresponding to a cross section for interaction of a meson and a nucleon of 40 ± 8 mb. The diffraction scattering has been calculated by Fernbach, Serber, and Taylor[3] and is a sensitive function of λ and of the "index of refraction" of the nucleus for mesons, defined as the ratio of the propagation vector inside the nucleus to that outside the nucleus, i.e., $(k_1+k)/k$. In Fig. 1 is plotted the diffraction scattering as a function of the opacity of the nucleus for various values[4] of k_1. In the same figure are plotted the Cornell experimental results[1] including the more recent measurements of Shapiro.[2] The rectangular box indicates the statistical standard error of the measurements.

Using the value of λ given above, one finds $k_1\lambda=0.3\pm0.3$ or $k_1=(0.8\pm0.8)\times10^{12}$ cm^{-1}. Now k_1 is related to energy of the meson

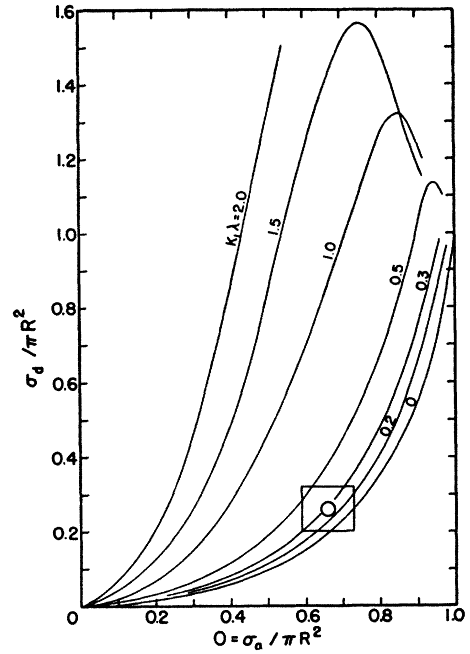

FIG. 1. The diffraction scattering is plotted as a function of the opacity of the nucleus for various values of the parameter $k_1\lambda$ which is related to the index of refraction of the nucleus as is explained in the text. The Cornell data are indicated by the plotted point.

and V_0, the average potential in the nucleus; i.e., $V_0=1.97\times10^{-11}$ $k_1\beta$. The experimental values above imply that V_0 is 11 ± 11 Mev.

The average potential in the nucleus is obviously related to the potential of a meson in the field of one nucleon, $V(r)$, by

$$V_0=\sum_{\text{all nucleons}}\int V(r)d\tau/(4\pi R^3/3). \tag{1}$$

If the potential is the same in the field of a proton and of a neutron, $V_0=\int Vd\tau(4\pi r_0^3/3)$ where[5] $r_0=RA^{-\frac{1}{3}}=1.47\times10^{-13}$ cm. Now the volume integral of the potential is related to the amplitude for scattering of a meson by a nucleon in the forward direction which is, according to the Born approximation,

$$a(10)=-\frac{1}{4\pi}\frac{2\mu}{\hbar^2}\int Vd\tau=-\frac{2\mu}{3\hbar^2}r_0^3V_0=-\frac{2}{3}r_0^3k_1k. \tag{2}$$

It is to be noted that this relation can be derived from general principles of wave interference, without using the average potential or the Born approximation.

The differential cross section for forward scattering of mesons by a nucleon is a^2, and the total scattering cross section is $\sigma_s=4\pi a^2$ if we assume isotropic scattering which may be a resasonable approximation for the energy of 45 Mev. It is convenient to express σ_s in terms of the "geometric cross section of a nucleon," writing

$$\sigma_s/\pi r_0^2=x^2,\quad x=(4/3)k_1kr_0^2. \tag{3}$$

Using the experimental value of k_1 and k, we get $x=0.13\pm0.13$. Thus the observed value of the diffraction scattering from carbon implies as the most probable value for the meson scattering by protons 1.8 percent of the "geometric cross section" πr_0^2 or 1 millibarn, and as the upper limit 7 percent or 5 millibarns.

Such a small cross section has been reported by Shutt *et al.*,[6] who exposed a cloud chamber filled with hydrogen at high pressure to the meson beam from the Nevis cyclotron and found only one recoil proton after scanning 1000 g/cm² of hydrogen. They conclude that the scattering cross section is less than 6 mb. On the other hand, Steinberger *et al.*[7] from transmission measurements, deduced a cross section for proton-meson scattering[8] of $0.2\pi r_0^2=13$ mb. Similarly, Skinner and Richman[9] find a large cross section (17±5 mb/sterad) for the differential scattering of mesons by carbon through 90°; at this angle, the nucleons in carbon should scatter essentially independently (and inelastically); if the nucleon scattering is isotropic, its total cross section would be 18±5 mb. However, the large scattering observed by Skinner and Richman is in contradiction with the experiments of Shapiro[2] who gets for the sum of the nuclear (not diffraction) scattering of mesons plus the emission of fast protons about 6 mb/sterad.

