the factor $1-E_1^{-1}(\frac{1}{2}c^2q^2+m_1^2c^4)(\frac{1}{4}c^2q^2+E_2^2(E_1+E_2)^{-2}c^2p^2+m_1^2c^4)^{-\frac{1}{2}}$. The corresponding energy distributions for a 100-Mev meson at $\psi=90°$ and $\psi=180°$ are shown in Fig. 1.

A comparison of the preceding calculations is shown in Table II. The only noteworthy effect introduced by Eq. (15) is the larger integrated cross section for π^- mesons on neutrons. The difference in the scattering of π^- mesons on neutrons and of π^- mesons on protons is most marked in the backward direction.

The experimental results for scattering of π^- mesons[2] indicate that in most inelastic collisions the meson loses 80 percent or more of its initial kinetic energy. It is quite evident that this result cannot be reconciled with the assumptions underlying Eq. (14). The relatively frequent occurrence of large energy losses suggest that a transfer of momentum from the struck nucleon to the rest of the nucleons takes place during the collision. If such a transfer tended to lower the kinetic energy of the struck nucleon before the re-emission of the meson, the qualitative features of the experimental results might be reproduced.

Note added in proof:—I am indebted to Mr. Petschek and Dr. Marshak for completely verifying the derivation of Eq. (14).

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Masses of Light Nuclei from Nuclear Disintegration Energies*

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Values of the atomic masses from n^1 to F^{20} have been derived from the Q-values of nuclear reactions with a procedure of statistical adjustment. Tables are given of several fundamental mass differences, the most probable Q-values, and the atomic masses. Some disparity with the mass spectroscopic results is noted.

I. INTRODUCTION

The large number of accurate Q-values that have become available in the past two years now make it possible for the first time to calculate the masses of the light nuclei directly in terms of O^{16}, without recourse to mass spectroscopic results. Since there are many more reactions than unknown masses, the masses are considerably overdetermined, and some adjustment procedure must be used to solve for the most probable masses. A general least-squares solution becomes exceedingly complex when so many independent variables are involved, and we have used the simpler but essentially equivalent procedure introduced by Tollestrup, Fowler, and Lauritsen.[1] The large number of reactions which interconnect the light nuclei provide many cross-checks on the internal consistency of the experimental data. By an approximate least-squares adjustment of the experimental Q-values we first obtain a numerically self-consistent set of Q-values which we regard as the most probable Q-values. The results are significant in the sense that the required amounts of adjustment are well within the experimental errors. This consistent set of Q-values determines a unique set of mass values which it seems reasonable to regard as the most probable masses. Probable errors in the masses are calculated by a straightforward compounding of gaussian errors.

II. EXPERIMENTAL Q-VALUES

The experimental Q-values used in deriving the masses are listed in the second column of Table I with a reference to the source of each entry in the last column. We have attempted to include as much data as possible for which high accuracy is claimed. Measurements of many different types are included, but all range measurements have been omitted because of the relatively large experimental uncertainties and the uncertainty of the empirical range-energy relation. The extensive magnetic analysis work by Buechner's group at the Massachusetts Institute of Technology accounts for more than one-fourth of the entries in Table I. The other values come from many different laboratories, and the good consistency is very gratifying.

Only those measurements with the smallest probable error have been included. The dividing line was arbitrarily set at 30 kev; with a few exceptions noted subsequently, all measurements with a probable error less than 30 kev are listed in Table I. With this criterion of selection, it has actually turned out that except for five cases, all of the measurements included have a probable error of 15 kev or less. The error of most of the measurements are much better than 1 percent except for those with Q-values below 1 Mev. But it should be mentioned that the calculation of the nuclear masses from Q-values is a linear and additive operation, and consequently absolute errors and not percentage errors are significant. A low energy reaction should not be excluded because of a large percentage error in its measured Q-value.

Several measurements have been omitted even though a small error was claimed; a list of references to these omitted values is appended to Table I. Many of these measurements, such as the early values for the photodisintegration threshold of deuterium, are known to

* Assisted by the joint program of the ONR and AEC.
[1] Tollestrup, Fowler, and Lauritsen, Phys. Rev. 78, 372 (1950).

involve experimental error. A few others have been omitted because of inconsistency with other direct and indirect measurements of the same Q-value by methods that are believed to be more reliable. For example, the β-spectrum end point for $C^{11}(\beta^+)B^{11}$ gives a Q-value of 2.003 ± 0.005 Mev (To 40). From the well-established n^1-H^1 mass difference and the accurately known threshold for $B^{11}(p,n)C^{11}$, we calculate a Q-value of 1.980 ± 0.003 Mev; the discrepancy is several times the probable errors. We believe that the neutron threshold measurement is more reliable and have omitted the beta spectrum measurement from the table. In addition, measurements of Q-value ratios, such as $H^2(\gamma,n)H^1/Be^9(\gamma,n)Be^8$, have been omitted. There are accurate measurements of each of these reactions alone and the ratios have been omitted to avoid increased complexity in the manipulation of the data.

