results in a space charge reduction of the field in the crystal. Since the electrons are probably more mobile than the holes, the temperature dependence of photoconductivity probably represents the variation with temperature of the rate of release of electrons from traps.

6. The excitation process responsible for the observed light absorption may be either the creation of a free electron and hole or the creation of an exciton. Mott[24] has attributed the first (lowest energy) fundamental absorption bands in the alkali halides to exciton creation. Such excitons in BaO would probably be thermally dissociated at room temperature.[25] The present experiments cannot distinguish between a band-to-band electronic transition, which enables an electron to move freely immediately after the absorption of light, and a transition to an exciton state, which would be quickly dissociated to yield an electron moving freely. Experiments with single, short light pulses (to reduce the effects of space charge) should be performed down to liquid hydrogen temperatures to distinguish between these processes.

Another way in which photoconductivity could result from the creation of excitons has recently been demonstrated by Apker and Taft.[26] They showed that excitons produced by absorption in the first fundamental absorption band in KI may ionize *F*-centers, giving rise to electrons in the conduction band. Such a process may contribute to photoconductivity in BaO, competing with thermal dissociation of excitons in the production of conduction electrons.

VI. ACKNOWLEDGMENTS

The authors are indebted to Dr. Arnold R. Moore and Mr. S. S. Stevens for valuable assistance in this work and are especially grateful for many discussions with Professor J. A. Krumhansl on the interpretation of these experiments.

[24] N. F. Mott, Trans. Faraday Soc. **34**, 500 (1938).

[25] Mott has estimated in Proc. Phys. Soc. (London) **A167**, 384 (1938), the extent to which dissociation of excitions may contribute to photoconductivity. If one inserts the value 4 for the optical dielectric constant and 34 for the static dielectric constant (recently measured in this laboratory) into Mott's estimation (p. 390), one finds that temperatures much below liquid air temperature would be required to prevent dissociation of excitons in BaO. This contrasts with the behavior of NaCl, which has a lower dielectric constant and in which optical absorption without photoconductivity has been observed at room temperature by Ferguson (Phys. Rev. **66**, 220 (1944)).

[26] L. Apker and E. Taft, Phys. Rev. **79**, 964 (1950).

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Low Energy Photomeson Production in Hydrogen

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Berkeley experiments have revealed the general features of the neutral and charged π-meson photoproduction cross sections for gamma-rays incident on protons. In particular, approximate equality of the π^0 and π^+ production cross sections has been observed. This has proved difficult to account for theoretically. Calculations based on a weak coupling perturbation treatment of the meson-nucleon interaction predict the π^0 production to be reduced by $\sim(\mu/M)^2$ relative to π^+ production for energies at which the Berkeley experiments are performed ($\sim$320 Mev). In this paper we avoid the perturbation theory approximation of weak nucleon-meson coupling. By means of a canonical transformation of the type introduced by Bloch and Nordsieck, we obtain a solution to the hamiltonian equation that involves no assumption concerning the largeness or smallness of $g^2/4\pi$. The approximations involved are neglect of nucleon recoil and treatment of spin and charge matrices as classical unit vectors. Handling the electromagnetic field by standard perturbation theory, we use this solution to calculate the matrix elements for photoproduction. In evaluating the matrix elements we must use a finite source cutoff to compensate neglect of nucleon recoil. We obtain qualitative agreement with the observed equality of π^0 and π^+ production cross sections with a choice of $g^2/4\pi\sim1-2$ and of source radius of the order of a nucleon Compton wavelength. Calculation of the anomalous nucleon magnetic moments corroborates a choice of these constants in this range. We also obtain an angular distribution and energy dependence for the production cross sections in accord with observation.

I. INTRODUCTION

EXTENSIVE experimental work at Berkeley has established the following general features of the photomeson production process, for gamma-rays of energy up to 320 Mev incident on protons:

1. The cross sections for production of neutral and charged pi-mesons by 320-Mev bremsstrahlung are approximately equal, ($\sigma_T\sim10^{-28}$ cm^2).[1,*]

2. The production cross sections are approximately isotropic in the center-of-mass frame.[2]

3. The excitation function for the production of neutral mesons is steeper than for the production of charged mesons.[3]

This paper attempts to arrive at an understanding of these features of neutral and charged photomeson production within the framework of the present formulation of meson field theory. From the point of view of

[1] Steinberger, Panofsky, and Steller, Phys. Rev. **78**, 802 (1950). J. S. Steller and W. K. H. Panofsky, Phys. Rev. **81**, 649 (1951). I wish to thank Professor R. Serber for calling my attention to this result.

* *Note added in proof.*—Later data indicate a reduction in π^0 production by a factor of roughly three relative to π^+ production.

[2] Steinberger, Panofsky, and Steller, Phys. Rev. **78**, 802 (1950); K. A. Brueckner, Phys. Rev. **79**, 641 (1950) (see p. 645).