The experimental evidence on meson scattering is thus conflicting. If the small value of Shutt *et al.* turns out to be correct, it would be in agreement with our result from the refractive index. If the larger values turn out to be right, we shall have to assume that the scattered amplitude has the opposite sign for scattering from neutrons and from protons. Since V_0 involves an average over all nucleons, the correct expression for (2) is

$$-(2\mu/3\hbar^2)r_0^3V_0=(Na_N+Za_P)/A, \tag{4}$$

where a_N and a_P are the scattering amplitudes from neutrons and protons, respectively. If these are nearly equal and opposite, V_0 can be small even if a_N and a_P themselves are large.

Theoretically, a_N and a_P should have the same sign for pseudoscalar mesons, now favored by all the experimental evidence, but opposite signs for scalar or pseudovector mesons. This can be seen as follows, e.g., for a positive scalar meson: If the scattering nucleon is a neutron, it must first absorb the incident meson, thus becoming a proton, and then emit the scattered meson. For scattering by a proton, the emission precedes the absorption. In the latter case, then, the intermediate state has a higher energy than the initial, and according to quantum-mechanical perturbation theory, this leads to an attractive potential between proton and meson, so that a_P is positive. Conversely, the potential between positive meson and neutron is repulsive and a_N is negative. For negative mesons, the argument is reversed. Thus for scalar mesons, we have approximately $a_N=-a_P$.

For pseudoscalar mesons, on the other hand, the emission and absorption of mesons is most likely if the nucleon makes, at the same time, a transition from a positive to a negative energy state. Therefore, regardless of the charge of the scattering nucleon, the intermediate state has essentially energy $-2Mc^2$, and the nucleon-meson potential is repulsive in all cases so that $a_N\approx a_P$. In hole theory, this result remains unchanged, just like the Klein-Nishina formula for Compton scattering; one should speak, however, of the creation of a pair of nucleons in the intermediate state. Although the result is based on second-order perturbation theory, it is likely to persist in higher orders because all the important intermediate states involve production of nucleon pairs.

Pseudovector mesons would behave like scalar ones, because their interaction with nucleons involves the nonrelativistic operator $\boldsymbol{\sigma}$.

The theoretical magnitude of the scattering cross section is, in the pseudoscalar theory,[10]

$$\sigma_s=\pi(g^2/\hbar c)^2(\hbar/Mc)^2=1.3(g^2/\hbar c)^2\text{ mb} \tag{5}$$

so that a value $g^2/\hbar c$ of 0.9 ± 0.9 would satisfy our result from (3). This is reasonable in view of other evidence, such as the strength of nuclear forces.[11] For the scalar theory,

$$\sigma_s=4\pi(g^2/\hbar c)^2(\hbar/\mu c)^2 \tag{6}$$

and in this case, $g^2/\hbar c$ can be deduced fairly accurately from the binding energy of the deuteron[12] and is 2.39 μ/M, giving $\sigma_s=30$ mb, about twice the highest experimental result. Pseudovector theory would give the same order of magnitude as scalar.

If the cross section for scattering of mesons is small, as the Cornell experiments plus theory would indicate, it becomes harder to understand the result of Bernardini,[13] who finds that most of the mesons scattering in photographic emulsions (presumably by Ag or Br) lose a large fraction of their energy, being degraded from the order of 50 to the order of 5 Mev. This result is most easily interpreted as indicating repeated scatterings by the nucleons in the nucleus; since in one scattering the energy might be reduced by a factor 2 at high and by 10 to 20 Mev at low meson energy, about 3 scatterings will be necessary to give Bernardini's result. But if the scattering cross section per nucleon is only a few millibarns, such repeated scattering would be most unlikely.

Further experimental evidence on meson scattering, especially by protons and deuterons, is clearly needed.

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$$D=\frac{\sigma_d(\lambda, k_1)}{\sigma_d(\lambda, k_1=0)}-1=4(k_1\lambda)^2\left(1-\frac{1}{18}\left(\frac{R}{\lambda}\right)^2+\cdots\right).$$

For carbon, $R/\lambda\approx 1$ so that the second term is only a small correction. The simplified formula, $D=4(k_1\lambda)^2$, was found by numerical calculation to hold within 5 or 10 percent for opacities up to 0.9 and $k_1\lambda$ up to about 1.
[5] This value follows from the energy difference between mirror nuclei which seems to us the most accurate determination of the nuclear radius. The mirror nuclei may be chosen in the neighborhood of carbon.
[6] Shutt, Miller, Thorndike, and Fowler (private communication).
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Ranges of High Energy Electrons in Water

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RANGE measurements of high energy electrons in various media have customarily been made with Geiger counters or cloud chambers as detectors. With the high intensity homogeneous electron beam available from the betatron, ionization measurements are employed to determine the distribution of dissipated energy in various materials. Such ionization measurements that can be interpreted to supply information on the ranges of high energy electrons in water are reported here.

