In writing the second member of (24) use is made of the fact that $U^a(t)$ is hermitian. The third member re-expresses the first with the aid of (19r) and its adjoint. The transition to the fourth involves (15). The fifth is reached with the aid of (19a) and its adjoint. Exploiting again Eq. (19r) we get to the sixth member of (24). Since $U^a(\infty)$ has always a reciprocal, we infer from (24) that

$$U^a(t)\bar{U}^a(t)=1. \tag{25a}$$

Applying the same identities in a somewhat different order and using (22) one similarly arrives at

$$U^r(t)\bar{U}^r(t)=1. \tag{25r}$$

Equations (25) and (8) imply the statement made at the beginning of the paragraph.

The author has enjoyed several instructive conversations with Dr. Hartland Snyder regarding the question discussed in this note.

* Research carried out at Brookhaven National Laboratory, under the auspices of the AEC.

[1] Equation (13) rather than (3) is essentially the definition adopted by Yang and Feldman for their "Heisenberg S-matrix." (Phys. Rev. **79**, 972, 1950.) The expressions

$$F(x)=U^r(t)F^{\rm in}(x)\bar{U}^r(t)\equiv F^{\rm in}(x)-i\int_{-\infty}^{t}dt'U^r(t')\,[H(t'),\,F^{\rm in}(x)]\,\bar{U}^r(t')$$

$$F(x)=U^a(t)F^{\rm out}(x)\bar{U}^a(t)\equiv F^{\rm out}(x)-i\int_{+\infty}^{t}dt'U^a(t')\,[H(t'),\,F^{\rm out}(x)]\,\bar{U}^a(t')$$

for the various Heisenberg field operators are equivalent to the integral equations employed by these authors. This may be seen by evaluating the commutator brackets explicitly for the various fields. The relation between $F^{\rm out}(x)$ and $F^{\rm in}(x)$ is then $F^{\rm out}(x)=\bar{U}^a(t)U^r(t)F^{\rm in}(x)\bar{U}^r(t)U^a(t)$.

[2] J. Schwinger, Phys. Rev. **74**, 1439 (1948).

[3] Equations (19) may be regarded as the definitions of the "Heisenberg reaction operator" in the same sense as (13) is taken to define the "Heisenberg collision operator." See reference 1.

The Mechanism of nD Capture

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THE thermal neutron-deuteron capture cross section has long been known to be very small (0.46×10^{-27} cm^2 for neutrons of velocity 2200 m/sec.[1] Several attempts at its calculation have been published, and the values obtained are: 3×10^{-27} cm^2 by Schiff,[2] 0.17×10^{-27} cm^2 by Höcker,[3] 0.73×10^{-27} cm^2 by Burhop and Massey,[4] while Verde[5] indicates that the cross section is very small but declines to give any numerical value.

It is generally considered that the H^3 wave function is mostly of ^2S character, and that capture from the continuum ^2S and ^4S states proceeds by the emission of a magnetic dipole gamma-ray. There are strong selection rules on this process. Two types of ^2S wave function must be distinguished, "symmetric," in which the neutrons are in a relative singlet state, and "antisymmetric," in which the neutrons are in a relative triplet state. Now if the radiation interaction involves only the spin magnetic moments of the nucleons then the transition can only link wave functions having the same neutron spin symmetry. Further, since ^2S wave functions of different energies are orthogonal, the magnetic transition must take place from the continuum ^4S state. Only the antisymmetric ^2S part of the ground-state wave function then has the neutron spin symmetry of the quartet state and can contribute to the matrix element of the transition. As this part of the ground state is believed to be very small, the capture cross section is correspondingly small. The above-mentioned authors all based their calculations on estimates of the amount of admixture. It is the purpose of the present note to indicate that their interpretation of the capture process is not a unique one.

Two additional mechanisms seem important: Magnetic dipole capture via the interaction moment[6] does not obey the selection rules which limited the possible transitions via the spin moments; electric quadrupole capture can occur from the continuum ^4S state to ground state ^4D terms.[7]

The capture cross section produced by the spin-antisymmetric interaction moment was computed for the present communication on the assumption that the ground state is a pure symmetric ^2S state. The matrix element for a transition between this state and the ^2S continuum state can be compared with the corresponding matrix element which appears in the calculation of the three-body magnetic moment anomaly. Then the numerical value of the capture matrix element follows from the known value of the three-body magnetic moment anomaly simply by renormalizing one wave function to an amplitude appropriate in the continuum. This method of approximation avoids any detailed assumptions about the interaction moment operator. The necessary renormalization was effected by comparison of the continuum and ground-state wave functions at the point of zero particle separation, a criterion which is reasonable because of the short-range nature of the interaction moment. In this manner a capture cross section was computed which was found to have about one-quarter of the experimental value. The transition involving the continuum ^4S state seems to contribute as much again, although its estimation is less reliable. Thus, in a first estimate, interaction moment magnetic dipole transitions to the main part of the H^3 wave function seem to yield about half the observed nD capture cross section.

