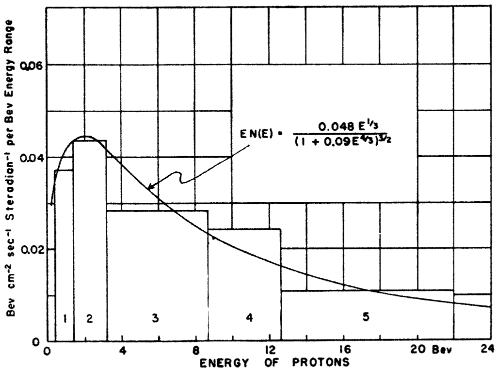

FIG. 1. Blocks 1, 2, 3, 4, and 5 result from differences in adjusted counter-telescope curves at various latitudes. The smoothed curve is a plot of an empirical relationship that fits the ionization data with experimental errors.

extrapolated behavior of known curves for lower minimum momenta of the primaries. This block is admittedly the least well determined of the set. The chief difference between the present results and those published by Bowen, Millikan, and Neher in 1938,[6] is in the abscissas.

An empirical expression that fits the experimental data is as follows:

$$EN(E)=0.048E^{1/3}/(1+0.09E^{4/3})^{3/2}, \qquad (1)$$

where E is measured in units of 10^9 ev. $EN(E)dE$ is the energy in Bev cm^{-2} sec^{-1} steradian^{-1} brought to the earth by protons whose energy lies between E and $E+dE$. The differential number distribution is then

$$N(E)=0.048/[E^{2/3}(1+0.09E^{4/3})^{3/2}]. \qquad (2)$$

The integral of this last equation, giving the numbers of primary particles with energies larger than E, is plotted in Fig. 2.

In justification of these expressions the following may be cited: (a) The expression (1) may be integrated directly and gives a total energy of 0.418×10^9 ev cm^{-2} sec^{-1} sterad^{-1} for all particles at the vertical in Peru. The experimental value is 0.413 in the same units. (b) A similar integration for Bangalore, India, yields 0.35 as against the experimental value 0.34. (c) From 0.4×10^9 ev to ∞, it gives 0.787 as compared with 0.774 for Saskatoon. (d) It gives a dependence on E of $E^{-2.67}$ for the differential number distribution at very large E. This is within the limits of the exponent found by Hilberry[8] to be necessary to explain extended showers. (e) It gives an effective dependence on E of $E^{-1.1}$ for the integral number spectrum in the range 2 to 12×10^9 ev. This is the distribution found necessary by Van Allen and Singer[9] to explain their results using rockets. (f) It gives a ratio in the total number of particles at 50°N and 30°N of 3.4, as compared with the value of 3.5 found for all primaries by Bradt and Peters.[10]

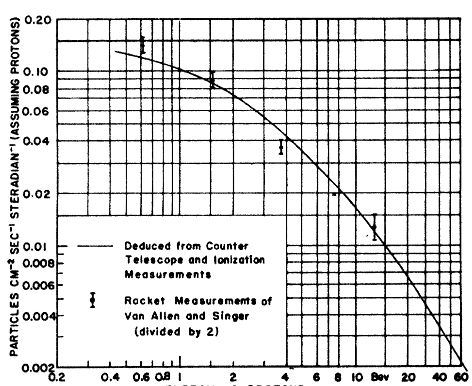

FIG. 2. The integral number distribution of the primary cosmic radiation deduced from the empirical relationship given in Fig. 1. For data of Van Allen and Singer, see reference 9.

The presence of particles heavier than protons in the primary radiation will affect only the constant in the numerator of Eqs. (1) and (2), provided the relative numbers of different particles are not dependent on the momentum, as it seems to be from the work of Bradt and Peters.[10]

The application of Liouville's theorem to be charged particles moving in the magnetic field of the earth implies that the found energy distribution of the primary cosmic-ray particles is also their distribution in space.

As has been pointed out by Van Allen and Singer,[9] and by Winckler *et al.*,[11] a discrepancy of about a factor of 2 exists between the numbers of primary particles determined directly near the top of the atmosphere and that found by taking the area under ionization curves. The, as yet undetermined, albedo effect will tend to make the directly measured value at high altitudes too large, while energy losses due to neutrinos will tend to make the numbers computed from ionization data too small. The small east-west effect measured at very high altitudes[9,11] is good evidence that the albedo, or general background, is important, at least at the Equator.

