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On Soft Photon Emission in Radiation Processes*

H. PRIMAKOFF† AND F. VILLARS
Department of Physics and Laboratory for Nuclear Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts
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IN the present note we have investigated the intensity of the soft photon spectrum emitted in ordinary hard photon radiation processes: e.g., x-ray emission and absorption, pair annihilation, etc.[1] The motivation for the work was the reported possibility of a discrepancy between the wavelength of the electron-positron annihilation gamma-ray, λ_{ann}, and h/mc: $\lambda_{ann}-h/mc \approx 10^{-3}\ h/mc$, a discrepancy[2] which could conceivably be interpreted on the basis of an extended energy conservation relation: $2mc^2$=energy of the two hard annihilation photons+energy of all the accompanying soft photons.

We first considered the equivalence of the two possible descriptions of the annihilation process: the configuration space treatment of the electron and positron in the sense of Fock,[3] and the customary one-particle treatment.[4] The two descriptions are essentially identical except for the impossibility of the inclusion of the electron-positron coulomb interaction in the latter. For the soft photon emission problem, the coulomb interaction has the role of determining the velocity spectrum of the annihilating particles; however, an appropriate velocity spectrum can always be assigned to the initial and final electron states in the one particle description. The two modes of treatment then become exactly equivalent and in particular, the intensity of the soft photon energy spectrum turns out to be determined by the average relative velocity v of the electron and positron. Quantitatively, the average energy $\bar{E}_{soft}$ carried off by this spectrum (per individual pair recombination) is approximately given by

$$(2/3\pi)(1/137)(v/c)^2\, 2\, mc^2 \approx 10^{-7}\, mc^2$$

for v/c values appropriate to the annihilation of slowed down positrons in matter. It is seen, therefore, that the soft photon effect is much too small to account for a discrepancy between hc/λ and mc^2 of the order of magnitude reported, even if the corresponding shift of the intensity maximum in the wavelength distribution of the annihilation line were as large as $\bar{E}_{soft}$ (see below). It is thus very satisfactory that the most recent precise absolute measurements of the energy of the 0.51-Mev ThC″ γ-ray line by Lindström,[5] taken together with the recent precise measurements by Hedgran[6] of the ratio of the energy of this line to that of the annihilation line, yield exact equality of λ_{ann} and h/mc within an experimental error of 3×10^{-4}.

In x-ray emission and absorption the average energy of the accompanying soft photon spectrum (per individual hard photon process) is again given by:

$$\bar{E}_{soft} \approx (2/3\pi)(1/137)(v/c)^2 E_{max}, \tag{1}$$

where E_{max} is the maximum available energy for the soft photons in a single hard photon process and v is the electron velocity appropriately averaged over the initial and final orbits. ($E_{max}\approx h\nu_{hard}$, $v/c \approx Z/137$ for mission; $E_{max}\approx$photoelectron kinetic energy, $v\approx$photoelectron velocity, for absorption.) It might therefore be thought that deviations of 1/10 to 1/100 percent could be observed from the Bohr frequency rule, the Ritz combination principle, and the Einstein photoelectric equation in such hard photon emission and absorption. The emitted soft photon spectrum however, is described by the almost flat intensity distribution:

$$\left(\frac{\bar{E}_{soft}}{E_{max}}\right)\left(\frac{E_{soft}}{E_{max}}\right)^{\bar{E}_{soft}/E_{max}} dE_{soft} \tag{2}$$

with the consequence that in the case of emission, for example, the usual hard photon line shape (derived with neglect of the soft photon effect)[7] is multiplied by a correction factor given approximately by

$$1-\pi(\bar{E}_{soft}/E_{max})[(E_{hard}-E_0)/\Gamma]. \tag{3}$$

Here, E_0 is the energy difference between the initial and final states involved in the hard photon emission, and Γ the hard photon natural line breadth. The energy of an emitted hard photon (or in a similar way, the kinetic energy of an ejected photoelectron) suffers, therefore, a most probable net displacement in the direction of lower energy of only

$$\frac{\pi}{8}\frac{\Gamma}{E_{max}}\bar{E}_{soft} \approx \frac{\pi}{8}(\bar{E}_{soft}/E_0)^2 E_0 \approx \frac{1}{50}\left(\frac{1}{137}\right)^2\left(\frac{v}{c}\right)^4 E_0, \tag{4}$$

Γ in the atomic x-ray region being $\approx(1/10)(1/137)(Z/137)^2E_0 \approx \bar{E}_{soft}$. The smallness of this net displacement as compared with $\bar{E}_{soft}$ is due to the extreme flatness of the soft photon energy spectrum.