The electric quadrupole capture cross section has been estimated very crudely, using the Gerjuoy-Schwinger value of 4 percent ^4D function. It is found nearly to equal the magnetic cross section, so that adding the two just gives the experimentally observed cross section. While this is certainly fortuitous, it does indicate that it is unnecessary to postulate an antisymmetric ^2S term in the ground state as the explanation of the capture process.

Accurate calculation of the quadrupole cross section would show what upper bound the small empirical value can set on the amount of ground-state ^4D function. It may be rather low. It is interesting that the interaction moment model of the three-body magnetic anomaly does not lead to any contradiction with the known small capture cross section.

This investigation was carried out as a direct consequence of conversations with Professor R. G. Sachs.

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[1] Sargent, Booker, Cavanagh, Hereward, and Niemi, Can. J. Research **A25**, 134 (1947).

[2] L. I. Schiff, Phys. Rev. **52**, 242 (1937).

[3] G. Höcker, Physik. Z. **43**, 236 (1942).

[4] Burhop and Massey, Proc. Roy. Soc. (London) **A192**, 156 (1947).

[5] Verde, Helv. Phys. Acta **XXIII**, 453 (1950); Nuovo cimento **7**, 283 (1950); **8**, 152 (1951).

[6] N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951); N. Austern, Phys. Rev. **81**, 307(A) (1951).

[7] The existence of such terms is known theoretically. Gerjuoy and J. Schwinger, Phys. Rev. **61**, 138 (1942); Pease and Feshbach, Phys. Rev. **81**, 142(L) (1951), also private communications from Dr. Pease.

Nuclear Magnetic Resonance Measurements on Be^9, Al^{27}, and Si^{29} in Beryl

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MEASUREMENTS have been made of nuclear magnetic resonance absorption in a single crystal of beryl ($Be_3Al_2Si_6O_{18}$) at a frequency of 2.3 Mc/sec and a temperature of 20°K.

The five lines due to electric quadrupole splitting of the Al^{27} resonance were clearly observed. With the hexagonal axis of the crystal parallel to the magnetic field, the separation between adjacent lines was 0.45 Mc/sec. Three lines due to Be^9 have been found. With the axis parallel to the field, the separation between the lines was 0.13 Mc/sec. A search was made for other lines in the regions where they would be expected to appear, if present, but nothing was observed. Since the signal-to-noise ratio for the outer lines of the triplet was greater than thirty to one, it seems certain that there were no other lines. This result is in agreement with the recent measurements of Schuster and Pake[1] and confirms the value $I=\frac{3}{2}$ for the spin of Be^9.

It is unfortunately not possible to obtain the quadrupole moment of Be^9 from the observed splitting since the electric field gradient at the nucleus is unknown. Since the observed quadrupole interaction energy $eQ\partial^2V/\partial z^2$ is about ten times smaller for Be than Al, and since the nuclei occupy fairly similar positions in the crystal,[2] it seems reasonable to suppose that the quadrupole moment of Be^9 is appreciably smaller than that of Al^{27} and probably of the order of 0.02×10^{-24} cm^2.

In addition to the lines due to Al and Be, a very weak resonance was also observed at a field corresponding to a g factor of 1.11. As there was no detectable shift of the resonance on rotation of the crystal axis with respect to the magnetic field it was due to a nucleus with very small or zero quadrupole moment. We have also observed the same resonance in two specimens of glass and we think it is probably the result of Si^{29}. According to the theory of Mayer[3] the spin of Si^{29} should be $\frac{1}{2}$, and experiments by Townes *et al.*[4] have shown that the quadrupole moment is small or zero. If we assume $I=\frac{1}{2}$, the magnetic moment is 0.55 nuclear magnetons which is in very good agreement with the value predicted for Si^{29} by the theory of Schawlow and Townes.[5]

[1] N. A. Schuster and G. E. Pake, Phys. Rev. **81**, 886 (1951).
[2] W. L. Bragg and J. West, Proc. Roy. Soc. (London) **A111**, 691 (1926).
[3] M. G. Mayer, Phys. Rev. **78**, 16 (1950).
[4] Townes, Mays, and Dailey, Phys. Rev. **76**, 700 (1949).
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The Effect of Scattering on Angular Correlation Measurements

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THE purpose of this note is to describe a simple method for correcting measurements of angular correlations (involving electrons) for multiple scattering in the source and for the finite solid angle of the detectors, which will allow the experimenter to determine specific activities needed for such measurements.

Consider an e-γ-experiment, where $W(\theta)$ is the correlation function and $F(\alpha)$ is the scattering function. The probability that a γ-ray will enter $d\Omega_1$ and that any electron correlated to it by $W(\theta)$ will be scattered through an angle α and enter $d\Omega_2$ is

$$P=d\Omega_1 d\Omega_2\int d\Omega_\theta W(\theta)F(\alpha). \tag{1}$$

The experimental correlation may be determined by the tedious process of evaluating $F(\alpha)$ numerically, expressing α in terms of θ, φ, etc., and carrying out the integral (1) numerically. However, in general,

$$W(\theta)=(1/2\pi)\Sigma\tfrac{1}{2}(2l+1)a_lP_l(\cos\theta) \tag{2}$$

$$F(\alpha)=(1/2\pi)\Sigma\tfrac{1}{2}(2k+1)b_kP_k(\cos\theta). \tag{3}$$

By expressing $P_k(\cos\alpha)$, via the addition theorem, in terms of θ and of β, the angle between $d\Omega_1$ and $d\Omega_2$, one obtains

$$P=d\Omega_1 d\Omega_2[(1/2\pi)\Sigma\tfrac{1}{2}(2l+1)a_lb_lP_l(\cos\beta)]. \tag{4}$$