[3] J. S. Steller and W. K. H. Panofsky, Phys. Rev. **81**, 649 (1951); J. Steinberger and A. Bishop, Phys. Rev. **78**, 494 (1950).

meson theory these are very simple processes, involving only the interaction of a single nucleon and its associated meson field with quanta. Thus, a theoretical understanding of the Berkeley observations would be very desirable. It is apparent from the experimental observation 1 (large magnitude of the π^0 production cross section) that any theoretical attempt at an explanation of this process must lie outside the perturbation-weak coupling approximation. This is because there is no direct interaction between neutral mesons and electromagnetic radiation. For charged meson production the gamma-ray can directly interact with and eject a π^+ meson, in a lowest order perturbation calculation ($\sim e^2g^2$). For neutral meson production in this same order the process is described by an interaction of the gamma-ray with the proton magnetic moment, followed by π^0 emission[1,4] For gamma-energies up to several times the production threshold of roughly 150 Mev, we would expect the π^0 photoproduction cross section to be reduced relative to that for π^+ mesons by $\sim(\mu/M)^2$. Perturbation calculations bear out this prediction.[5]

However, we may avoid the assumption of weak coupling between the proton and meson field and view this process along lines more closely related with the notion of the compound nucleus, as applied to the calculation of low energy photoprocesses in heavy nuclei. The physical nucleon—a Dirac particle with its associated meson field—is described in its initial state by momentum, spin, and isotopic spin vectors $\mathbf{p}_i$, $\boldsymbol{\sigma}_i$, and $\boldsymbol{\tau}_i$. After absorbing a gamma it emits a π-meson, recoiling with $\mathbf{p}_f$, $\boldsymbol{\sigma}_f$, and $\boldsymbol{\tau}_f$, as directed by conservation of energy, momentum, angular momentum, and charge. The difference between π^+ and π^0 production is derived from the fact that the proton changes its charge state upon emitting a π^+, but $\boldsymbol{\tau}$ remains unchanged for π^0 emission. However, if the meson field couples with the nucleon spin, we may expect an appreciable spin change $\Delta\boldsymbol{\sigma}$ to accompany both the π^+ and π^0 photoproduction processes and hence comparable cross sections for them. Nucleon velocity effects are considerably smaller at this energy. This is analogous to the considerations that apply in treating the (γ,p) and (γ,n) processes in heavy nuclei.[6] In the nuclear photoprocess, on the other hand, there is a coulomb barrier that is responsible for a drastic reduction in the number of protons emitted, relative to the number of neutrons. No coulomb effects operate in this process of meson production from protons. On the basis of the preceding discussion, we are motivated to renounce perturbation theory, at least as far as meson-nucleon coupling is concerned. In order to obtain the desired coupling of the meson field with the nucleon spin, we resort to a pseudoscalar formulation of the meson field. Brueckner's[2] analysis of the charged meson production data argues strongly for this choice. Our calculation of the π^0 production process on the basis of a scalar meson field, which does not directly couple with the nucleon spin, further substantiates it by predicting much too small a cross section. Yang[7] and others have presented convincing arguments against a vector meson field in a charge symmetric formulation.

[4] G. Araki, Prog. Theor. Phys. **5**, 507 (1950).

[5] K. A. Brueckner and K. M. Watson give a more complete discussion of this point in Phys. Rev. **79**, 187 (1950).

[6] MDDC 1175 (LLA 24), U. S. Government Printing Office.

II. DESCRIPTION OF CALCULATIONS AND RESULTS

We give a brief description of the method, approximations, and results of this calculation. A nucleon, described by the one-particle Dirac equation, is coupled with a quantized, charge symmetric pseudoscalar meson field. Pseudovector coupling is assumed. No approximation is made concerning the largeness or smallness of the coupling strength $g^2/4\pi$. Following the Bloch-Nordsieck[8] argument, we replace the hamiltonian by its positive energy part. The approximation involved here is neglect of the change in the nucleon velocity upon interaction with the quantum and meson fields.[9] Treating the spin and charge (isotopic spin) vectors $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ classically, we can write down an exact solution to the equation of motion. The wave function is obtained by means of a canonical transformation of the type introduced by Bloch and Nordsieck[8] for the radiation field, and applied by Lewis, Oppenheimer, and Wouthuysen[10] to meson problems.

Handling the electromagnetic field by standard perturbation theory, we use this solution to calculate the matrix elements for transitions from an initial state in which one gamma-ray is present to a final state with one free meson. The results are readily compared with Foldy's[11] perturbation calculation of π^+ production on the basis of pseudoscalar meson theory with pseudovector coupling, in which nucleon recoil is neglected. Expanding our matrix element for π^+ production in a power series in g, we find that the leading term, of the order eg, agrees with Foldy's calculation. The matrix element we calculate for π^0 production is of the order eg^3. It diverges linearly unless a finite-source cutoff is used to compensate the neglect of nucleon recoil. We obtain qualitative agreement with the observed equality of π^0 and π^+ photoproduction cross sections with a choice of nucleon-meson coupling constant of the order of unity ($g^2/4\pi\sim1-2$) and of the source radius of the order of the nucleon Compton wavelength. In agreement with observations 2 and 3 above, the calculated cross sections are roughly spherically symmetric in the center-of-mass system, and the excitation function for π^0 production is steeper than for π^+ production. Brueckner[2] has given a detailed discussion of the rela-

[7] C. N. Yang, Phys. Rev. **77**, 242 (1950).

[8] F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937).

[9] The effect of recoil is reduced by the order of the meson, proton mass ratio for energies at which the Berkeley experiments operate.

[10] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).

