For $|\alpha|<2\pi$, the contour of Fig. 1, which is indented at the (simple) poles of the integrand, can be used. (The cut along the negative axis of reals makes possible immediate use of the Stirling asymptotic expansion of $\log\Gamma(s)$[3] to show that the integrand vanishes on the arcs as the contour recedes to infinity.) The function $\zeta(s+\sigma)$ has a simple pole at $s=1-\sigma$ with residue $+1$, and $\Gamma(s)$ has simple poles at $s=-n$ with residues $(-1)^n/n!$ Consequently, we have

$$F(\sigma,\alpha)=\Gamma(1-\sigma)\alpha^{\sigma-1}+\sum_{n=0}^{\infty}\frac{(-)^n}{n!}\zeta(\sigma-n)\alpha^n. \quad (4)$$

This is patently an analytic function of σ if $\sigma\leqslant 0$ and for all non-integral σ.

If now $\sigma=m$, a positive integer, although $\Gamma(1-\sigma)\alpha^{\sigma-1}$ and one term of the series in Eq. (4) become infinite separately, their sum remains finite. We have

$$F(m,\alpha)=\lim_{\sigma\to m}\left\{\Gamma(1-\sigma)\alpha^{\sigma-1}+\frac{(-)^{m-1}}{(m-1)!}\zeta(\sigma-m+1)\alpha^{m-1}\right\}$$
$$+\sum_{n=m-1}^{\infty}\frac{(-)^n}{n!}\zeta(\sigma-n)\alpha^n=\frac{(-)^{m-1}}{\Gamma(m)}\left\{C+\frac{\Gamma'(m)}{\Gamma(m)}-\log\alpha\right\}\alpha^{m-1}$$
$$+\sum_{n=m-1}^{\infty}\frac{(-)^n}{n!}\zeta(\sigma-n)\alpha^n,$$

where C is Euler's constant. Therefore, by the principle of analytic continuation, Eq. (4) holds for *all* σ. The series converges absolutely if $|\alpha|\leqslant 2\pi$.

Equation (4) readily yields the differentiation property of the Bose functions:

$$\partial^n F(\sigma,\alpha)/\partial\alpha^n=(-)^n F(\sigma-n,\alpha).$$

When $\alpha\to 0$, it is seen that $F(\sigma,\alpha)$ diverges as $\alpha^{-|\sigma-1|}$ if $\sigma<1$, and as $\log(1/\alpha)$ if $\sigma=1$, and of course converges toward $\zeta(\sigma)$ if $\sigma>1$. If $1<\sigma\leqslant 2$, then $F(\sigma,\alpha)$ has an infinite slope at the origin although the function itself remains finite. Clearly, the origin $\alpha=0$ is a branch point for all the $F(\sigma,\alpha)$.

The series in Eq. (4) converges quite rapidly in the neighborhood of $\alpha=0$ for positive σ which are not too large. For example, with an accuracy of at least 1 percent when $\alpha\leqslant 1$, we have

$$F(\tfrac{1}{2},\alpha)=1.77\alpha^{-\frac{1}{2}}-1.46+0.208\alpha-0.0128\alpha^2,$$
$$F(\tfrac{3}{2},\alpha)=-3.54\alpha^{\frac{1}{2}}+2.61+1.46\alpha-0.104\alpha^2+0.00425\alpha^3,$$
$$F(5/2,\alpha)=2.36\alpha^{\frac{3}{2}}+1.34-2.61\alpha-0.730\alpha^2+0.0347\alpha^3.$$

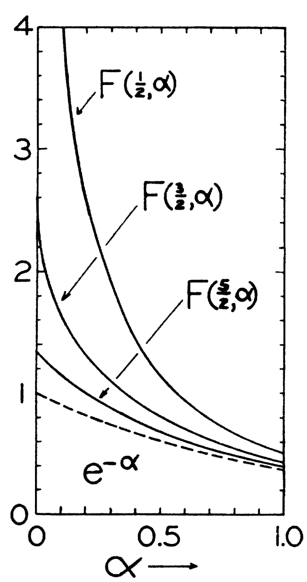

FIG. 2. The Bose-Einstein integral functions $F(\frac{1}{2},\alpha)$, $F(\frac{3}{2},\alpha)$, and $F(5/2,\alpha)$ for the range $0\leqslant\alpha\leqslant 1$.

These functions, together with $e^{-\alpha}$, are shown graphically in Fig. 2 for the range $\alpha\leqslant 1$. For $\alpha>1$, the $F(\sigma,\alpha)$ are conveniently evaluated by the familiar series of exponentials.