Equations (3) and (4) show that any measurement of the emitted hard photon or ejected photoelectron energy, by extrapolation of the corresponding line shape on the high energy side, will be independent of the actual presence of the soft photon emission to about one part per million. Essential agreement with the usual energy conservation relations (neglecting soft photons) is thus always to be expected within this precision.

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† On leave from Washington University, St. Louis, Mo.
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The Isomeric Level of Cd^{111m}

CARL L. MCGINNIS
Department of Physics, University of California, Berkeley, California
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IN a previous investigation of the radioactivity of the 111-isobars,[1] spin and parity were assigned to the several levels, with the result that eight of the nine assignments agreed with the predictions of the shell model.[2] The 48-min isomeric level of Cd^{111} at 396 kev was the exception, being given spin 13/2 and even parity rather than the predicted $h_{11/2}$. This assignment was made on the basis of the usual γ-ray half-life formula,[3] row 3, Table I, and the ratio of 149/247 internal conversion electrons, row 4.

TABLE I. Data for assigning spin and parity to the 396-kev level of Cd^{111}.

		Theory		Expt.
396-kev level	13/2 even	9/2 odd	11/2 odd	
149-kev γ-ray	elec. 4	mag. 2	elec. 3	
149-kev γ, $T_{\frac{1}{2}}$ (sec)	117	8×10^{-4}	8×10^{-4}	2916
149/247 conv. e^-	15.7	10.2	11.4	14.5 ± 1
149/247 γ-rays	0.085	0.43	0.35	0.33 ± 0.06
In^{111} decay ratio	1.9×10^{-8}	2.9×10^{-4}	2.9×10^{-4}	1×10^{-4}

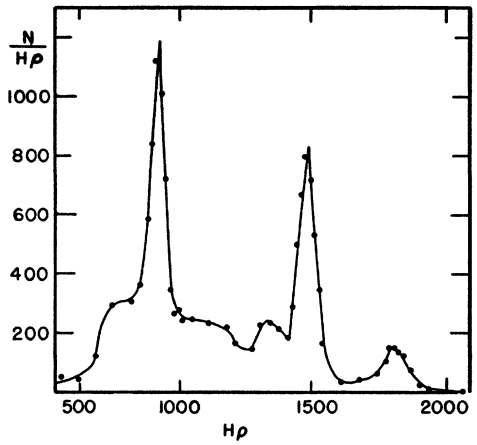

FIG. 1. Cd^{111m} photoelectron spectrum from a 1.1-mg/cm² gold radiator.

As the evidence for spin 13/2 even parity was not conclusive, it was decided to reinvestigate this assignment.

Cd^{111m} decays by the emission of a 149-kev γ-ray followed by a 247-kev γ. The latter is known to be electric quadrupole and 6.0 percent internally converted. The ground state of Cd^{111} is $s_{\frac{1}{2}}$, and this first excited state is $d_{5/2}$. On this basis, spin and parity are assigned to the 396-kev level, row 1, Table I, assuming a multipole order for the 149-kev γ listed in row 2. The possible ratios 149/247-kev γ-rays, row 5, were computed on the basis of the latter being 6 percent internally converted; and the percent the 149-kev γ is converted was computed from the measured K/L ratio, 2.0, and the theoretical K internal conversion coefficients.[4] In a similar manner the possible ratios of 149/247 conversion electrons, row 4, were calculated. It is seen from Table I that the γ-ray ratio is more sensitive to the character of the 149-kev γ than is the ratio of conversion electrons.

This γ-ray ratio was measured by observing the Cd^{111m} photoelectron spectrum from a 1.1 mg/cm² gold radiator, Fig. 1. The measured ratio of 149/247 of photoelectrons was 0.9 ± 0.1. The correction for absorption[5] in an 0.4 mg/cm² Nylon window of the spectrograph increased the ratio to 1.0 and estimating the loss in the gold foil makes the ratio 1.2 ± 0.2. According to the experimental data of Jones[6] a 149-kev γ is 3.67 times more efficient photoelectrically than a 247-kev γ. Hence the γ-ray ratio is 0.33 ±0.06.