[11] L. L. Foldy, Phys. Rev. **76**, 372 (1949). See also Feshbach and Lax, Phys. Rev. **76**, 134 (1949).

tion between the angular distribution and the nature of the interaction of the gamma-ray with the nucleon spin, or moment, on the basis of a pseudoscalar meson field theory. The difference in behavior of the excitation function for charged and neutral meson production indicates that different production mechanisms dominate near threshold. For π^+ production the quantum field interacts with the meson charge and nucleon spin, as indicated by the first term of Eq. (6) following. This matrix element is independent of emitted meson momentum, and the cross section increases linearly with the π^+ momentum. This mechanism does not operate for π^0 production. In the latter case the nucleon spin recoils as the emitted π^0 field carries away angular momentum. Hence, the matrix element varies with a first power and the cross section with the cube of the emitted π^0 momentum at threshold.

We also use the same calculational procedure to determine the anomalous nucleon magnetic moments. We wish to learn from this calculation if we obtain qualitative agreement with the observed value of $|\Delta\mu_N|\sim 2\mu_0$ with the choice of $k_{\max}$ and $g^2/4\pi$ made in the photoproduction calculation. Within the uncertainties in the values of these parameters and in our classical treatment of spin, we find that we do.

III. CALCULATION

A. Wave Funtion

We write the hamiltonian for a Dirac particle of mass M' and momentum $\mathbf{p}$, interacting via pseudovector coupling with a charge symmetric pseudoscalar meson field,

$$H=\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M'+\tfrac{1}{2}\sum_{\alpha=1}^{3}\sum_{\mathbf{k}}\omega_k(P_{\mathbf{k}\alpha}{}^2+Q_{\mathbf{k}\alpha}{}^2)+(g/2^{\frac{1}{2}}\mu L^{\frac{3}{2}})\sum_{\alpha=1}^{3}\sum_{\mathbf{k}}(\boldsymbol{\sigma}\cdot\mathbf{k}+\omega_k\gamma_5)(\tau_\alpha/\omega_k^{\frac{1}{2}})\times(Q_{\mathbf{k}\alpha}\cos\mathbf{k}\cdot\mathbf{r}-P_{\mathbf{k}\alpha}\sin\mathbf{k}\cdot\mathbf{r}).\quad(1)$$

In Eq. (1), $P_{\mathbf{k}\alpha}$ and $Q_{\mathbf{k}\alpha}$ are the canonically conjugate momentum and amplitude for mode $(\mathbf{k}\alpha)$ of the meson field. They are defined by a fourier expansion of the meson field,

$$\varphi_\alpha(\mathbf{r})=(1/L^{\frac{3}{2}})\textstyle\sum_{\mathbf{k}}(1/\omega_k^{\frac{1}{2}})(P_{\mathbf{k}\alpha}\cos\mathbf{k}\cdot\mathbf{r}+Q_{\mathbf{k}\alpha}\sin\mathbf{k}\cdot\mathbf{r}).\quad(2)$$

The meson mass is μ; ω_k is $(k^2+\mu^2)^{\frac{1}{2}}$; $g^2/4\pi$ is the dimensionless coupling constant, L^3 the normalization volume; $\boldsymbol{\alpha}$, β, $\boldsymbol{\sigma}$, and $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$ are the usual Dirac matrices; τ_α is the isotopic spin vector; $\alpha=3$ describes the neutral charge state of the meson field. We use units of $\hbar=c=1$. We neglect the nucleon recoil by replacing the velocity operator $\boldsymbol{\alpha}$ by the value $\mathbf{v}$ of the nucleon velocity. If we assume that the nucleons travel with slow, nonrelativistic velocities $v<1$, as is the case for the Berkeley experiments, we may in addition replace β by one and neglect the γ_5 term in the interaction. In this approximation, the hamiltonian expression (1) reduces to[12]

$$H=\mathbf{v}\cdot\mathbf{p}+M'+\tfrac{1}{2}\sum_{\alpha,\mathbf{k}}\omega_k(P_{\mathbf{k}\alpha}{}^2+Q_{\mathbf{k}\alpha}{}^2)+(g/2^{\frac{1}{2}}\mu L^{\frac{3}{2}})\times\sum_{\alpha,\mathbf{k}}(\tau_\alpha\boldsymbol{\sigma}\cdot\mathbf{k}/\omega_k^{\frac{1}{2}})(Q_{\mathbf{k}\alpha}\cos\mathbf{k}\cdot\mathbf{r}-P_{\mathbf{k}\alpha}\sin\mathbf{k}\cdot\mathbf{r}).\quad(3)$$

