Thermal Conductivity of Liquid Helium. I

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(Received April 9, 1951)

Direct measurements have been made of the thermal conductivity of liquid helium over the temperature range from its normal boiling point at 4.2°K to a temperature within a few hundredths of a degree of the λ-point. The results show that the thermal conductivity decreases linearly with temperature from a value of 6.5×10^{-5} cal/deg cm sec at 4.2°K to about 4.5×10^{-5} cal/deg cm sec close to the λ-point. No large deviation from linearity can be detected in the heat conductivity until about 0.2 degree from the transition temperature of 2.18°K.

I. INTRODUCTION

THE thermal conductivity of liquid helium between 2.2°K and its normal boiling point at 4.2°K was first studied by Keesom and Keesom.[1] These authors only reported one quantitative result, *viz.*: 6×10^{-5} cal/deg cm sec at the temperature 3.3°K. The experiments by Pellam and Squire[2] on the ultrasonic absorption in liquid helium make it possible to calculate the dependence on temperature of the heat conductivity if the classical theory of sound attenuation is assumed valid over the entire temperature range of liquid helium I. Such a calculation predicts an enormous increase of the thermal conductivity starting at 3°K as the temperature is lowered to the transition point of liquid helium II. Thus, it was of interest to make a direct investigation of the thermal conductivity. The results clearly indicate that the heat conductivity remains nearly constant with temperature even quite close to the transition point.

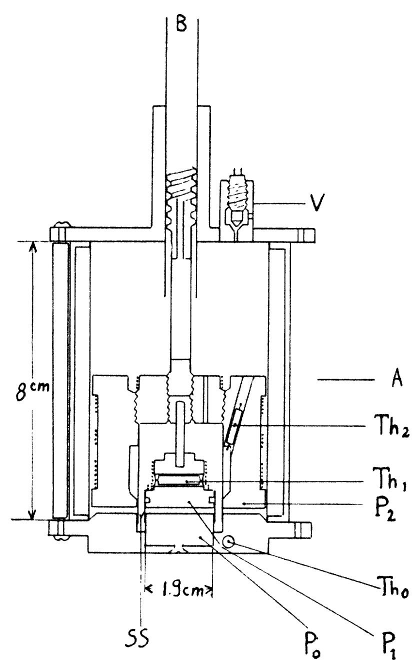

FIG. 1. Experimental apparatus for heat conductivity of liquid helium I: *A*—level of the effective temperature; *B*—tube leading to helium ballast tank; *V*—valve; *SS*—stainless steel walls; P_1—central plate; P_2—guard ring; P_0—lower plate; Th_0, Th_1, Th_2—carbon resistance thermometers.

II. APPARATUS

For this research an apparatus, Fig. 1, with horizontal plates P_0 and P_1 was constructed with a guard ring P_2. This is different from the apparatus of Ubbink and de Haas[3] in that the guard ring, P_2, extends around the sides of the central plate, P_1, as well as on top of it. Both P_0, P_1, and P_2 are made from aluminum whose thermal conductivity at these low temperatures is about 10^5 greater than liquid helium I. The liquid helium column between P_0 and P_1 is confined by a thin stainless steel wall, marked *SS* in Fig. 1. The wall thickness is 0.003 inch and the metal is 18–8 stainless steel. A similar stainless steel wall connects the base plate with P_2.

The central plate, P_1, was heated during the measurements by means of current passing through a coil of wire whose resistance at these low temperatures was 210 ohms. The wire was small diameter (size No. 35) cupron metal, and its resistance is very nearly independent of temperature in the liquid helium region. The guard ring, P_2, was similarly heated with a 450-ohm coil. Good thermal contact between the coil and aluminum metal was assured by a coat of lacquer. The coils were insulated thermally from the liquid helium by a layer of paper.