The author wishes to thank Professor F. London for his interest in this work.

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Radioactive Decay of I^{131}

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THE principal radiations emitted in the disintegration of I^{131} have been organized into schemes by Metzger and Deutsch (M.D.)[1] and by Kern, Mitchell, and Zaffarano (K.M.Z.).[2] These are shown in Fig. 1, and it will be seen that they include essentially the same features and differ only in the soft beta-ray branch. However, a number of less prominent features remain to be accounted for. Brosi *et al.*[3] have found that a small fraction of the disintegrations lead to the 12-day metastable level of Xe^{131}; a gamma-ray of an energy approximately 720 kev occurring in about 5 percent of disintegrations has been discovered by Cavanagh[4], and subsequently reported by Cork *et al.*[5], and by Zeldes *et al.*;[6] Cork[5] has produced a convincing photographic spectrum showing K and L conversion lines due to a 177-kev gamma; Zeldes[6] has produced evidence of a weak 810-kev beta-ray. All these radiations, except the 810-kev beta-ray, have been assembled by Cork[5] into a scheme which must, however, be very far from the truth. As the authors point out, there are wide anomalies in the intensities. For instance, in one branch the 600-kev beta-ray which arises from 85 percent of the disintegrations is shown followed by the 723-kev gamma-ray (5 percent) leading to the metastable level (1 percent). Moreover, the scheme places both the 637- and the 364-kev gamma-rays in the soft beta-branch, although M.D. have shown that only the first of these is associated with the soft beta-ray, the other being in the 600-kev beta-

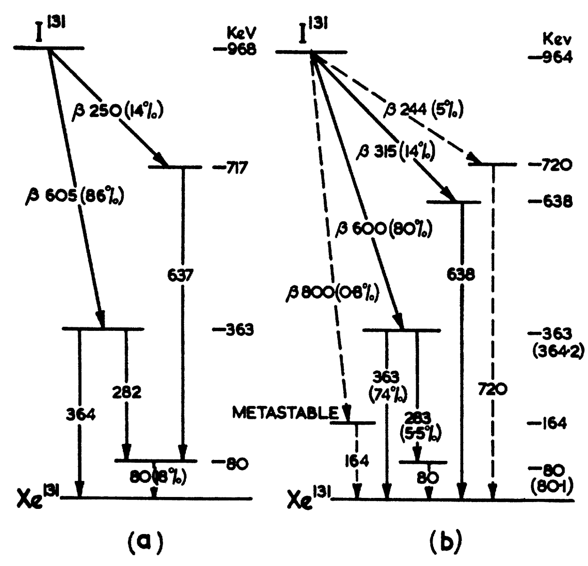

FIG. 1. Decay schemes for I^{131}: (a) Kern, Mitchell, and Zaffarano (K.M.Z.); (b) Metzger and Deutsch (M.D.) with proposed additional branches (dotted).

branch. A further objection to this decay scheme is that it indicates an energy difference between I^{131} and Xe^{131} nuclei of 1.479 Mev, a value which can be shown to be too great. For, since the dose rate in air produced by a gamma-emitting source is closely proportional to the quantum energy over the range 0.1–1.0 Mev, it follows that a measurement of the gamma-ray ionization from a source of known disintegration rate gives an estimate of the energy emitted per disintegration as gamma-rays; and, since the ratio of mean beta-energy to the maximum does not vary greatly among isotopes of a given atomic number, a measurement of the beta-energy emitted from such a source gives an estimate of the total energy carried by beta-particles and neutrinos. The sum of these two quantities is the total energy difference between parent and daughter nucleus. Such energy measurements have been carried out by Gray,[7] and by comparison with the I^{131} samples of the standard disintegration rate now distributed by the National Bureau of Standards, which are based on 4π solid angle beta-counting and are therefore independent of the assumed decay scheme, they yield values of the beta- and gamma-energy which are in complete agreement, within experimental error, with either M.D. or K.M.Z., and demonstrate that the total energy of disintegration cannot differ from 0.964 Mev by more than about 10 percent.