If we treat the spin and charge vectors classically, so that we can commute the different components of $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ among themselves, we can write down an exact solution to the wave equation,

$$H\psi(Q,\mathbf{r})=E\psi(Q,\mathbf{r}).$$

The normalized eigenfunction is

$$\psi(Q,\mathbf{r})=L^{-\frac{3}{2}}\exp i\{(g/2^{\frac{1}{2}}\mu L^{\frac{3}{2}})\sum_{\mathbf{k},\alpha}(\tau_\alpha\boldsymbol{\sigma}\cdot\mathbf{k}/\omega_k^{\frac{3}{2}})\sin\mathbf{k}\cdot\mathbf{r}\times[Q_{\mathbf{k}\alpha}+\tfrac{1}{2}(g/2^{\frac{1}{2}}\mu L^{\frac{3}{2}})(\tau_\alpha\boldsymbol{\sigma}\cdot\mathbf{k}/\omega_k^{\frac{3}{2}})\cos\mathbf{k}\cdot\mathbf{r}]+M\mathbf{v}\cdot\mathbf{r}\}\prod_{\alpha\mathbf{k}}h_{m_{\alpha\mathbf{k}}}[Q_{\mathbf{k}\alpha}+(g/2^{\frac{1}{2}}\mu L^{\frac{3}{2}})\times(\tau_\alpha\boldsymbol{\sigma}\cdot\mathbf{k}/\omega_k^{\frac{3}{2}})\cos\mathbf{k}\cdot\mathbf{r}],\quad(4)$$

and the corresponding eigenvalue is

$$E=M+\sum_{\mathbf{k},\alpha}\omega_k(m_{\alpha\mathbf{k}}+\tfrac{1}{2}).\quad(5)$$

The method of obtaining this solution by means of a canonical transformation is given in references 8 and 10. The $m_{\alpha\mathbf{k}}$ are the numbers of free mesons of wave number $\mathbf{k}$, energy ω_k, and charge state α. The $h_{m_{\alpha\mathbf{k}}}$ are normalized harmonic oscillator wave functions. The nucleon-meson field coupling manifests itself in a displacement of the origin of the field oscillators whose coordinates now depend on the spin and charge states. Neglected in Eqs. (4) and (5) are terms of order $(\mathbf{v}\cdot\mathbf{k})/\omega_k<1$. The mass parameter M in Eqs. (4) and (5) represents the mass of the physical nucleon—1836 m_0. It is related to the mass constant M' in Eq. (3) by

$$M=M'-(g^2/4\mu^2L^3)\sum_{\mathbf{k},\alpha}[\tau_\alpha{}^2(\boldsymbol{\sigma}\cdot\mathbf{k})^2/\omega_k{}^2].$$

This mass renormalization is discussed for the electromagnetic case in reference 8 and by Pauli and Fierz.[13] The infinite sum is related to the self-energy problem of field theory. We do not discuss this further here, since our nonrelativistic treatment is not valid for large k. We shall employ cutoffs when infinite sums arise, corresponding to a finite size nucleon of radius $\sim 1/k_{\max}$. These infinities are the same as those which obscure standard perturbation calculations with neglect of recoil.

B. Matrix Element

Interaction with the electromagnetic field will induce transitions of the nucleon-meson system between dif-

[12] A more detailed treatment following the Bloch-Nordsieck argument (Eqs. (4) through (9) of reference 8) yields this same result for slow nucleons.

[13] W. Pauli and M. Fierz, Nuovo cimento **15**, 1 (1938).

ferent stationary states as described by Eq. (4). In contrast with the situation for the meson field, the coupling with the quantum field is weak ($e^2=1/137$), and we treat it by perturbation theory. We calculate the matrix element for the system to absorb a quantum of momentum $\mathbf{K}$ and make a transition from an initial state with velocity, spin, and isotopic spin $\mathbf{v}_i$, $\boldsymbol{\sigma}_i$, $\boldsymbol{\tau}_i$, and with no free mesons (all $m_{\alpha\mathbf{k}}=0$) to a final state with $\mathbf{v}_f$, $\boldsymbol{\sigma}_f$, $\boldsymbol{\tau}_f$, and with one free meson of momentum $\mathbf{k}'$ and charge α' ($\alpha'=3$ for π^0 production). The perturbing terms in the hamiltonian are

$$H'=(ge)/(2^{\frac{1}{2}}\mu)\mathbf{A}\cdot\boldsymbol{\sigma}(\varphi_1\tau_2-\varphi_2\tau_1)$$
$$-e\int\mathbf{A}\cdot(\varphi_2\,\mathbf{grad}\varphi_1-\varphi_1\,\mathbf{grad}\varphi_2)d\mathbf{r}. \quad (6)$$

We neglect the interaction of the nucleon with quanta because of its heavy mass and slow velocity. This is consistent with our other approximations. We introduce the fourier analysis, Eq. (2), for the meson field. Expanding the vector potential $\mathbf{A}$ in plane waves, we write $L^{-\frac{3}{2}}((2\pi)^{\frac{1}{2}}/K^{\frac{1}{2}})\boldsymbol{\varepsilon}_{\mathbf{K}}{}^{\lambda}\exp(i\mathbf{K}\cdot\mathbf{r})$ for annihilation of a photon of momentum $\mathbf{K}$ and polarization $\lambda(=1, 2)$, so that Eq. (6) reads