Because of the method of weighting the data, the inclusion of additional data with large probable errors would have a negligible effect on the average Q-values used. For lack of a better alternative, the stated errors have been assumed in every case to have similar significance as an indication of experimental accuracy. They are all regarded as the conventional 50 percent "probable error." On the basis of this assumption, different measurements of the same Q-value have been averaged together, weighting each value inversely as the square of the probable error. Inverse reactions or reactions giving the same Q-value have been averaged together; for example, items 2, 3, and 4 in Table I all give the binding energy of the deuteron and have been averaged together. These weighted average values are listed in column 3 of Table I and are used as the experimental Q-values in subsequent calculations. The probable error $\bar{P}$ in the average Q-value $\bar{Q}\pm\bar{P}$ is calculated by both internal and external consistency, and the larger of the two is used:

$$1/\bar{P}_{\mathrm{int}}^2=\textstyle\sum_i(1/P_i^2),$$

$$\bar{P}_{\mathrm{ext}}=0.67\left(\frac{\sum_i w_i(Q_i-\bar{Q})^2}{(n-1)\sum_i w_i}\right)^{\frac{1}{2}};\quad w_i=\frac{1}{P_i^2}.$$

For only two reactions in Table I is $\bar{P}_{\mathrm{ext}}$ greater than $\bar{P}_{\mathrm{int}}$; this is interpreted as an indication that experimental physicists are overly cautious in assigning probable errors.

Energy standards and fundamental constants used in the calculation of a Q-value from the experimental data change slightly with time. Corrections should be made to conform to the best values of these constants, but a complete revision of this kind has not been undertaken in the present work. In a few cases which came to our attention, this correction has been made as noted in the footnotes to Table I.

III. NUCLEAR CYCLES AND FUNDAMENTAL MASS DIFFERENCES

Figure 1 illustrates graphically the interconnections between the light nuclei which are of interest in this discussion. Each line connecting two nuclei represents a reaction in which one of the nuclei is the target nucleus, the other the residual nucleus. The reactions can be divided into two classes. The first class, indicated by dotted lines, contains reactions which are independent, at the present stage of investigation, in the sense that they are not equivalent to any combination of other reactions. The second class, indicated by solid lines, contains those reactions any one of which can be constructed by a suitable combination of two or more of the other reactions in this class. For example, the reaction $Be^9(d,\alpha)Li^7$ is equivalent to the sum of the two reactions $Be^9(p,\alpha)Li^6$ and $Li^6(d,p)Li^7$ and belongs in the second class: $C^{13}(d,\alpha)B^{11}$ belongs in the first class because there is no other path between C^{13} and B^{11} at the present time.

From these reactions in the second class one can construct many equivalent nuclear cycles or combinations of reactions with the same sum. These cycles are useful in that: (1) They give better experimental values of certain fundamental mass differences than the direct determination. For example, compare the direct determination of the $n-H^1$ mass difference from the neutron beta-decay with the equivalent cycles listed in Table II, Group 2. (2) These cycles provide a test of the internal consistency of the Q-values. This is useful in judging the statistical consistency of experimental input data and is used in Sec. IV as a basis for a statistical adjustment of the Q-values.

Table II contains all of the independent and simplest nuclear cycles in addition to three direct determinations: $n(\beta^-)H^1$, $H^1(n,\gamma)H^2$, and $H^2(d,p)H^3$. The cycles fall into five groups with the respective sums: (1) zero, (2) $n-H^1$, (3) $n+H^1-H^2$, (4) $2H^2-H^1-H^3$, and (5) $2H^2-He^4$. We emphasize the fact that the cycles we have used are linearly independent, which means that none of the cycles are obtained by a combination of two others. The first choice of the independent cycles is arbitrary, although it is desirable that they be as simple as possible to keep the probable errors small. However, the cycles which can subsequently be constructed by a combination of the original cycles should not be used for statistical reasons. A cycle which is used more than once is thereby given a statistical weight greater than is justified by its probable error.

With the exception of Group 1, each of the groups of cycles in Table II determines the most probable value of a fundamental mass difference. The eight independent cycles in Group 2 each determine an experimental value of the $n-H^1$ mass difference when the experimental Q-values are substituted in the cycle. This value is listed opposite each cycle, with a probable error computed from the probable error P_i of the N Q-values in the cycle:

$$P_{\mathrm{cycle}}=\left(\sum_{i=1}^{N}P_i^2\right)^{\frac{1}{2}}.$$

From these eight determinations of $n-H^1$ the weighted average is calculated, weighting each value inversely as the square of its probable error. This average value of $n-H^1$ is then assumed to be the most probable value of this fundamental mass difference. The second cycle, for which the best accuracy is claimed, largely determines the weighted mean value. Omitting this cycle, the weighted average is 782.7 kev. The arithmetic average of the eight values is 784.0 kev. Leaving out the fifth cycle, which seems high, the arithmetic average becomes 782.2 kev.

The probable error in the weighted average value of $n-H^1$ has been calculated from internal consistency and external consistency; both values are listed in Table II. A closer examination of the data reveals that the weighted average is mainly determined by cycles 2, 3, and 4, each of which contains a threshold measurement calibrated against the $Li^7(p,n)Be^7$ threshold at 1.882 ± 0.002 Mev. In view of this correlation of the input data the probable error of the weighted average has been set at 1 kev.

The remaining groups of cycles in Table II have been treated in the same way. The only difference is that in the last two cycles of Group 5 which give the $2H^2-He^4$ mass difference, the quantity $n+H^1-H^2$ occurs. The weighted mean of this difference from Group 3, 2.225 ±0.002 Mev has been substituted as the experimental value of this quantity in these two cycles.

The good internal consistency of the nuclear data is evident from the good agreement of the cycle sums. Each reaction in the second class appears in at least one cycle, and each cycle is a check on the consistency of the data.