It therefore seems reasonable to assume that either the M.D. or K.M.Z. scheme disposes correctly of the radiations with which they both deal, and that the other radiations are to be accounted for by further branching of the spectrum. It is possible to differentiate between these two schemes by measurement of the 80-kev gamma-ray. Both groups detect this gamma-ray by the conversion line in the beta-spectrum, and they are in substantial agreement as to the number of conversion electrons produced; they do, however, differ widely in the values assigned to the conversion coefficient, K.M.Z. giving 15.5 unconverted quanta per 100 disintegrations and M.D. only 3.4. The actual intensity can be conveniently assessed experimentally by measurement of the ionization in a copper chamber. The effect of the 80-kev radiation is greatly enhanced by the photoelectric effect and can be separated out by absorption in tin (Fig. 2). Correction having been made for the relative stopping power of copper and for the effect of secondary electrons arising within the air cavity, it is found that the intensity is 2.6 quanta per 100 disintegrations. So small a value could not occur if the 80-kev line were in cascade with the 637-kev line, and it follows that the M.D. scheme is to be preferred, even though this rules out the tempting hypothesis that the 720-kev gamma-ray arises from the 717-kev level shown in the K.M.Z. scheme.

As regards the presence of further branches in the spectrum, it is probable that one such branch is formed by Zeldes' 810-kev beta-ray leading to the metastable level at 164 kev. The sum of the energies is correct within reasonable experimental error; and, so far as can be judged, the intensities are in agreement. The author has extracted the Xe^{131} from three I^{131} samples, and by estimating the activity by means of a cavity ionization chamber it was found that the frequency of disintegrations leading to the metastable level is 0.8 ± 0.1 percent. This compares reasonably with the statement in Zeldes' paper that the frequency of the beta-ray is "rather less than 1 percent." Another branch might be formed by the 720-kev gamma-ray associated with a 240-kev beta-ray, for the latter could quite easily have escaped detection in the presence of the more intense 315-kev branch. The inclusion of these features leads to the decay scheme given in Fig. 1(b), and it is suggested that this scheme is in reasonable accordance with the known facts concerning the disintegration of I^{131}. However, the 177-kev gamma-ray is still not accounted for. Its intensity must be quite small, since there is no sign of it in any of the spectra published by M.D. and K.M.Z.; and moreover, if it occurred in more than 5 percent of disintegrations it would certainly have been apparent in the tin absorption curve (Fig. 2). This gamma-ray might conceivably occur in a separate branch with a weak 787-kev beta-ray; or alternatively, if it were in cascade with the 720-kev line, the associated beta-branch would have an energy of 80 kev, and in this region of the spectrum could easily have escaped detection.

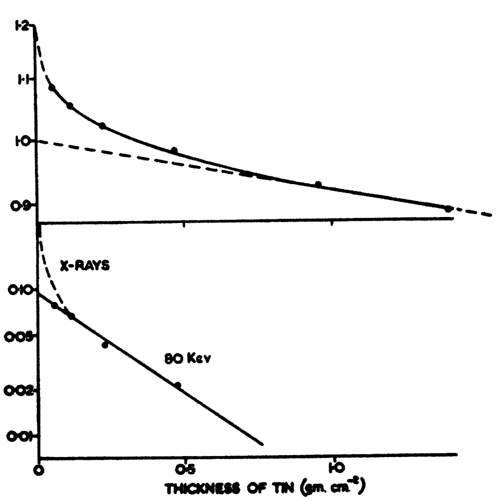

FIG. 2. Absorption of I^{131} gamma-rays in tin.

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Capture of μ-Mesons in Heavy Elements*

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We have used large liquid scintillation counters[1] to study the processes associated with the stopping of negative μ-mesons in heavy elements. The reasonable efficiency of the liquid scintillators for gamma-rays and neutrons of a few Mev has permitted us to use the non-ionizing radiations following nuclear capture for mean life measurements in Cu and Sb.

The experimental arrangement is shown in Fig. 1. A coincidence circuit (resolving time 3 μsec) selects events (AS_1S_2-X); and for each such event the relative lag between S_1 and S_2 is measured by photographing an oscilloscope trace on which are presented the scintillator pulses and the output of a chronotron timing circuit.[2] Thus we look for events where a penetrating cosmic-ray particle

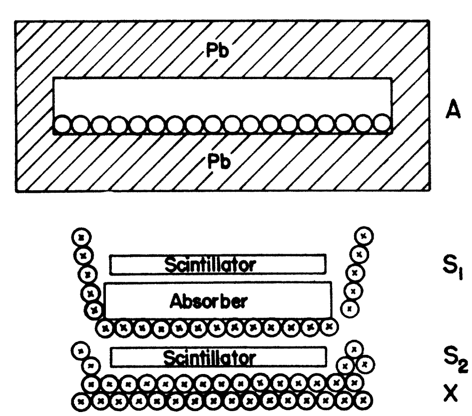

FIG. 1. Experimental disposition: Only the actual volume of scintillator liquid is shown; each tank is 12 in. by 12 in. by 1 in.