All these aluminum plates and guard rings are housed in a brass can having double walls (see Fig. 1). Between the walls there is helium gas at very small absolute pressure so that there is adequate thermal insulation across the double wall container. At the top of the brass can is a valve *V* which makes it possible to open the experimental chamber into contact with the liquid helium bath in the outer surrounding Dewar flask. The calibration of the carbon thermometers was done with the helium around P_0, P_1, P_2, in contact with the outer bath through the open valve. During the experiments the valve was closed and the pressure on the

[1] W. H. Keesom and A. P. Keesom, Physica **3**, 359 (1936).
[2] J. R. Pellam and C. F. Squire, Phys. Rev. **72**, 1245 (1947).
[3] I. B. Ubbink and W. J. de Haas, Physica **10**, 451, 465 (1943).

liquid helium inside the experimental chamber was kept very accurately constant. Most of the experiments were made with a pressure of 76 cm (barometric conditions) even though the temperature of the bath in the outer Dewar was cooled well below 4.2°K. In this way, the heat conductivity studies were made on the liquid helium between P_0 and P_1 in such way that the liquid helium was well away from the liquid-vapor transition. Thus no heat supplied to the liquid could produce a vapor. Some studies were made in which the pressure was held constant on the experimental liquid at as high as 3 atmospheres and for this a large tank (approximately 500 liters) was placed in the supply line so that fluctuations of pressure would be at a minimum.

The vapor pressure of the liquid helium in the outer bath was kept constant to 0.1 mm of Hg by means of a hand operated manostat and a differential oil manometer. These techniques are commonly practiced in low temperature laboratories.

The distance, d, between the plates was controlled from the outside by means of a screw having a pitch of $\frac{1}{32}$ inch per revolution (distances given by number of turns are to be multiplied by 0.79 mm). The rod from the top of the apparatus to the movable plate P_1 was of necessity a thin wall tube and so was flexible. This made the zero distance, corresponding to contact between P_0 and P_1, a little uncertain. Thus, when the outside vernier scale read zero, the plates were actually separated a distance, d_0.

The apparatus was constructed with the following factors in mind: (a) the thermometers must be sensitive and reliable, (b) the separation of plates P_0 and P_1 must be accurately known and must be parallel, (c) the flow of heat energy must be accounted for.

III. TEMPERATURE MEASUREMENT

The thermometers were of standard type resistors used in electronic circuits and which are composed of carbon surrounded by a ceramic or by an insulating lacquer. These thermometers are shown in Fig. 1 as Th_0, Th_1, and Th_2 where they give the temperature of the aluminum plates P_0, P_1, and the guard ring P_2. Simple wheatstone bridges were used to measure their resistance and the current was kept small; with a current of 10^{-5} ampere the heating was negligible and current fluctuations caused no trouble. The current was maintained constant by using compensatory resistances in the bridge. Thermal contact between the thermometer and the aluminum metal was made with paraffin.

Two factors limit the accuracy of the calibration curve, Fig. 2, giving electrical resistance as a function of temperature; (a) hysteresis effects when the thermometer is warmed back up to room temperature and then cooled to liquid helium, (b) precision of determining the temperature from the vapor pressure of the helium bath and taking into account the hydrostatic head of liquid helium above the location of the thermometer. As shown in Fig. 3, marked "B," there is a

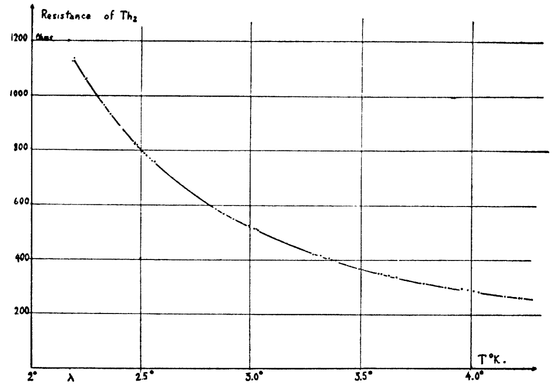

FIG. 2. Calibration curve of the carbon-resistor thermometer Th_2.

change of 30 ohms out of 1200 ohms at 2.17°K if the thermometer has been warmed back up to room temperature between measurements. The thermometers were therefore freshly calibrated each time they were cooled to liquid helium temperatures. After more than twenty different experiments the deviations due to hysteresis are only ±0.01°K. Any hysteresis effects while keeping the thermometers within the temperature range of liquid helium were too small to be significant.