IV. THE ADJUSTED Q-VALUES

The 43 reactions in the second class contain only 25 nuclei, including O^{16}. Thus the masses are overdetermined, and some adjustment procedure must be adopted to solve for a unique set of mass values. We have assumed that the most probable set of Q-values is that numerically self-consistent set which is obtained by the least squares adjustment of the experimental Q-values. This self-consistent set of Q-values determines a unique set of mass values which we regard as the most probable

TABLE I. Nuclear reaction energies used in evaluating masses

Reaction	Experimental Q value (Mev)	Weighted mean of experimental Q (Mev)	Adjusted value of Q (Mev)	Ref.*
$n(\beta^-)H^1$	0.783 ± 0.013[a]		0.7823 ± 0.001	Ro 50p
$H^1(n,\gamma)H^2$	2.230 ± 0.007			Be 50g
$H^2(\gamma,n)H^1$	-2.226 ± 0.003	$H^1(n,\gamma)H^2=$	2.225 ± 0.002	Mo 50p
$H^2(p,n)2H^1$	-2.225 ± 0.010	2.227 ± 0.003		Sm 50b
$H^2(n,\gamma)H^3$	6.251 ± 0.008		6.257 ± 0.004	Ki 50p
$H^2(d,n)He^3$	3.265 ± 0.009[b]		3.268 ± 0.004	To 49a
$H^2(d,p)H^3$	4.036 ± 0.012[b]	4.031 ± 0.005	4.032 ± 0.004	To 49a
	4.030 ± 0.006			St 51
$H^3(\beta^-)He^3$	0.0186 ± 0.0002	0.0185 ± 0.0002	0.0185 ± 0.0002	Je 49, Sl 49
	0.0183 ± 0.0003			Cu 49b
	0.0180 ± 0.0005			Gr 49
	0.0190 ± 0.0005			Ha 49b
$H^3(p,n)He^3$	-0.7637 ± 0.001	$H^3(p,n)He^3=$		Ta 49c
$He^3(n,p)H^3$	0.766 ± 0.010	-0.7637 ± 0.001	-0.7638 ± 0.001	Fr 50
$He^6(\beta^-)Li^6$	3.215 ± 0.015		(unadjustable)	Pe 50
$Li^6(p,\alpha)He^3$	4.017 ± 0.012[b]	4.019 ± 0.005	4.016 ± 0.005	To 49b
	4.021 ± 0.006			St 51
	3.97 ± 0.03			Bu 50e
$Li^6(d,p)Li^7$	5.019 ± 0.007		5.020 ± 0.006	St 51
$Li^7(p,n)Be^7$	-1.6457 ± 0.002	-1.6453 ± 0.001[c]		He 49
	-1.6450 ± 0.002		-1.6452 ± 0.001	Sh 49d
$Li^7(p,\alpha)\alpha$	17.340 ± 0.014	17.339 ± 0.009	17.337 ± 0.007	St 51
	17.338 ± 0.011			Wh 50e
$Li^7(d,p)Li^8$	-0.187 ± 0.010	-0.188 ± 0.006	(unadjustable)	Pa 50
	-0.188 ± 0.007			St 51
$Be^8(\alpha)\alpha$	0.101 ± 0.010[d]	0.091 ± 0.004	0.096 ± 0.004	He 49b
	0.089 ± 0.005			To 49b
$Be^9(\gamma,n)Be^8$	-1.666 ± 0.002		-1.666 ± 0.002	Mo 50p
$Be^9(n,\gamma)Be^{10}$	6.797 ± 0.008		6.810 ± 0.006	Ki 50a
$Be^9(p,n)B^9$	-1.852 ± 0.002		(unadjustable)	Ri 50
$Be^9(p,d)Be^8$	0.558 ± 0.003	0.559 ± 0.002	0.559 ± 0.002	To 49b
	0.562 ± 0.004			St 51
$Be^9(p,\alpha)Li^6$	2.121 ± 0.007[b]	2.133 ± 0.007	2.132 ± 0.006	To 49b
	2.142 ± 0.006			St 51
$Be^9(d,p)Be^{10}$	4.585 ± 0.008	4.588 ± 0.006	4.585 ± 0.005	St 51
	4.591 ± 0.008			Kl 51
$Be^9(d,t)Be^8$	4.597 ± 0.013		4.591 ± 0.004	St 51
$Be^9(d,\alpha)Li^7$	7.150 ± 0.008	7.153 ± 0.006	7.152 ± 0.005	St 51
	7.151 ± 0.010			Wh 50e
	7.191 ± 0.024			Kl 51
$Be^{10}(\beta^-)B^{10}$	0.553 ± 0.015	0.556 ± 0.003	0.556 ± 0.003	Fu 49b
	0.545 ± 0.010			Be 50