$$H'=(ge/2^{\frac{1}{2}}\mu L^3)(2\pi/K)^{\frac{1}{2}}\boldsymbol{\varepsilon}_{\mathbf{K}}{}^{\lambda}\cdot\boldsymbol{\sigma}\sum_{\mathbf{k}}1/\omega_k{}^{\frac{1}{2}}$$
$$\times[(\tau_2P_{\mathbf{k}1}-\tau_1P_{\mathbf{k}2})\cos\mathbf{k}\cdot\mathbf{r}+(\tau_2Q_{\mathbf{k}1}-\tau_1Q_{\mathbf{k}2})\sin\mathbf{k}\cdot\mathbf{r}]$$
$$-(ie/2L^{\frac{3}{2}})(2\pi/K)^{\frac{1}{2}}\sum_{\mathbf{k}}[\boldsymbol{\varepsilon}_{\mathbf{K}}{}^{\lambda}\cdot\mathbf{k}/(\omega_{\mathbf{k}}\omega_{\mathbf{k}+\mathbf{K}})^{\frac{1}{2}}]$$
$$\times(P_{\mathbf{k}1}-iQ_{\mathbf{k}1}+P_{-\mathbf{k}1}+iQ_{-\mathbf{k}1})$$
$$\times(P_{(-\mathbf{k}-\mathbf{K})2}-iQ_{(-\mathbf{k}-\mathbf{K})2}$$
$$+P_{(+\mathbf{k}+\mathbf{K})2}+iQ_{(+\mathbf{k}+\mathbf{K})2}). \quad (7)$$

We use the following properties of canonical coordinates u and v and of hermite polynomials of zero order:

$$[v, e^{iau}]=ae^{iau},\quad vh_0(u+c)=(i/\sqrt{2})h_1(u+c),$$
$$uh_0(u+c)=(1/\sqrt{2})h_1(u+c)-ch_0(u+c).$$

We then write for the matrix element for production of a meson of momentum $\mathbf{k}'$ and charge α'

$$\text{M.E.}=(e/L^{9/2})(2\pi/K)^{\frac{1}{2}}\int d\mathbf{r}\prod_{\mathbf{k}\alpha}dw_{\mathbf{k}\alpha}\exp[-iM(\mathbf{v}_f-\mathbf{v}_i)\cdot\mathbf{r}-2i\sum_{\mathbf{k}\alpha}\beta_{\mathbf{k}\alpha}w_{\mathbf{k}\alpha}]h_{1\mathbf{k}'\alpha'}(f)\prod_{\mathbf{k}\alpha}{}'h_{0\mathbf{k}\alpha}(f)$$
$$\times\Bigg\{(g/2^{\frac{1}{2}}\mu L^{\frac{3}{2}})\boldsymbol{\varepsilon}_{\mathbf{K}}{}^{\lambda}\cdot\boldsymbol{\sigma}\sum_{\mathbf{q}}\omega_{\mathbf{q}}{}^{-\frac{1}{2}}\exp[i(\mathbf{K}-\mathbf{q})\cdot\mathbf{r}][\tau_2h_{1\mathbf{q}1}(i)\prod_{\mathbf{k}\alpha}{}'h_{0\mathbf{k}\alpha}(i)-\tau_1h_{1\mathbf{q}2}(i)\prod_{\mathbf{k}\alpha}{}'h_{0\mathbf{k}\alpha}(i)]$$
$$-i\sum_{\mathbf{q}}\frac{\boldsymbol{\varepsilon}_{\mathbf{K}}{}^{\lambda}\cdot\mathbf{q}}{(\omega_{\mathbf{q}}\omega_{\mathbf{q}+\mathbf{K}})^{\frac{1}{2}}}\left(\frac{g\tau_1\boldsymbol{\sigma}\cdot\mathbf{q}}{\mu(\omega_{\mathbf{q}}L)^{\frac{3}{2}}}\exp[-i\mathbf{q}\cdot\mathbf{r}]\prod_{\mathbf{k}\alpha}h_{0\mathbf{k}\alpha}(i)+h_{1-\mathbf{q}1}(i)\prod_{\mathbf{k}\alpha}{}'h_{0\mathbf{k}\alpha}(i)\right)$$
$$\times\left(\frac{g\tau_2\boldsymbol{\sigma}\cdot(\mathbf{q}+\mathbf{K})}{\mu(\omega_{\mathbf{q}+\mathbf{K}}L)^{\frac{3}{2}}}\exp[i(\mathbf{q}+\mathbf{K})\cdot\mathbf{r}]\prod_{\mathbf{k}\alpha}h_{0\mathbf{k}\alpha}(i)-h_{1(\mathbf{q}+\mathbf{K})2}(i)\prod_{\mathbf{k}\alpha}{}'h_{0\mathbf{k}\alpha}(i)\right)\Bigg\}, \quad (8)$$

where

$$w_{\mathbf{k}\alpha}=Q_{\mathbf{k}\alpha}+(g/2^{\frac{1}{2}}\mu L^{\frac{3}{2}})\frac{[(\tau_\alpha\boldsymbol{\sigma})_f+(\tau_\alpha\boldsymbol{\sigma})_i]\cdot\mathbf{k}}{\omega_k{}^{\frac{3}{2}}}\cos\mathbf{k}\cdot\mathbf{r},$$

$$\beta_{\mathbf{k}\alpha}=(g/2^{\frac{1}{2}}\mu L^{\frac{3}{2}})\frac{[(\tau_\alpha\boldsymbol{\sigma})_f-(\tau_\alpha\boldsymbol{\sigma})_i]\cdot\mathbf{k}}{\omega_k{}^{\frac{3}{2}}}\sin\mathbf{k}\cdot\mathbf{r}.$$

Above, $h(i)\{h(f)\}$ refers to initial {final} spin and charge values in the arguments of the hermite polynomials, and $\prod_{\mathbf{k}\alpha}{}'$ denotes a product for all values of $\mathbf{k}$ and α excluding the one written immediately to the left of it.