In order to avoid any errors during an experiment, it was preferable to calibrate the thermometers at a given temperature over a range of approximately 0.1°K and make all heat measurements in this range. Under these conditions the electrical resistance thermometers are so accurate and reproducible that their error is smaller than other factors which enter into the heat flow measurements. We refer to Appendix I for more details on the thermometers.

The vapor pressure at the surface of the bath of liquid helium was measured by a mercury manometer and cathetometer. The vapor pressure determined the absolute temperature by using the 1947 tables prepared

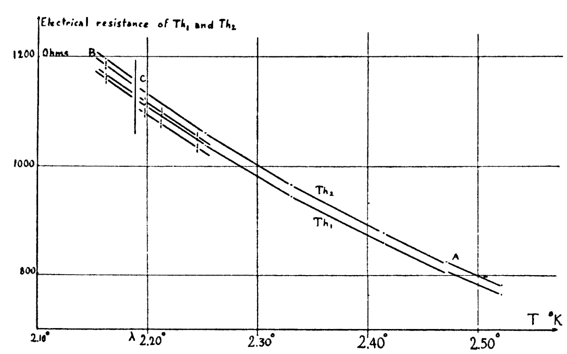

FIG. 3. Calibration curves of the thermometers Th_1 and Th_2 near the lambda-point. The dots represent experimental points. The dots marked with vertical bars represent a later measurement. (The thermometers returned to 290° between measurements.) A. Jump: bridge current changed from 5×10^{-5} to 10^{-5} amp. B. Hysteresis in carbon resistance thermometer (Sec. III). C. Jump in temperature: effect of hydrostatic pressure in liquid helium 1.

by the Mond Laboratory. The hydrostatic pressure due to the level of the bath above the thermometer requires a small correction. It was found that the brass experimental chamber was such a good heat conductor as to cause the thermometers to appear at an effective height of 4 to 5 cm above the bottom of the apparatus. Details on this point are left to Appendix II. The experiment on the heat conductivity of liquid helium requires that the difference of temperature between two thermometers be known with great precision, rather than great accuracy on the absolute value of the temperature. This considerably eased the task of making corrections due to the very slight change in level of the bath.

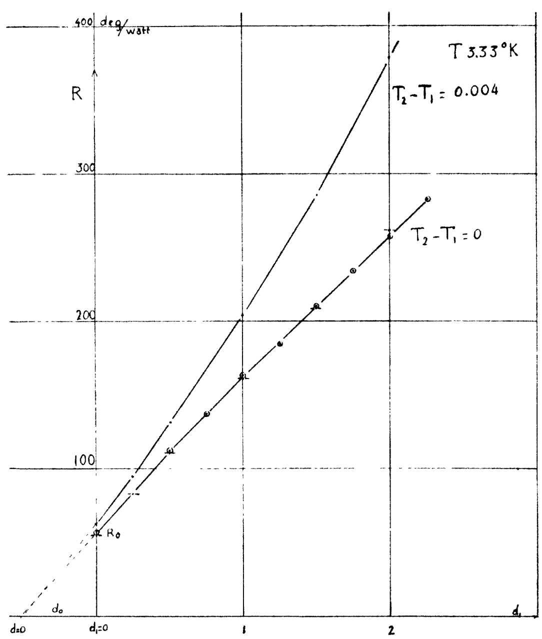

FIG. 4. Thermal resistance between the plates P_1 and P_0. The points on the lower curve are for $T_2-T_1=0$. The points on the upper curve are for $T_2-T_1=0.004$°K.