Similar expansions of $P_l(\cos\beta)$ and integrations over $d\Omega_1$ and $d\Omega_2$ yield:

$$P=S_1S_2(1/2\pi)\Sigma\tfrac{1}{2}(2l+1)a_lb_lg_lh_lP_l(\cos\theta), \tag{5}$$

where now θ is the angle between the centers of two circular counters of solid angle S_1 and S_2 and half-angles g_0 and h_0 respectively, and

$$g_l\equiv\int_0^{g_0}P_l(\cos\alpha)d(\cos\alpha)\Big/\int_0^{g_0}d(\cos\alpha) \tag{6}$$

with a similar definition for h_l. We note that S_1g_l is the coefficient in (3) for $F(\alpha)=1$, $0\leq\alpha\leq g_0$; $F(\alpha)=0$, $g_0\leq\alpha\leq\pi$, so that the solid angle correction is identical with a scattering correction. The bracket in (4) represents a new correlation function so that similar treatment extends the result to many "scatterings." Thus, given n events such that the $k+1$th event is correlated to the kth event by $W_k(\theta_k)=(1/2\pi)\Sigma\frac{1}{2}(2l+1)a_{lk}P_l(\cos\theta)$, then the correlation between the first and the nth event is just

$$W_n(\theta_n)=\frac{1}{2\pi}\Sigma\frac{2l+1}{2}\Big(\prod_{k=1}^{n-1}a_{lk}\Big)P_l(\cos\theta_n) \tag{7}$$

where θ_n is the angle between the first and the nth events.

(1) Equation (7) extends (5) to $e-e$ angular correlations. (This formula is valid only if the correlation between the $k+1$th and kth events, W_k, is independent of the previous events. Thus it should not give the angular correlation between the first and last events of a cascade transition.)

(2) The form of the correlation is unaffected by the presence of scattering or finite φ-symmetric detectors. For example, if only P_2 appears in the experimental data, (say $W(\theta)=1+A\cos^2\theta=1+aP_2$) then only P_2 is present in the correlation function. This is of interest since often the highest power of l appearing in $W(\theta)$ is of importance even when the a_l are not accurately known. (Where lens spectrometers are used to detect the electrons the form is still unaltered but g_1 and g_2, the acceptance angles of the spectrometer, replace 0 and g_0 in the limits in (6).)

(3) The multiple scattering coefficients b_l are just the correction factors for a_l so that one need not evidence $F(\alpha)$ explicitly. Usually only b_2 or b_2 and b_4 are needed. In the Goudsmit and Saunderson[1] treatment of multiple scattering

$$b_l=\exp[-q(1-C_l)/b_1]\equiv\exp(-mt) \tag{8}$$

where $q=Nst$ (N=atomic density; s=total single scattering cross section; t=effective source thickness) and c_l is just the coefficient of the P_l expansion of $f(\theta)$, the single scattering function. The reader is referred to this treatment for a discussion of assumptions inherent in (8) and for simple expressions for b_l in terms of electron energy, source thickness, atomic number, etc.

(4) Electrons originating near the surface of a source are scattered less than Eq. (8) indicates. (8) applies to a collimated electron beam traversing a foil. A first-order approximation valid when (8) is valid and applicable to thin sources would replace b_l by $(1-b_l)/mt$.

(5) Order of magnitude results using the above approximations: for a source thickness of 200 micrograms ($Z=52$; $A=120$; $E=100$ kev) $b_2=0.9$, $b_4=0.77$. For a half-angle of 18° $g_2=0.9$, $g_4=0.75$.

[1] S. Goudsmit and J. L. Saunderson; Phys. Rev. **57**, 24 (1940); **58**, 36 (1940).

Emission of Long-Range Particles in the Fast Neutron Ternary Fission of U-238 and Th-232

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FOLLOWING a detailed investigation of the long-range particles emitted in the slow neutron ternary fission of U-235 by the photographic plate method,[1] experiments were undertaken to determine whether similar phenomena occur in the fast neutron fission of U-238 and Th-232. Although Tsien *et al.*[2] reported that U-238 gave no such long-range particles when irradiated by neutrons from the bombardment of beryllium by 6.7-Mev deuterons and Tsien and Faraggi[3] obtained a similar result for Th-232, yet the observation of such a mode of photofission in uranium by Titterton and Goward[4] and in thorium by Titterton and Brinkley[5] threw doubt on the validity of the argument used by the French group to explain the absence of the long-range particles in their experiments.

In the present experiments two sets of plates, each including Ilford C_2 and D_1 emulsions, were loaded with uranium acetate and thorium nitrate respectively, as described elsewhere[4,5] and were exposed to neutrons of energy 2.5 Mev obtained from the