In evaluating this matrix element an essential difference is observed between the charged and neutral meson production calculations. The operation of charged meson field amplitudes φ_1, φ_2 in Eq. (6) on the initial state function has introduced $h_{1\mathbf{k}1}(i)$ and $h_{1\mathbf{k}2}(i)$ into the integrand of Eq. (8). They can connect directly with $h_{1\mathbf{k}'\alpha'}(f)$ for $\mathbf{k}=\mathbf{k}'$ and $\alpha'=1$ or 2. This set of direct terms is present for charged meson production but does not contribute to the π^0 process, for which $\alpha'=3$.

We write

$$\text{M.E.}(\gamma,\pi_1)=\text{M.E.}_D+\text{M.E.}_I, \quad (9)$$

$$\text{M.E.}(\gamma,\pi_0)=\text{M.E.}_0, \quad (10)$$

where M.E.$_D$ denotes these direct terms, and M.E.$_I$ and M.E.$_0$ differ only in isotopic spin $\boldsymbol{\tau}$. We use the generating functions for hermite polynomials[8] to carry through analytically the integrations in Eq. (8). We observe the integrals to dictate momentum conservation[14]

$$M\mathbf{v}_i+\mathbf{K}=M\mathbf{v}_f+\mathbf{k}'. \quad (11)$$

Energy conservation reads

$$\tfrac{1}{2}Mv_i{}^2+K=\tfrac{1}{2}Mv_f{}^2+\omega_{k'}. \quad (12)$$

[14] For this reason it is preferable not to treat the nucleon as a delta-function source at the origin. The calculational result would be the same, since we neglect velocity terms.

We obtain

$$\mathrm{M.E.}_D=(gei/\mu L^3)(\pi/2K\omega_{k'})^{\frac{1}{2}}\times\{\exp\textstyle\sum_{k\alpha}(-g^2/8\mu^2L^3\omega_k^3)[\boldsymbol{\Delta}(\tau_\alpha\boldsymbol{\sigma})\cdot\mathbf{k}]^2\}\times\left[\boldsymbol{\varepsilon}_K{}^\lambda\cdot\boldsymbol{\sigma}\tau_2-\frac{2\boldsymbol{\varepsilon}_K{}^\lambda\cdot\mathbf{k}'\boldsymbol{\sigma}\cdot(\mathbf{k}'-\mathbf{K})}{\omega_{\mathbf{k}'-\mathbf{K}}{}^2}\tau_2-\frac{\boldsymbol{\varepsilon}_K{}^\lambda\cdot\mathbf{k}'\boldsymbol{\Delta}(\tau_2\boldsymbol{\sigma})\cdot(\mathbf{k}'-\mathbf{K})}{\omega_{\mathbf{k}'-\mathbf{K}}{}^2}\right]; \quad (13)$$

$$\mathrm{M.E.}_I=\boldsymbol{\Delta}(\tau_1\boldsymbol{\sigma})\cdot\mathbf{k}'\mathrm{M.E.}'; \quad (14)$$

$$\mathrm{M.E.}_0=\boldsymbol{\Delta}(\tau_3\boldsymbol{\sigma})\cdot\mathbf{k}'\mathrm{M.E.}'; \quad (15)$$

$$\mathrm{M.E.}'=-(g^3ei/\mu^3L^3)(\pi/2K\omega_{k'}{}^3)^{\frac{1}{2}}\times\{\exp\textstyle\sum_{k\alpha}(-g^2/8\mu^2L^3\omega_k^3)[\boldsymbol{\Delta}(\tau_\alpha\boldsymbol{\sigma})\cdot\mathbf{k}]^2\}\times L^{-3}\sum_{\mathbf{k}}\frac{\boldsymbol{\varepsilon}_K{}^\lambda\cdot\mathbf{k}}{\omega_{\mathbf{k}}{}^2\omega_{\mathbf{k}+\mathbf{K}}{}^2}[\tau_1\boldsymbol{\sigma}\cdot\mathbf{k}+\tfrac{1}{2}\boldsymbol{\Delta}(\tau_1\boldsymbol{\sigma})\cdot\mathbf{k}]\times[\tau_2\boldsymbol{\sigma}\cdot(\mathbf{k}+\mathbf{K})+\tfrac{1}{2}\boldsymbol{\Delta}(\tau_2\boldsymbol{\sigma})\cdot(\mathbf{k}+\mathbf{K})]. \quad (16)$$

We must introduce a high momentum cutoff $k_{\max}$ to compensate neglect of nucleon recoil in the hamiltonian expression, Eq. (3), in order to prevent the exponential factor in Eqs. (13) and (16) from vanishing. The exponential factor is quite sensitive to the cutoff, since the exponent diverges quadratically for large $k_{\max}>\mu$. However it will not affect our discussions of the relative π^0 and π^+ production rates, or of the angular distributions and excitation functions. We see quite clearly in this exponential factor the essential difference between this work and the Bloch-Nordsieck calculation for soft bremsstrahlung in electron scattering. The exponent in the quantum field calculation—Eq. (28) of reference 8—diverges logarithmically for long wavelengths ($k\rightarrow 0$) so that the matrix elements vanish even when a finite source cutoff is introduced. Hence, the probability for emission of any finite number of quanta is zero. However, we observe that the exponents in Eqs. (13) and (16) above vanish for $k\rightarrow 0$. Because of the finite rest mass μ of a meson, this would be true even without the factor k^2 that appears in the numerator of the exponent as a consequence of the derivative coupling in our choice of hamiltonian Eq. (1). Energy conservation as expressed by Eq. (12) limits the number of mesons emitted by an externally perturbed nucleon, in contrast to the Bloch-Nordsieck result for massless quanta. The ultraviolet catastrophe associated with $k_{\max}\rightarrow\infty$ is ameliorated by methods taking the nucleon recoil into account, and it is removed by the renormalization techniques. We do not investigate this question further but remark here that a reasonable choice for $k_{\max}$ suffices to establish qualitative accord with experimental observation 1.