IV. OPERATION AND THEORY OF THE APPARATUS

As Ubbink and de Haas[3] have shown, one must measure the heat flow and temperature gradient for at least three different distances, d, of separation of plate P_1 from plate P_0, so that by extrapolation one can deduce the effective cross section of the helium column and other parameters of heat loss may be accounted for. In the present work a series of such measurements was made.

An experiment began with the calibration of thermometers. If a measurement on heat conductivity was to be made at a constant bath temperature, θ_0, then the thermometers were calibrated from a temperature 0.06°K above the final temperature on down to θ_0. With the pressure on the liquid helium within the experimental chamber held constant at 76 cm of Hg, the plates P_1 and P_2 were electrically heated until they remained at a constant temperature 0.06° above the temperature θ_0 of the plate P_0. The distance, d, was then changed and a new set of values of electric power in the heating coils, of temperatures, etc., was taken.

If the temperature of the guard ring, P_2, is equal to the temperature of the central plate, P_1, then all the power, E_1, generated in P_1 will flow to the bottom plate P_0. The gradient of temperature between P_1 and P_0 is $(T_1-T_0)/d$ and the effective cross-sectional area of the helium column is S so that the heat conductivity coefficient is:

$$K=\frac{E_1/S}{(T_1-T_0)/d}\ \text{watts/deg cm.} \tag{1}$$

The effective cross section, S, depends on the distance, d, and the latter has an uncertainty in absolute value. Let us examine this difficulty.

We define the thermal resistance between plates P_1 and P_0 by:

$$R=(T_1-T_0)/E_1 \tag{2}$$

which from the above equation defining the heat conductivity is also the ratio:

$$R=d/SK. \tag{3}$$

Now the distance $d=d_0+d_1$, so that we may write the ratio as:

$$R=d_0/SK+d_1/SK=R_0+R_1. \tag{4}$$

The distance, d_1, on the vernier scale of the movable screw mechanism is zero when the gap between the plates P_1 and P_0 is just d_0, which we shall see is about 0.5 mm. For d_1 equal to zero the thermal resistance is R_0 as is shown in Fig. 4 where we have plotted R against d_1. Extrapolation of R to zero defines experimentally the distance d_0 which is shown in Fig. 4.

The actual geometric area of plates P_0 and P_1 is S_1 which is smaller than the effective cross section, S, used in the expression for the heat conductivity, K. We shall define a preliminary heat conductivity, K_1, by use of the geometric cross section, S_1, as follows:

$$R_1=d_1/S_1K_1=\left(\frac{T_1-T_0}{E_1}\right)_{d_1}-\left(\frac{T_1-T_0}{E_1}\right)_{d_1=0}. \tag{5}$$

This preliminary value, K_1, is listed in Table I.

Examining Fig. 5 which shows schematically the two plates P_1 and P_0 with the stainless steel wall, we see the heat flux lines drawn for one-half of the cylinder. Some of the heat is conducted straight across the area S_1, some is conducted down the stainless steel wall, and some of the heat passes by the fringe of flux shown in the drawing. In this way we can write the heat conductance:

$$\frac{1}{R_1}=\frac{S_1K_1}{d_1}=\frac{S_1K}{d_1}+\frac{sK_{ss}}{d_1}+\frac{\Delta SK}{d_1} \tag{6}$$

where "s" is the area determined by the product of the stainless steel wall thickness by its circumference, K_{ss} is the heat conductivity of the stainless steel,[4] and "ΔS" is the equivalent cross section of the fringe flux. The term, K, is the true heat conductivity of liquid helium. Thus we write simply:

$$K_1 = K + (s/S_1)K_{ss} + (\Delta S/S_1)K. \tag{7}$$

We now assert that as the plates P_1 and P_0 get closer together the last term in the above expression becomes relatively less important. Thus for the plates touching, $d=0$, which is to say, $d_1 = -d_0$:

$$K_1 = K + (s/S_1)K_{ss}. \tag{8}$$

The true value of the liquid helium heat conductivity, K, is thus obtained by the extrapolation of K_1 to the distance $d=0$, and the small correction (10 percent at 4.0°K) term due to the stainless steel is then used.[5] Such extrapolation, as shown in Fig. 6, has the additional advantage of reducing to a minimum any error in the results caused by the guard ring, P_2, being at a slightly different temperature than the plate P_1. This latter point is brought out in Appendix III.