TABLE I.—*Continued.*

Reaction	Experimental Q value (Mev)	Weighted mean of experimental Q (Mev)	Adjusted value of Q (Mev)	Ref.*
	0.555 ±0.005			Fe 50
	0.560 ±0.005			Hu 50*b*
$B^{10}(n,\alpha)Li^{7}$	2.793 ±0.027	2.789 ±0.009	2.793 ±0.003	Ha 50*p*
	2.788 ±0.010			Je 50, El 48
$B^{10}(p,\alpha)Be^{7}$	1.148 ±0.006	1.150 ±0.003	1.148 ±0.003	Br 50*a*
	1.152 ±0.004			Va 50
	1.147 ±0.010			Bu 50
$B^{10}(d,p)B^{11}$	9.235 ±0.011		9.234 ±0.009	St 51
$B^{11}(p,n)C^{11}$	−2.762 ±0.003		(unadjustable)	Ri 50
$B^{11}(p,\alpha)Be^{8}$	8.567 ±0.011	8.570 ±0.009	8.575 ±0.006	St 51
	8.574 ±0.014			Li 51
$B^{11}(d,p)B^{12}$	1.136 ±0.005		(unadjustable)	St 51
$B^{11}(d,\alpha)Be^{9}$	8.018 ±0.007		8.016 ±0.006	Va 51*p*
$C^{12}(n,\gamma)C^{13}$	4.948 ±0.008		4.948 ±0.004	Ki 50*a*
$C^{12}(d,n)N^{13}$	−0.281 ±0.003		−0.280 ±0.003	Bo 49*c*
$C^{12}(d,p)C^{13}$	2.716 ±0.005	2.723 ±0.005	2.723 ±0.004	St 51
	2.732 ±0.006			Kl 51
$C^{13}(p,n)N^{13}$	−3.003 ±0.003		−3.003 ±0.002	Ri 50
$C^{13}(d,p)C^{14}$	5.91 ±0.03	5.941 ±0.004	5.944 ±0.004	Cu 50
	5.948 ±0.008			St 51
	5.940 ±0.004			Li 51
$C^{13}(d,t)C^{12}$	1.310 ±0.006	1.310 ±0.003	1.309 ±0.003	St 51
	1.310 ±0.003			Li 51
$C^{13}(d,\alpha)B^{11}$	5.160 ±0.010	5.163 ±0.005	(unadjustable)	St 51
	5.164 ±0.006			Li 51
$C^{14}(\beta^{-})N^{14}$	0.154 ±0.003	0.155 ±0.001	0.155 ±0.001	Le 47*a*
	0.152 ±0.005			Le 48*a*
	0.1563±0.001			Co 48*c*
	0.155 ±0.002			Be 48*b*
	0.1575±0.005			An 49*b*
	0.155 ±0.001			Fe 49
	0.155 ±0.001			Wa 50*b*
$C^{14}(p,n)N^{14}$	−0.620 ±0.009	$C^{14}(p,n)N^{14}$=		Sh 49*a*
$N^{14}(n,p)C^{14}$	0.630 ±0.006	−0.628 ±0.004	−0.627 ±0.001	Fr 50
	0.630 ±0.010			St 48*a*
$N^{13}(\beta^{+})C^{13}$	2.220 ±0.006	2.222 ±0.004	2.221 ±0.002	Ly 39
	2.224 ±0.005			Ho 50
$N^{14}(n,\gamma)N^{15}$	10.823 ±0.012		10.833 ±0.007	Ki 50*a*
$N^{14}(d,p)N^{15}$	8.615 ±0.009		8.608 ±0.007	St 51
$N^{15}(p,\alpha)C^{12}$	4.960 ±0.007	4.961 ±0.005	4.961 ±0.005	St 51
	4.961 ±0.006			Li 51
$N^{15}(d,\alpha)C^{13}$	7.681 ±0.009		7.684 ±0.006	St 51
$O^{15}(\beta^{+})N^{15}$	2.705 ±0.005		(unadjustable)	Pe 49*p*
$O^{16}(d,n)F^{17}$	−1.614 ±0.010[e]		(unadjustable)	He 48*a*
$O^{16}(d,p)O^{17}$	1.917 ±0.005	1.917 ±0.004	1.918 ±0.004	St 51
	1.918 ±0.008			Kl 51
$O^{16}(d,\alpha)N^{14}$	3.112 ±0.006	3.116 ±0.004	(unadjustable)	St 51
	3.119 ±0.005			Wh 51
$O^{18}(p,n)F^{18}$	−2.453 ±0.002		−2.453 ±0.002	Ri 50*p*
$F^{18}(\beta^{+})O^{18}$	1.657 ±0.015		1.671 ±0.002	Bl 49*a*
$F^{19}(p,\alpha)O^{16}$	8.113 ±0.030	8.118 ±0.009	8.124 ±0.007	Ch 50
	8.118 ±0.009			St 51
$F^{19}(d,p)F^{20}$	4.373 ±0.007		(unadjustable)	St 51
$F^{19}(d,\alpha)O^{17}$	10.050 ±0.010		10.042 ±0.007	St 51[f]

[a] The recoil energy of the proton included.
[b] Probable error recalculated according to the systematic procedure outlined in Brown, Snyder, Fowler, and Lauritsen, Phys. Rev. **82**, 159 (1951).
[c] −1.6457±0.002 Mev has been used as a standard in many of the experimental Q-values in this table. This corresponds to a threshold energy of 1.882±0.002 Mev.
[d] Recalculated with recent values of ThC″ gamma-ray energy and $Be^{9}(\gamma,n)Be^{8}$ threshold.
[e] Corrected to $Li^{7}(p,n)Be^{7}$ threshold = 1.882 Mev.
[f] References to values omitted from the table: Me 49*a*, Ki 39*d*, My 42, Wi 45, Ar 48, Al 40*a*, Ro 48*b*, To 40, Si 44, Si 45*a*.
* The designation in the last column of the table refers to the reference list in Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 364 (1950). In addition:

Ha 50*p* Hanna, Phys. Rev. **80**, 530 (1950).
Ki 50*p* Kinsey and Bartholomew, Phys. Rev. **80**, 918 (1950).
Kl 51 Klema and Phillips, Phys. Rev. **83**, 212 (1951), and thesis, Rice Institute (1950).
Li 51 Li and Whaling, Phys. Rev. (to be published), and Phys. Rev. **82**, 122 (1951).
Mo 50*p* Mobley and Laubenstein, Phys. Rev. **80**, 309 (1950).
Pe 49*p* Perez-Mendez and Brown, Phys. Rev. **76**, 689 (1949).
Ri 50*p* Richards and Smith, Phys. Rev. **80**, 524 (1950).
Ro 50*p* Robson, Phys. Rev. **81**, 297 (1951).
St 51 Strait, Van Patter, Buechner, and Sperduto, Phys. Rev. **81**, 747 (1951).
Va 51*p* Van Patter, Sperduto, Huang, Strait, and Buechner, Phys. Rev. **81**, 233 (1951).
Wh 51 Whaling and Li (private communication).