Further details are being published elsewhere.

* Assisted in part by the joint program of the ONR and AEC.
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Observations of Zener Current in Germanium *p-n* Junctions*

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IN 1934 Zener[1] published a theory of excitation of electrons directly from the valence band to the conduction band under the influence of high electric fields. For this purpose the energy gap of width $\mathcal{E}_G$ is treated as a region of negative kinetic energy in which the wave function is attenuated, so that the probability of penetrating the gap is approximately

$$f=\exp[-(\pi^2/h)(2m)^{\frac{1}{2}}\mathcal{E}_G^{\frac{3}{2}}/eE], \qquad (1)$$

where m is the effective mass and E the electric field; this formula differs from Zener's by being extended to larger energy gaps. The number of oscillations per second in the valence band is

$$\nu=eaE/h, \qquad (2)$$

so that the current per unit cell, containing z/a^3 electrons, is $e\nu fz$. If the field is uniform over a certain region and produces a voltage drop V, then the Zener current per unit area is

$$I=e^2Vzf/a^2h=V\exp[\alpha-(\beta/E)]$$
$$=V10^{(11-10^7E)}\text{amp/cm}^2, \qquad (3)$$

the last form corresponding to the constants for germanium and an effective mass equal to the electron mass, V being expressed in volts, and E in volts/cm.

Measurements of the Zener current have been made across *p-n* junctions in germanium, formed in a single crystal by using arsenic as the donor impurity and gallium as the acceptor.[2–4] Figure 1 shows the reverse $i-e$ characteristic of the junction plotted on a log-log scale over five decades of current. The critical voltage gradient across the junction was measured by determining the behavior of the capacitance of the junction against the reverse bias voltage. The slope of the $\log V$ *versus* $\log C$ plot for the junc-

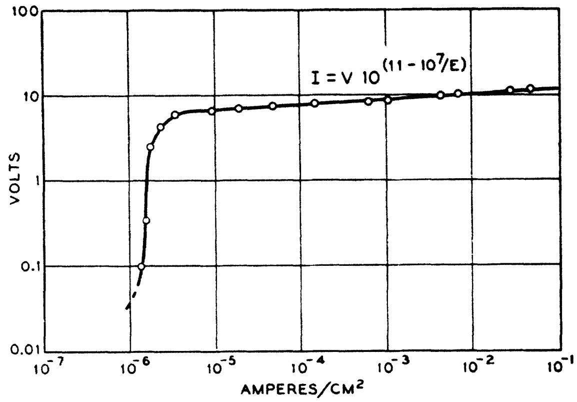

FIG. 1. Current-voltage characteristic for a *p-n* junction.

tion was 3, as is expected for a constant gradient of impurity concentration across the junction.[5] From the measured capacitance of the junction it is possible to calculate the maximum voltage gradient across the junction, which occurs at the midpoint.

The general behavior of the curve is in agreement with theory, and the rapid increase in current with voltage is consistent with Eq. (3), which gives

$$d\ln I/d\ln V = 1+(2\beta/3E) \qquad (4)$$

for $E\propto V^{\frac{1}{3}}$. For $V=8$ and $I=2.5\times10^{-3}$ Eq. (4) becomes 23, whereas the slope in Fig. 1 is 24. In agreement with out observations, the Zener current should cause no permanent damage, since it corresponds to field-induced generation of hole-electron pairs in the junction which are separated by the field, leaving an undisturbed valence bond structure.

The field for appreciable currents deduced from Fig. 2 is 2.2×10^5 volts/cm, whereas Eq. (3) gives 6.9×10^5 volts/cm. The discrepancy may be due to the poorness of the approximation (1). A group of junctions having critical voltages varying from 1 to 10^3 volts due to differences in concentration gradient all have critical fields of about 2×10^5 volts/cm. The high critical voltage junctions were measured under pulse conditions to avoid heating effects; heating is negligible in the low voltage units.