We are now in a position to understand why the stainless steel wall was used to confine the helium column. In the first place it cuts down on convection currents which for a liquid with such small viscosity can become a large disturbing factor in heat flow measurements. Less important reasons are that (a) it permits the plates P_0 and P_1 to be more nearly parallel, and (b) the fringe flux is reduced.

V. RESULTS

The values of the heat conductivity, K, for liquid helium between the temperatures 4.15°K and 2.27°K are given in Table I and they are plotted in Fig. 7. These values come from the extrapolation of the experimental results with the correction of the stainless steel wall taken into account as explained in Sec. IV of this paper. Taking into account the errors involved in extrapolating experimental values and the errors discussed in the Appendix, the absolute values of the heat conductivity are within 10 percent accurate. Figure 7 shows that the heat conductivity decreases with temperature to 2.4°K and shows a very small increase below that temperature towards the transition point at 2.18°K. The points on the curve of Fig. 7 have lines to each side and parallel to the temperature axis which indicate the range of temperature over which the measurement was made.

[4] The values of the heat conductivity of stainless steel 18–8, K_{ss}, were reported by J. E. Zimmerman of The Carnegie Institute of Technology at the Cryogenics Conference held by the ONR at the Georgia Institute of Technology, March 1950. These values are reproduced in Table I.

[5] The distance, d_1, used for the stainless steel wall is slightly shorter than the actual distance because of the construction of the apparatus. The correction for the heat conduction by the stainless steel is, therefore, an upper limit and the helium heat conductivity in Table I represents a lower limit.

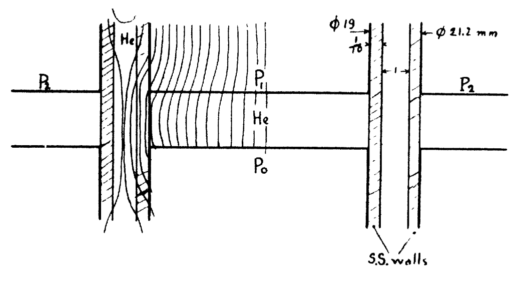

FIG. 5. Schematic representation of the thermal flux lines between P_1 and P_0.

VI. DISCUSSION AND CONCLUSIONS

At the temperature 3.3°K we have very good agreement with the value given by Keesom and Keesom.[1] The experimental values of K are in accord with the calculated values from the classical theory of sound absorption used by Pellam and Squire[2] for liquid helium above 3.0°K. Below 3.0°K the ultrasonic absorption becomes so large that the heat conductivity would have to increase enormously and the present experiments indicate that this does not happen. We must conclude that the classical theory of sound attenuation, which permits the sound energy to be absorbed by viscosity and by heat conduction, is no longer valid for liquid helium in the temperature range below 3.0°K. Table II compares these results.