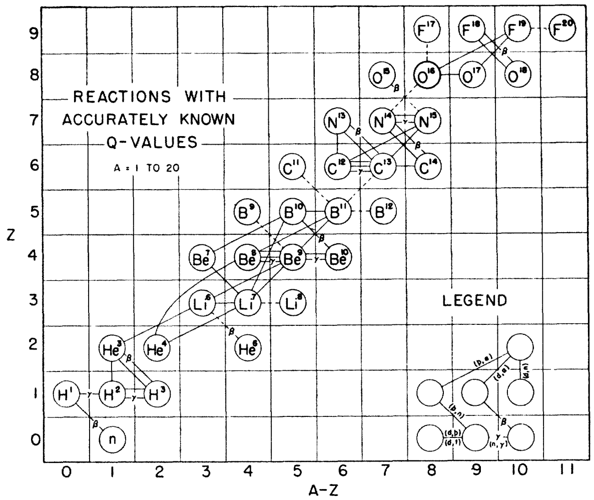

FIG. 1. The nuclear reactions with accurately known Q-values at the present time are represented on this chart by lines connecting target nucleus and residual nucleus.

masses. By numerical self-consistency we mean that the Q-values satisfy all the conditions set by the cycles in Table II, that all equivalent cycles have the same sum.

TABLE Ia. Summary of the adjustments of the interlinked Q-values below O^{16}.

Amount of adjustment		Number of cases
0 (kev)		11
1		10
2		4
3		5
4		1
5		2
6		2
7		1
8		0
9		0
10		1
11		0
12		0
13		1
unadjustable		2
	Total	40
Sign of adjustment		**Number of cases**
0		11
+		11
−		16
unadjustable		2
	Total	40
Ratio of the adjustment to the probable error of the experimental Q		**Number of cases**
0 or up to 1/5		17
between 1/5 and 1/2		10
between 1/2 and 1		9
5/4		1
13/8		1
unadjustable		2
	Total	40

More specifically, we have assumed that the set of most probable Q-values, $[Q_i^{\text{adj}}]$ is that numerically self-consistent set which satisfies the condition $\sum_i(1/P_i^2)(Q_i^{\text{adj}}-Q_i^{\text{exp}})^2$ be a minimum, where $[Q_i^{\text{exp}}]$ is the set of experimental Q-values. For the most general treatment the sum above would be taken over all of the reactions in the second class. Because of the large number of independent variables we have found it convenient to consider the cycles one at a time. The sum is taken over only those reactions included in one cycle, and the cycles are adjusted for numerical consistency one at a time. This treatment deviates from a complete least-squares adjustment in that the sum above is broken up into many partial sums to be minimized separately, and in disregarding the fact that many of the Q-value measurements are not observationally independent; for example, many measurements have used a common energy standard, such as the Po-alpha energy.

TABLE Ib. Q-values adjustable by more than one cycle.

Reaction	Experimental $\bar{Q}$	Q adjusted from one cycle	From which cycle	Final adjusted value
$H^2(d,n)He^3$	3.265±0.009	3.268±0.004 3.267±0.008	$n-H^1$ $2H^2-He^4$	3.268±0.004
$Li^6(d,p)Li^7$	5.019±0.007	5.019±0.006 5.020±0.006	zero $2H^2-He^4$	5.020±0.006
$Li^7(p,\alpha)\alpha$	17.339±0.009	17.334±0.007 17.341±0.008	$2H^2-He^4$ $2H^2-He^4$	17.337±0.007
$Be^9(d,p)Be^{10}$	4.588±0.006	4.588±0.005 4.582±0.006	$n-H^1$ $n+H^1-H^2$	4.585*±0.005
$Be^9(d,\alpha)Li^7$	7.153±0.006	7.153±0.005 7.152±0.005 7.151±0.006 7.152±0.006	zero $n-H^1$ $2H^2-He^4$ $2H^2-He^4$	7.152±0.005
$B^{10}(n,\alpha)Li^7$	2.789±0.009	2.795±0.003 2.790±0.006 2.791±0.008	zero $n-H^1$ $2H^2-He^4$	2.793*±0.003
$B^{11}(d,\alpha)Be^9$	8.017±0.006	8.015±0.006 8.017±0.006	zero $2H^2-He^4$	8.016±0.006
$C^{12}(d,p)C^{13}$	2.723±0.005	2.722±0.005 2.723±0.004 2.723±0.004 2.722±0.004	zero $n-H^1$ $n+H^1-H^2$ $2H^2-H^1-H^3$	2.723±0.004
$N^{14}(n,p)C^{14}$	0.628±0.004	0.627±0.001 0.627±0.004	$n-H^1$ $2H^2-He^4$	0.627±0.001
$N^{13}(\beta^+)C^{13}$	2.222±0.004	2.221±0.002 2.222±0.003	$n-H^1$ $n-H^1$	2.221±0.002
$N^{14}(n,\gamma)N^{15}$	10.823±0.012	10.834±0.007 10.832±0.009	$n+H^1-H^2$ $2H^2-He^4$	10.833±0.007
$N^{15}(d,\alpha)C^{13}$	7.681±0.009	7.683±0.006 7.686±0.008	zero $2H^2-He^4$	7.684±0.006

* Shifted by half-kev from the weighted mean.