We consider matrix element (13) for production of π_1 mesons. Charge vector τ_2 goes to $\sqrt{2}\tau^+=(\tau_1+i\tau_2)/\sqrt{2}$ for π^+ production. Noting that g in this calculation corresponds to Foldy's[11] $(4\pi)^{\frac{1}{2}}g$, we see that the first two terms of Eq. (13), apart from the exponential factor, agree with his matrix element calculated with straightforward application of second-order perturbation theory and with neglect of nucleon recoil. Their absolute square, averaged over quantum polarizations and summed over spin $\boldsymbol{\sigma}$, treated as a classical unit vector, yields

$$\frac{4\pi^2e^2(g^2/4\pi)}{\mu^2K\omega_{k'}L^6}\left(1-\frac{2\mu^2k'^2\sin^2\theta}{\omega_{\mathbf{k}'-\mathbf{K}}{}^4}\right)\times\exp\sum_{\mathbf{k}\alpha}(-g^2/4\mu^2L^3\omega_k^3)[\boldsymbol{\Delta}(\tau_\alpha\boldsymbol{\sigma})\cdot\mathbf{k}]^2, \quad (17)$$

where θ is the angle between the emitted meson and the incident photon. Performing the final state sum, we obtain a π^+ cross section in qualitative agreement with the experimental observations 1, 2, and 3 on π^+ production. For low meson energies, the cross section is roughly isotropic in the center-of-mass frame and has a linear excitation function. Brueckner[2] gives a more detailed discussion of the main features of π^+ production. These general qualitative features of π^+ production will not be seriously altered by inclusion of the third term in Eq. (13). As indicated in the work of Foldy and Brueckner, choice of $(g^2/4\pi)\sim 0.1$ to 0.4 gives a general agreement with the experimentally observed production cross section of $\sim 10^{-28}$ cm². We see here that

$$\Gamma^2\equiv(g^2/4\pi)\exp\sum_{\mathbf{k}\alpha}(-g^2/4\mu^2L^3\omega_k^3)[\boldsymbol{\Delta}(\tau_\alpha\boldsymbol{\sigma})\cdot\mathbf{k}]^2\approx(g^2/4\pi)\exp\{-(g^2/4\pi)(k_{\max}/\mu)^2|\boldsymbol{\Delta}(\tau\boldsymbol{\sigma})|^2/4\pi\}$$

is roughly $\sim 1/10$ for a reasonable choice of $(g^2/4\pi)\sim 1-2$, and of $k_{\max}|\boldsymbol{\Delta}(\tau\boldsymbol{\sigma})|\sim 5\mu$. We calculate for the square of matrix element (15), averaged over quantum polarization and summed over spin,

$$\langle|\mathrm{M.E.}_D|^2\rangle_{\mathrm{Av}}=\frac{\pi^2e^2\Gamma^2}{2\mu^2K\omega_{k'}L^6}\left(\frac{\boldsymbol{\Delta\sigma}\cdot\mathbf{k}'}{\omega_{k'}}\right)^2(g^2/4\pi)^2\times\left(\frac{1}{3\pi}\frac{K}{\mu}\frac{k_{\max}}{\mu}\right)^2.$$

We see this to indicate a meson production mechanism of magnitude comparable with that operating in charged meson production as described in Eq. (17). It is responsible for the entire contribution to neutral meson production,[15] which process is thus predicted to have an isotropic cross section in the center-of-mass system and a rapidly rising excitation function proportional to the cube of the emitted π^0 momentum.

In this paragraph we attempt to get a hold on the error introduced by the treatment of the spin and

[15] A contribution to neutral meson production from the interaction of a gamma-ray with the Dirac magnetic moment of the nucleon is neglected. The contribution of this matrix element is proportional to eg and increases with the cube of the emitted π^0 momentum near threshold. It is reduced relative to Eq. (15) by the ratio μ/M, and its neglect is consistent with our approximations.

charge vectors, $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, as classical unit vectors in this calculation. The first two terms of matrix element (13) which contribute to charged meson production are linear in $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$. We verify by comparison with a perturbation calculation, in which $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are treated quantum mechanically as matrices with noncommuting components, that, to lowest order in g, the classical treatment gives the same results. However, in expressions (15) and (16) we see that the matrix element for π^0 production contains three $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ vectors. It may be thought that neglect of the commutation relations might introduce serious error. In order to attempt a rough estimate we calculate the π^0 cross section by fourth-order perturbation theory, with neglect of nucleon recoil. To order eg^3, the matrix element thus calculated is five times as large as our results here. In connection with these higher order processes in a perturbation calculation, Brueckner and Watson[5] have observed that they give the predominant contribution for pseudoscalar coupling, yielding a cross section compatible with experiment, for $(g^2/4\pi)\sim 10$.