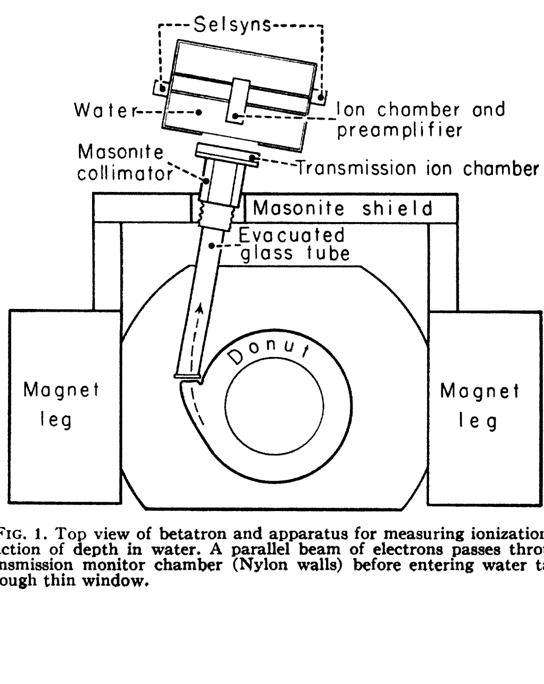

Fig. 1. Top view of betatron and apparatus for measuring ionization as function of depth in water. A parallel beam of electrons passes through transmission monitor chamber (Nylon walls) before entering water tank through thin window.

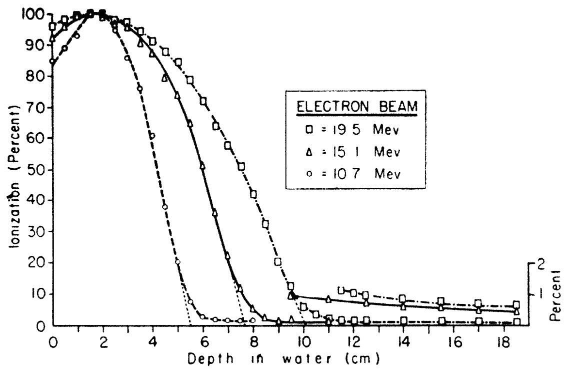

Fig. 2. Relative distribution of ionization as function of depth in water. Extrapolated ranges are indicated. Ionization produced in absorption of the bremsstrahlung was measured with increased amplification and is plotted against ordinate scale at right.

The energy of the electron beam can be varied continuously from 5 Mev to 22 Mev. The energy is determined by controlling the time at which the electrons are "expanded" from their orbit. The expansion timing circuit was calibrated directly against electron energy with known electrodisintegration thresholds. Thin foils of copper (10.9 Mev) and polythene (carbon, 18.7 Mev) were employed with the direct electron beam for this calibration.

A schematic diagram of the apparatus arrangement is shown in Fig. 1. The electron beam emerged from a thin window in the doughnut and passed in an evacuated tube through the fringing field of the magnet. Different field sizes and lengths of evacuated extension tubes were employed, but in all cases the electrons were in a parallel beam at the measuring apparatus. The ionization was measured in a small ion chamber (0.381 cm^3) immersed in a water tank. Its position could be accurately varied by remote control. The inside diameter and height dimensions of the chamber were 5 mm. It was mounted on a stem on a preamplifier leading to a dc amplifier. Details of this measuring apparatus have been described elsewhere.[1,2]

Typical distributions of ionization as a function of depth in water produced by a 6-cm diameter electron beam are shown in Fig. 2. The increase in ionization beyond the surface which is due to multiple scattering is less marked with increasing energy. The ionization distribution can be made proportional to the number of electrons by correcting for the specific ionization of the electrons as a function of energy. Brode's plot of specific ionization against energy[3] was used to correct distributions such as those in Fig. 2 to a plot of number of electrons *versus* depth in water. This correction did not prove to be important, and the

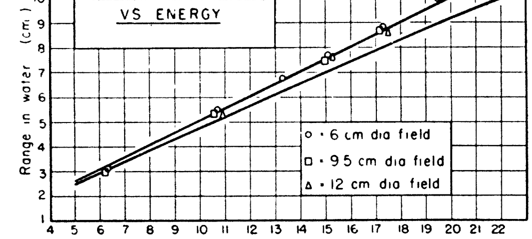

Fig. 3. Experimental points are the extrapolated ranges obtained from data plots such as those in Fig. 2. The maximum experimental uncertainty in the points is ±0.2 Mev in energy and ±2 mm in range. The upper curve represents maximum ranges predicted on the basis of Halpern and Hall's equation including density effects. The lower curve represents the predictions of the Bethe-Bloch equation ignoring density effects.