In Table II the gas theory is used to compute the heat conductivity; $K = 2.5\eta c_v$, where the specific heat per gram at constant volume is taken from the work of Keesom and Clusius[6] and the viscosity, η, is taken from the recent results of Bowers and Mendelssohn.[7] The best agreement, so far as temperature dependence is concerned, is between the gas theory and the experimental work reported here. As this manuscript was being prepared, the preliminary results of Bowers and

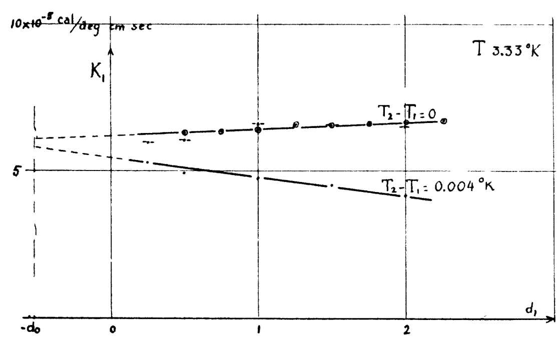

FIG. 6. Uncorrected conductivity of liquid helium and extrapolation to the distance $d=0$. The points on the upper curve are for $T_2 - T_1 = 0$. The points on the lower curve are for $T_2 - T_1 = 0.004$°K.

[6] W. H. Keesom and K. Clusius, Comm. Leiden **192**; W. H. Keesom, *Helium* (Elsevier Press, New York (London), 1942), p. 218.

[7] R. Bowers and K. Mendelssohn, Proc. Phys. Soc. (London) **A62**, 394 (1949).

TABLE I. Heat conductivity in liquid helium.

$\frac{T_0+T_1}{2}$ °K	T_0-T_1 in °K/1000	T_2-T_1 in °K/1000	Pressure in cm of Hg	R_0 °K/w	K_1 in cal/°K cm sec, d_1 in turns—1 turn = 0.79 mm: $\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	K_{ss} $\frac{\text{cal}}{\text{°K cm sec}}$	K $\frac{\text{cal}}{\text{°K cm sec}}$
2°.24−	37	0	76	80.5		5.51	5.75		5.85		15	4.8
...	...	+1.6	...	75.4		5.63	6.05		6.53		...	4.9
...	...	+3.2	...	70.5		6.32	6.58		7.58		...	5
2°.24+	50	(−)	...	78.7		4.67	4.23		4.60	4.59	...	4.4
2°.30	70	(−)	...	74.4		5.07	5.13		5.04		16	4.6+
2°.38	75		...	75.1		5.04	5.17		5.05		17	4.6−
2°.45	53		...	74.2		4.85	5.13		5.24	5.11	18	4.4
2°.50	47		...	71.9		5.23	5.50	6.00	5.51	5.77	19	4.6
2°.85	19		...	67.0	6.05	5.67	6.14		5.96		25	5.2
2°.86	37	(+)	...	60.0	5.82	6.01	6.31	6.58	6.68		...	5.0
2°.87	57		...	61.6	5.91	5.99	6.00	6.07	6.14		...	5.2
2°.88	80		...	60.1	5.83	5.81	6.14	6.39	6.39		...	4.9
3°.33	80		95	57.7		6.27	6.37		6.39	6.75	33	5.4−
...	...		140	58.6		6.37	6.85		6.82	6.83	...	5.8
...	...		76	57.7		6.31	6.43	6.62	6.75		...	5.4
...	20		...	68.7	6.51	6.26	7.17	6.93	6.79		...	5.6
3°.34	52	−4.	...	62.6	5.29	4.90	4.74	4.52	4.24		...	5.1
3°.35	70		...	54.9	5.94	5.97	6.66	6.55	6.52		...	5.2
3°.65	31		...	55.1		7.34	7.03		7.30		38	6.4
3°.63	64		...	51.0		7.34	7.26		7.18		...	6.5
4°.18	50		205	47.7		7.23	7.41		7.34		48	6.3
...	...		145	46.1		7.10	7.10		7.10		...	6.1
...	...		115	46.3		7.15	7.05		7.05		...	6.2
...	67		93	42.6		7.72	8.70		9.43	10.13	...	

Mendelssohn[8] on the heat conductivity of liquid helium appeared in *Nature*. Their work is in agreement with our experimental values.