This ratio eliminates the possibility that the 396-kev level has spin 13/2 even parity and favors the assignment $h_{11/2}$. The previously measured[1] ratio of conversion electrons, 14.5, is considered to be in error because of the difficulties in making the corrections for scattering and self-absorption in the measurement of such a large ratio.

The $g_{9/2}$ ground state of In^{111} decays by allowed K-capture to the $g_{7/2}$ level of Cd^{111} at 419 kev. The possible ratios (In^{111} transitions to the 396-kev level)/(In^{111} transitions to the 419-kev level), are listed in row 6. The theoretical ratios are those of the Fermi theory f values. The experimental ratio is, within a factor of two, 1×10^{-4}, which was determined by extracting chemically the 48-min Cd from the 2.84-day In. This measurement is consistent with the assignment $h_{11/2}$ to the 396-kev level and is not consistent with the previously assigned spin 13/2 and even parity.

Hence, it is concluded that the 396-kev level of Cd^{111} is $h_{11/2}$. This places the nine levels of the 111-isobars in agreement with the predictions of the shell model.

The usual γ-ray half-life formula assigns a multipole order of four to the 149-kev γ rather than the measured three. This result indicates that such a formula cannot be relied upon to assign a multipole order, despite the apparent correlation with energy and half-life as is shown by the plot of Axel and Dancoff.[3] Sunyar and Goldhaber[7] find that this formula yields spin differences one unit too high for electric 2^3 and 2^4 transitions. The description of the 149-kev γ-ray as electric 2^3 agrees with their conclusion.

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The Anomalous Spin Gyromagnetic Ratio of the Electron*

S. KOENIG, A. G. PRODELL, AND P. KUSCH
Columbia University, New York, New York
(Received June 20, 1951)

PRELIMINARY results of a new measurement of the anomalous spin gyromagnetic ratio[1] of the electron will be presented in this letter.

By a measurement of transitions between Zeeman levels of the hfs of atomic hydrogen in a fixed magnetic field, and a measurement of the precession frequency of protons in a water sample placed in the same magnetic field, it has been possible to find an experimental value for $g_J(^2S_{\frac{1}{2}}, H)/g_I$. Here g_J is the Landé g factor of the electron configuration of the $^2S_{\frac{1}{2}}$ state of hydrogen, and g_I that of the proton, except for small corrections due to diamagnetic effects.

Inasmuch as the ground state of hydrogen is pure to considerably better than a part in a million, it is apparent that g_J equals gs', the spin g value of the bound electron in atomic hydrogen. It can be shown that application of the correction for the relativistic mass change due to binding yields[2] for the g value of a free electron, g_S:

$$g_S=g_S'(1+\alpha^2/3).$$

Essentially, then, what is measured is the ratio of the spin magnetic moment of the electron to that of the proton.

The recent measurement by Gardner and Purcell[3] of the ratio of the cyclotron frequency of the electron and the nuclear resonance frequency of the proton in a magnetic field establishes the ratio of the orbital gyromagnetic ratio, g_L, of the electron, and g_I, the nuclear g value, of the proton, again within the limits of the internal diamagnetic effects of the hydrogen bearing molecule. A combination of the two results yields a value for g_S/g_L, or effectively the spin magnetic moment of the electron in units of the Bohr magneton.

Preliminary results for the value of g_S/g_I have been obtained. They depend essentially on the measurement of the frequency of the transition ($F=1, m=0\leftrightarrow F=1, m=-1$) in a field of 1500 gauss, on a prior knowledge[4] of the hyperfine separation, $\Delta\nu(H)$, of hydrogen, and on the measurement of the proton resonance frequency in the same magnetic field. A mechanism is provided in the present experiments for interchanging the positions of the water sample together with the associated rf coil mount and the uhf circuit in which the atomic transitions occur. A crucial point of the experiment is, of course, the accuracy with which the interchange may be made.

A cylindrical water sample is used in the experiment to insure optimum coincidence of water sample and atomic beam. This requires a correction[5] of +1.5 parts in 10^6 to the measured value of g_J/g_I, due simply to the bulk diamagnetism of the water. No such correction is necessary for the spherical sample of mineral oil used by Gardner and Purcell. A correction of −0.4 in 10^6 must be applied because of the presence of a known concentration of paramagnetic ions in the water sample. Finally, to combine our result with that of Gardner and Purcell, the known discrepancy between the resonant frequency of the proton in mineral oil and water requires an additional correction of +2.2 parts in 10^6.

After the application of the total correction of 3.3 parts in