An alternative explanation of the "breakdown" phenomenon might propose the production of secondary electrons by the normal saturation current flowing through a critical field. This mechanism was eliminated by comparing the high field current when the specimen was dark and when illuminated sufficiently to change the low field saturation current[4] by a factor of about two. It was found that

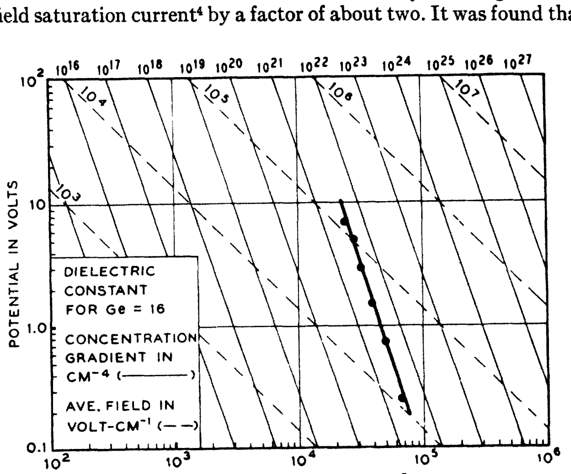

FIG. 2. Capacity-voltage characteristic for a *p-n* junction. (A dielectric constant of 16 is assumed.)

the increase in current was the same both below and above the critical breakdown voltage, i.e., that the photocurrent was not multiplied in traversing a barrier biased at greater than critical field strength.

Investigations are now being carried out to determine to what extent "patch effects" are important and to compare temperature and pressure coefficients with theory.

We are indebted to our colleagues H. R. Moore, G. L. Pearson, W. J. Pietenpol, G. K. Teal, and W. van Roosbroeck in connection with this study and to F. Seitz for a stimulating question regarding secondary currents.

* This material was presented on March 9, 1951, at the 304th meeting of the American Physical Society, Phys. Rev. **82**, 765 (1951).

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Erratum: Radioactivity of Ag^{111}, Cd^{111}, In^{111}, and Sn^{111}

[Phys. Rev. **81**, 734 (1951)]

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THE formulas of Eqs. (1) and (2) should read $N_{ee}/N_{e247}=\Omega x(1-e^{-\lambda t})$ and $N_{ee}/(N_{e172}+N_{e247})=2\Omega xy(1-e^{-\lambda t})/(x+y)$, respectively.

Asymptotic Expression of the Thomas-Fermi Function for a Packed Atom*

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THE Thomas-Fermi function $\phi(x)$ for an atom packed in a finite volume, i.e., the TF function with the initial slope $B<B_0$ ($=1.58808$), has recently become of interest because of its role in the theory of solids under high pressure. Since it is given hitherto merely numerically and only for several values of the atomic radius, its approximate expression in a closed form is strongly desired.

For a free atom ($B=B_0$) as well as for free ions ($B>B_0$), we have indeed the well-known Sommerfeld asymptotic formulas.[1] Now, since a packed atom probably has a finite radius, like an ion, it seems reasonable to treat it by a procedure wholly analogous to that used by Sommerfeld for an ion, provided that the indeterminate constant A in Eq. (36a) of the Sommerfeld paper is determined by means of an alternative boundary condition at the surface, $x=x_0$, of the atom:

$$\phi'(x_0)=\phi(x_0)/x_0. \qquad (1)$$

By this method, we have obtained a definitive asymptotic expression for the TF function for a packed atom with a finite radius x_0, in TF units,

$$\phi(x)=[1/(1+z)^{\lambda_1/2}]\{1+A_0[(1+z)/(1+z_0)]^{\lambda_1/\lambda}\}, \qquad (2)$$

where

$$A_0=(4z_0+1)/[(\lambda+3)z_0-1],\quad z\equiv(x/12^{\frac{1}{3}})^{\lambda},$$
$$\lambda_1\equiv7.772,\quad \lambda\equiv0.772,\quad \lambda_1/\lambda=10.067.$$

The coefficient A_0 converges to 1.06 with increasing radius and can be taken in practice as 1.2 for $x_0>5$.

In spite of the fact that our method seems at first sight to be not so successful, the values of Eq. (2) for $x_0>5$ are in a surprisingly good accord with the exact solutions computed numerically by Slater and Krutter.[2] For the atom packed as extremely as $x_0<2.5$, however, Eq. (2) departs too much to be useful, as would be expected, since for the smaller values of x_0 the starting assumptions, i.e., the adequacy of the asymptotic treatment and the