We have indicated in the text of Sec. IV that the plates P_0 and P_1 are separated by a very small distance, d_0, which is about 0.5 mm when the vernier scale reading on the external screw is zero. If we take the average value of d_0 to be 0.53 mm then the thermal resistance R_0 can be considered to be caused by the liquid helium in that distance. The heat conductivity, K_0, which gives this value of R_0 for the distance d_0, is listed in the last row of Table II. These values are in good accord with the values K obtained by the experimental methods described in Sec. IV. At temperatures below 2.4°K, the values of K_0 may even be more accurate than the values of K. The values of K_0 appear on Fig. 7 as single points.

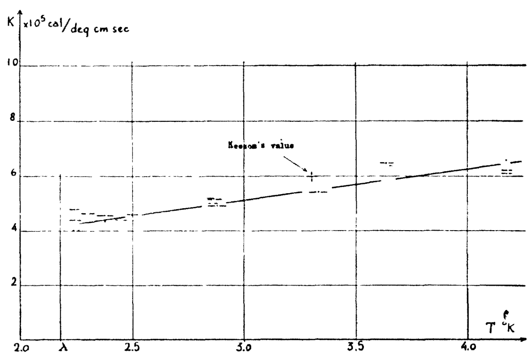

FIG. 7. Heat conductivity of liquid helium I. The horizontal bar through an experimental point indicates the temperature range in which the measurement was made.

[8] R. Bowers and K. Mendelssohn, Nature 167, 111 (1951).

Table I shows that with a change of pressure on the liquid there was no observable change in the heat conductivity. In most respects liquid helium between 2.2°K and 4.5°K behaves more like a gas than a normal liquid. It is well understood that this is caused by quantum effects of the uncertainty principle which tend to keep the atoms separated from one another.

The author is pleased to acknowledge the research guidance of Professor Charles F. Squire and for the very considerable assistance given generously by the members of the low temperature laboratory. During the research period, the author held a fellowship given by The Rice Institute to a student from France.

APPENDIX I

Referring to Fig. 4 which gives the thermal resistance as a function of the distance of separation of the plate P_0 and P_1, we see that the values obtained have a very small scatter about the curve for $T_2-T_1=0$. One obtains an entirely different curve, for example, with the guard ring temperature slightly higher than the central plate P_1; thus with $T_2-T_1=0.004$°K a new curve is shown in Fig. 4. We may therefore conclude that the stability of the thermometers as indicated by the small scatter of points on the ideal curve is such that the same scatter could be produced by the guard ring being about 0.0004°K higher than the plate P_1.

APPENDIX II

Referring to Fig. 3 which gives the electrical resistance of the carbon thermometer as a function of temperature, we see that at the λ-point of the liquid helium there is a sudden shift of the value of the electrical resistance. The magnitude of this shift was found to be dependent on the height of the liquid helium in the Dewar flask. One may understand that the temperature of the thermometer is influenced by the hydrostatic head of liquid helium, and that when the superfluid is obtained at the λ-point the enormous heat conductivity of the bath brings the thermometer to the temperature of the surface of the bath. Temperature gradients simply cannot be supported in the superfluid bath. Indirectly the shift in the resistance of the carbon thermometers was a measure of the height of the helium level. Observing the height of the level through the clear windows of the Dewar flasks made it clear that the actual level of the carbon thermometers was effectively raised to about 5 cm above the bottom of the brass apparatus because of the high heat conductivity of the brass metal.

APPENDIX III

Suppose that not all of the power, E_1, produced in plate P_1 went to the plate P_0, but that a small quantity, e, went to the guard ring because of a temperature difference T_2-T_1. The error caused by this heat flux will be minimized as the distance of separation of the plates P_1 and P_0 vanishes. To see this let us define the thermal resistance of the path between P_1 and the guard ring as "r," such that:

$$r=(T_2-T_1)/e. \tag{9}$$

The ratio of the power, e, which goes to the guard ring to the power, E_0, which goes to the plate P_0 will then be:

$$e/E_0=[(T_1-T_2)/r]\times[R/(T_1-T_0)]. \tag{10}$$

As the thermal resistance R of the helium column between P_1 and P_0 vanishes with the distance $d=0$, we may conclude that the ratio becomes small and that the relative importance of the small heat leak to the guard ring is minimized. The apparatus was constructed in such a way as to make the thermal resistance. r, as large as possible.