Since all of the reactions in the second class can be brought into a numerically consistent system simply by altering the Q-values until all the equivalent cycles have a common sum, this adjustment procedure is very simple in practice. For example, consider the first cycle in Group 1 of Table II. The cycle sum should be zero, but the sum of the experimental Q-values is 7 kev. This 7-kev discrepancy has been divided into three parts, proportional to the square of the probable error of the three Q-values in the cycle, and these increments have been subtracted from the respective Q-values. The sum of the adjusted Q-values is now zero. Similarly, the other zero cycles are adjusted so that the sum is zero for each cycle.

A similar adjustment procedure has been applied to the cycles in Group 2 of Table II. In this case the cycle sum is $n-\mathrm{H}^1$, and the weighted average value of this difference, discussed in Sec. III, is the value to which the cycle is fitted. The remaining groups of cycles are treated in exactly the same way. With a few exceptions noted below, these adjusted Q-values are listed in the fourth column of Table I.

The probable error of an adjusted Q-value is a function of the probable errors of all of the Q-values in the cycle as well as the probable error of the cycle sum. It can be shown that P_1^*, the probable error in the adjusted value of $Q_1 \pm P_1$, is given by

$$(P_1^*)^2 = P_1^2\left[1-\frac{P_1^2}{\sum_i P_i^2}+\frac{P_1^2}{\sum_i P_i^2}\frac{P_c^2}{\sum_i P_i^2}\right],$$

where P_c is the probable error in the weighted mean of the cycle sum, and P_i refers to the probable error in the experimental Q-value for one of the reactions in the cycle. The sums in the denominator are taken over all of the reactions in the cycle. P_c is, of course, zero for the zero cycles and is negligible for the $n-\mathrm{H}^1$ and $n+\mathrm{H}^1-\mathrm{H}^2$ cycles. As can be seen from the expression for P^* above, the probable error in an adjusted Q-value may be much smaller than the probable error in the corresponding experimental Q-value. For example, the probable error in the adjusted Q-value $\mathrm{F}^{18}(\beta^+)\mathrm{O}^{18}$ is only 2 kev, although the probable error in the experimental value is 15 kev. The adjusted value and its probable error are determined largely by the inverse reaction $\mathrm{O}^{18}(p,n)\mathrm{F}^{18}$ for which the probable error is only 2 kev.

It should be noted that in calculating Q-values for reactions not listed in the table, smaller probable errors can usually be obtained by using combinations of reactions listed in the table rather than by using the masses and their probable errors. For example, the Q-value for $\mathrm{N}^{14}(d,\alpha)\mathrm{C}^{12}$ can be calculated directly from

TABLE II. Nuclear cycles and fundamental mass differences.