IV. SUMMARY AND DISCUSSION

To summarize, we have performed a nonperturbation calculation of the photomeson production process. We treat the interaction of a quantized charge-symmetric pseudoscalar meson field pseudovectorially coupled with a Dirac nucleon. Nucleon recoil is neglected, and spin and charge operators $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are treated as classical unit vectors. We use a finite source cutoff with a radius of approximately the nucleon Compton wavelength in order to compensate neglect of nucleon recoil. Because of uncertainties connected with the choice of the coupling parameter $g^2/4\pi$, the cut-off radius $1/k_{\max}$, and the spin reorientation $\Delta\boldsymbol{\sigma}$, we can give no reliable quantitative results. However, it appears possible to establish qualitative agreement with the experimental observations listed in the opening paragraph. No physical assumptions outside of the structure of standard meson theory are necessary. The results supply additional evidence in favor of coupling between nucleons and mesons that is of intermediate strength ($g^2/4\pi \sim 1-2$) and that involves the nucleon spin. If we consider a scalar meson with no spin coupling, there occurs a $\Delta\mathbf{v}<1$ in place of $\Delta\boldsymbol{\sigma}$ in matrix element (15) for π^0 production, and the cross section for π^0 production is calculated to be considerably too small to agree with experiment. In addition, the electric dipole angular distribution calculated on the basis of a scalar theory conflicts with observation.

We can further corroborate our results by considering the anomalous magnetic moments of the neutron and proton. We calculate the matrix element of Eq. (6) between initial and final states with no free mesons and with the same $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$. We represent the vector potential with a slow space variation $\mathbf{A}_q \exp(i\mathbf{q}\cdot\mathbf{r})$. Under these conditions the entire contribution to the matrix element (8) results from the second term of Eq. (6) and is calculated to be

$$-(g^2/4\pi)(4/15\pi)(M/\mu)(k_{\max}/\mu)(i\tau_1\tau_2\boldsymbol{\sigma}\cdot\mathbf{A}_q\boldsymbol{\sigma}\cdot\mathbf{q})\mu_0, \quad (18)$$

where $\mu_0=e/2M$ is one nuclear magneton. Because of our classical treatment of $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, we cannot assign an unambiguous numerical value to this matrix element. We note, however, that the magnitude of the magnetic field strength is $B=qA_q$, so that we can set an upper limit of $(g^2/4\pi)(4/15\pi)(M/\mu)k_{\max}/\mu)\mu_0$ for the calculated anomaly. For the choice of constants specified in the discussion of the photoprocess ($k_{\max}\sim M$; $(g^2/4\pi)\sim 1$ to 2), the anomalous magnetic moments are calculated to be not greater than four to eight nuclear magnetons.[16]

The value of the method of calculation discussed in this paper lies in the fact that it avoids the often unjustified perturbation approximation. It is severely limited to low energy processes, in which it is possible to neglect nucleon recoil, and to discussions not totally obscured by the self-energy infinities which can be handled only by the new covariant subtraction techniques. With neglect of nucleon recoil, we get the familiar $1/r^3$ singularity when we calculate the two-nucleon interaction. We write the Bloch-Nordsieck transformed equation, for two nucleons of mass M_1 and M_2 at $\mathbf{r}_1$ and $\mathbf{r}_2$ (and a pseudoscalar meson field),

$$\begin{aligned}\{\mathbf{v}_1\cdot\mathbf{p}_1+M_1+\mathbf{v}_2\cdot\mathbf{p}_2+M_2+\tfrac{1}{2}\sum_{\mathbf{k}\alpha}\omega_k(P_{\mathbf{k}\alpha}{}^2+Q_{\mathbf{k}\alpha}{}^2)\\ +(g^2/2\mu^2L^3)\tau_1\tau_2(\boldsymbol{\sigma}_1\cdot\mathbf{grad}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{grad}_2)\\ \times\textstyle\sum_{\mathbf{k}}\omega_k{}^{-2}\exp[i\mathbf{k}\cdot(\mathbf{r}_2-\mathbf{r}_1)]-E\}u(Q,\mathbf{r})=0.\end{aligned}$$

This is just the wave equation with the potential term calculated on the basis of second-order perturbation theory. Transforming back to the original canonical coordinates, we have the exact solution. However, one must take into account the nucleon recoil[17] in order to remove the objectionable $1/r^3$ singularity in the potential and to prevent the system from collapsing. The method should prove of value in application to other simple scattering, absorption, and production processes involving a one-nucleon system or a phenomenologically described many-nucleon configuration.

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[16] Matrix element (18) may be in error because of a classical treatment of spin. However, the same spin factor occurs in the matrix elements for π^0 production and for the anomalous magnetic moment. (See Eq. (16).) We may expect this classical approximation to introduce a similar error into both calculations and thus not affect the above argument.

[17] G. Araki, Phys. Rev. **75**, 1101 (1949); L. Van Hove, Phys. Rev. **75**, 1519 (1949); S. M. Dancoff, Phys. Rev. **78**, 382 (1950).