TABLE II. Comparison of results on heat conductivity.

T°K	$10^5\times K$(exp) cal/deg cm sec	K Keesom	K Sound absorb.	$2.5c_v\eta$ Elementary gas theory	K_0 Zero resistance
2°.24−	4.8				4.05
2°.24+	4.4			3.7	4.15
2°.30	4.6+		100		4.4
2°.38	4.6−				4.35
2°.45	4.4				4.4
2°.50	4.6			3.2	4.6
2°.58			17		
2°.86	4.9 5.2				5.2 5.3
3°.08			5.5	3.3	
3°.35	5.1 5.7	6	6		5.4 5.3
3°.50			7	3.6	
3°.63	6.4		6.7		5.6 6.1
4°			5.5	4.4	
4°.2	6.1 6.3		5		6.4 6.6

PHYSICAL REVIEW VOLUME 83, NUMBER 3 AUGUST 1, 1951

Injected Light Emission of Silicon Carbide Crystals

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(Received April 5, 1951)

Recombination of carriers injected through $P-N$ boundaries in silicon carbide crystals may lead to light emission ("injected light emission"). This light emission was investigated as a function of temperature and of current through the crystal by use of a photomultiplier. The emission spectrum extends from 4500A to 6500A at room temperature and is found to be nearly independent of current from 0.1 ma to 50 ma. The light intensity increases approximately proportionally to current (efficiency about 10^{-6} quanta per electron at room temperature for a particular crystal).

I. INTRODUCTION

SOME silicon carbide crystals emit "cold" light while current passes.[1–7] Two types have been reported: (a) a bluish light and (b) a yellow light, the type emitted depending on the direction of current flow. The parts of the crystal that emit yellow light do not coincide, in general, with those emitting blue light on current reversal; nor does the same crystal necessarily emit both types. We conclude that the mechanism of excitation differs for the two cases.

Previously published data on the intensity and spectral distribution of the light are of a qualitative nature. In this paper quantitative information is presented on the spectral distribution of the yellow light and its dependence on current density and temperature.

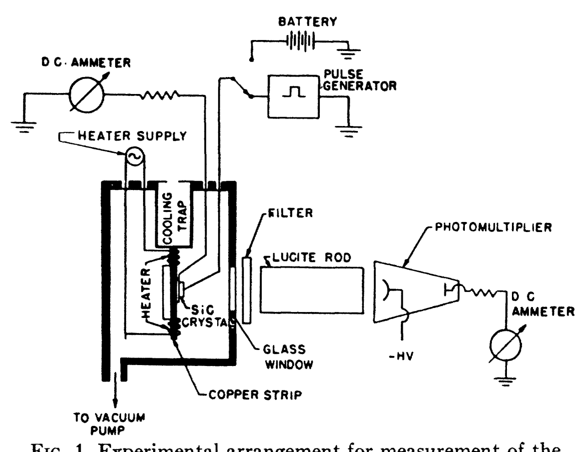

FIG. 1. Experimental arrangement for measurement of the light emission from silicon carbide crystals.

[1] O. Lossew, Wireless World and Radio Review **271**, 93 (1924).
[2] O. Lossew, Z. Fernmeldetechnik **7**, 97 (1926).
[3] O. Lossew, Phil. Mag. **6**, 1028 (1928).
[4] O. Lossew, Physik. Z. **30**, 920 (1929).
[5] O. Lossew, Physik. Z. **32**, 692 (1931).
[6] O. Lossew, Physik. Z. **34**, 397 (1933).
[7] B. Claus, Ann. Physik **11**, 331 (1931).