Cycle	Mass difference from experimental Q (Mev)
Group 1. Nuclear cycles giving a sum of zero	
$\mathrm{B}^{11}(p,\alpha)\mathrm{Be}^8$, $\mathrm{Be}^9(p,d)\mathrm{Be}^8$, $\mathrm{B}^{11}(d,\alpha)\mathrm{Be}^9$	0.007 ±0.012
$\mathrm{Be}^9(p,\alpha)\mathrm{Li}^6$, $\mathrm{Li}^6(d,p)\mathrm{Li}^7$, $\mathrm{Be}^9(d,\alpha)\mathrm{Li}^7$	0.001 ±0.012
$\mathrm{N}^{15}(p,\alpha)\mathrm{C}^{12}$, $\mathrm{C}^{12}(d,p)\mathrm{C}^{13}$, $\mathrm{N}^{15}(d,\alpha)\mathrm{C}^{13}$	0.003 ±0.011
$\mathrm{B}^{10}(n,\alpha)\mathrm{Li}^7$, $\mathrm{Li}^7(p,n)\mathrm{Be}^7$, $\mathrm{B}^{10}(p,\alpha)\mathrm{Be}^7$	0.006 ±0.010
$\mathrm{F}^{19}(p,\alpha)\mathrm{O}^{16}$, $\mathrm{O}^{16}(d,p)\mathrm{O}^{17}$, $\mathrm{F}^{19}(d,\alpha)\mathrm{O}^{17}$	0.015 ±0.014
Group 2. $n-\mathrm{H}^1$	
$n(\beta^-)\mathrm{H}^1$	0.783 ±0.013
$\mathrm{H}^3(p,n)\mathrm{He}^3$, $\mathrm{H}^3(\beta^-)\mathrm{He}^3$	0.7822±0.001
$\mathrm{C}^{13}(p,n)\mathrm{N}^{13}$, $\mathrm{N}^{13}(\beta^+)\mathrm{C}^{13}$	0.781 ±0.005
$\mathrm{C}^{14}(p,n)\mathrm{N}^{14}$, $\mathrm{C}^{14}(\beta^-)\mathrm{N}^{14}$	0.783 ±0.004
$\mathrm{O}^{18}(p,n)\mathrm{F}^{18}$, $\mathrm{F}^{18}(\beta^+)\mathrm{O}^{18}$	0.796 ±0.015
$\mathrm{H}^2(d,p)\mathrm{H}^3$, $\mathrm{H}^2(d,n)\mathrm{He}^3$, $\mathrm{H}^3(\beta^-)\mathrm{He}^3$	0.7845±0.010
$\mathrm{C}^{12}(d,p)\mathrm{C}^{13}$, $\mathrm{C}^{12}(d,n)\mathrm{N}^{13}$, $\mathrm{N}^{13}(\beta^+)\mathrm{C}^{13}$	0.782 ±0.007
$\mathrm{B}^{10}(n,\alpha)\mathrm{Li}^7$, $\mathrm{Be}^9(d,\alpha)\mathrm{Li}^7$, $\mathrm{Be}^9(d,p)\mathrm{Be}^{10}$, $\mathrm{Be}^{10}(\beta^-)\mathrm{B}^{10}$	0.780 ±0.013
Weighted mean of $n-\mathrm{H}^1$ =	0.7823±0.001
(p_e=0.24 kev, p_i=0.95 kev, p_e/p_i=0.25)	
Group 3. $n+\mathrm{H}^1-\mathrm{H}^2$	
$\mathrm{H}^1(n,\gamma)\mathrm{H}^2$	2.227 ±0.003
$\mathrm{H}^2(d,p)\mathrm{H}^3$, $\mathrm{H}^2(n,\gamma)\mathrm{H}^3$	2.220 ±0.009
$\mathrm{Be}^9(p,d)\mathrm{Be}^8$, $\mathrm{Be}^9(\gamma,n)\mathrm{Be}^8$	2.225 ±0.003
$\mathrm{Be}^9(d,p)\mathrm{Be}^{10}$, $\mathrm{Be}^9(n,\gamma)\mathrm{Be}^{10}$	2.209 ±0.010
$\mathrm{C}^{12}(d,p)\mathrm{C}^{13}$, $\mathrm{C}^{12}(n,\gamma)\mathrm{C}^{13}$	2.225 ±0.009
$\mathrm{N}^{14}(d,p)\mathrm{N}^{15}$, $\mathrm{N}^{14}(n,\gamma)\mathrm{N}^{15}$	2.208 ±0.015
Weighted mean of $n+\mathrm{H}^1-\mathrm{H}^2$ =	2.225 ±0.002
(p_e=1.2 kev, p_i=1.9 kev, p_e/p_i=0.63)	
Group 4. $2\mathrm{H}^2-\mathrm{H}^1-\mathrm{H}^3$	
$\mathrm{H}^2(d,p)\mathrm{H}^3$	4.031 ±0.005
$\mathrm{Be}^9(p,d)\mathrm{Be}^8$, $\mathrm{Be}^9(d,t)\mathrm{Be}^8$	4.038 ±0.013
$\mathrm{C}^{12}(d,p)\mathrm{C}^{13}$, $\mathrm{C}^{13}(d,t)\mathrm{C}^{12}$	4.033 ±0.006
Weighted mean of $2\mathrm{H}^2-\mathrm{H}^1-\mathrm{H}^3$ =	4.032 ±0.004
Group 5. $2\mathrm{H}^2-\mathrm{He}^4$	
$\mathrm{Li}^7(p,\alpha)\mathrm{He}^4$, $\mathrm{Be}^8(\alpha)\alpha$, $\mathrm{Be}^9(p,d)\mathrm{Be}^8$, $\mathrm{Be}^9(d,\alpha)\mathrm{Li}^7$	23.842 ±0.012
$\mathrm{N}^{15}(d,\alpha)\mathrm{C}^{13}$, $\mathrm{C}^{13}(d,p)\mathrm{C}^{14}$, $\mathrm{C}^{14}(p,n)\mathrm{N}^{14}$, $\mathrm{N}^{14}(n,\gamma)\mathrm{N}^{15}$	23.817 ±0.016
$\mathrm{B}^{10}(n,\alpha)\mathrm{Li}^7$, $\mathrm{Be}^9(d,\alpha)\mathrm{Li}^7$, $\mathrm{B}^{11}(d,\alpha)\mathrm{Be}^9$, $\mathrm{B}^{10}(d,p)\mathrm{B}^{11}$ with $n+\mathrm{H}^1-\mathrm{H}^{2*}$	23.842 ±0.017
$\mathrm{Li}^7(p,\alpha)\mathrm{He}^4$, $\mathrm{Li}^6(d,p)\mathrm{Li}^7$, $\mathrm{Li}^6(p,\alpha)\mathrm{He}^3$ $\mathrm{H}^2(d,n)\mathrm{He}^3$ with $n+\mathrm{H}^1-\mathrm{H}^{2*}$	23.829 ±0.015
Weighted mean of $2\mathrm{H}^2-\mathrm{He}^4$ =	23.834 ±0.007

* $n+\mathrm{H}^1-\mathrm{H}^2$ = 2.225 ±0.002 Mev from the weighted mean in Group 3.

TABLE III. Table of atomic masses.

	A mass number	$M-A$, mass defect (Mev)	M, atomic mass from nuclear data (amu)[a]	Atomic mass from mass spectroscopy[b]	Bethe[c]
n	1	8.3638 ±0.0029	1.008 982 (±3)		1.008 93
H	1	7.5815 ±0.0027	1.008 142 (±3)	1.008 165 (±4)	1.008 123
H	2	13.7203 ±0.006	2.014 735 (±6)	2.014 778 (±8)	2.014 708
H	3	15.8271 ±0.010	3.016 997 (±11)		3.017 02
He	3	15.8086 ±0.010	3.016 977 (±11)		3.017 00
He	4	3.6066 ±0.014	4.003 873 (±15)	4.003 944 (±19)	4.003 90
He	6	19.065 ±0.025	6.020 474 (±27)		6.020 90
Li	6	15.850 ±0.021	6.017 021 (±22)		6.016 97
Li	7	16.969 ±0.024	7.018 223 (±26)		7.018 22
Li	8	23.296 ±0.028	8.025 018 (±30)		8.025 02
Be	7	17.832 ±0.024	7.019 150 (±26)		7.019 16
Be	8	7.309 ±0.027	8.007 850 (±29)		8.007 85
Be	9	14.007 ±0.028	9.015 043 (±30)		9.015 03
Be	10	15.560 ±0.026	10.016 711 (±28)		10.016 77
B	9	15.076 ±0.029	9.016 190 (±31)		9.016 20
B	10	15.004 ±0.026	10.016 114 (±28)		10.016 18
B	11	11.909 ±0.022	11.012 789 (±23)		11.012 84
B	12	16.912 ±0.020	12.018 162 (±22)		12.019 0
C	11	13.889 ±0.022	11.014 916 (±24)		11.014 95
C	12	3.542 ±0.015	12.003 804 (±17)	12.003 842 (±6)	12.003 82
C	13	6.958 ±0.013	13.007 473 (±14)		13.007 51
C	14	7.153 ±0.010	14.007 682 (±11)		14.007 67
N	13	9.179 ±0.013	13.009 858 (±14)		13.009 88
N	14	6.998 ±0.010	14.007 515 (±11)	14.007 564 (±7)	14.007 51
N	15	4.528 ±0.011	15.004 863 (±12)		15.004 89
O	15	7.233 ±0.012	15.007 768 (±13)		15.007 8
O	16		16.000 000 (standard)		16.000 000
O	17	4.221 ±0.006	17.004 533 (±7)		17.004 50
F	17	6.970 ±0.011	17.007 486 (±11)		17.007 5
F	19	4.149 ±0.014	19.004 456 (±15)		19.004 50
F	20	5.914 ±0.017	20.006 352 (±19)		

[a] 1 amu = 931.152 Mev.
[b] A. O. Nier, Phys. Rev. **81**, 624 (1950).
[c] H. A. Bethe, *Elementary Nuclear Theory* (John Wiley and Sons, New York, 1947). Errors omitted here.

TABLE IV. Fundamental mass spectroscopic doublets.

	Computed from nuclear data (mMU)[a]	From mass spectroscopy
$2H^1-H^2$	1.5494 ± 0.0024	1.5519 ± 0.0017[b]
$2H^2-He^4$	25.596 ± 0.008	25.612 ± 0.009[c], 25.604 ± 0.009[d]
$3H^2-\frac{1}{2}C^{12}$	42.302 ± 0.016	42.373 ± 0.040[e]
$C^{12}H_4{}^1-O^{16}$	36.372 ± 0.019	36.478 ± 0.022[c]
$C^{12}H_2{}^1-N^{14}$	12.573 ± 0.012	12.586 ± 0.013[c]

[a] 1 amu = 931.152 Mev, J. W. M. DuMond and E. R. Cohen, Phys. Rev. **82**, 555 (1951).
[b] T. R. Roberts, Phys. Rev. **81**, 624 (1951).
[c] A. O. Nier and T. R. Roberts, Phys. Rev. **81**, 507 (1951).
[d] H. Ewald, Z. Naturforsch. **5**, 1 (1950).
[e] A. O. Nier (private communication, computed from other doublets, not measured directly).

the masses of the four nuclei involved: 13.5697 ± 0.024 Mev; using the Q-values for $N^{14}(d,p)N^{15}$ and $N^{15}(p,\alpha)C^{12}$, one obtains 13.569 ± 0.009 Mev.

Some reactions appear in more than one cycle and in some cases the adjusted value from one cycle does not agree with the adjusted value from another cycle. Table Ib lists these reactions, with the adjusted values obtained from the different cycles containing the reaction. For the final adjusted value we have taken the weighted mean of the several adjusted values from the different cycles. The probable error assigned to this final adjusted value is the probable error of the most accurate preliminary adjusted value. These final adjusted values, listed in the last column of Table Ib, have been substituted back in the original cycles, and the remaining Q-values in the cycles readjusted to yield numerical consistency as before. This last adjustment is very small, never more than 4 kev, and we have neglected any small effect this small adjustment might have on the probable error of the adjusted Q-value. The final adjusted values are listed in the fourth column of Table I, together with the ones not requiring readjustment, and are used in subsequent calculation, referred to as "adjusted values."

V. THE ATOMIC MASSES

The calculation of the mass values from the adjusted Q-values is straightforward. The H^1 mass is given in terms of O^{16} by

$$H^1=\tfrac{1}{16}O^{16}+\tfrac{1}{16}[-9Q_a+10Q_b+5Q_c-(Q_1-Q_2-Q_3+Q_4+Q_5+Q_6+Q_7-Q_8)]\times1.07394\text{ mMU}$$

$$Q_a=n-H^1(\text{Mev})\quad Q_2=C^{14}(\beta^-)N^{14}\quad Q_6=Be^9(p,\alpha)Li^6$$

$$Q_b=n+H^1-H^2\quad Q_3=C^{13}(d,p)C^{14}\quad Q_7=Li^6(p,\alpha)He^3$$

$$Q_c=2H^2-He^4\quad Q_4=C^{13}(d,\alpha)B^{11}\quad Q_8=H^2(d,n)He^3$$

$$Q_1=O^{16}(d,\alpha)N^{14}\quad Q_5=B^{11}(d,\alpha)Be^9.$$

The probable error is the square root of the sum of the squares of the probable errors of all the reactions in the chain above, with the appropriate factors. The error will depend slightly on the particular chain chosen; in general, the most direct chain gives the smallest probable error. The value of the mass however, does not depend on the particular chain, all chains are equivalent when the adjusted Q-values are used. The four mass differences in Table II give immediately the n^1, H^2, H^3, and He^4 masses, and the remaining masses are calculated from the remaining Q-values. The results are listed in Table III. It is clear from the foregoing expression for the proton mass that the quantity determined by the Q-values is $M-A$, the mass defect in energy units. These mass defects are included in Table III. They are convenient to use in calculating Q-values, and are independent of the conversion factor from energy units to mass units.

The most recent mass spectroscopic values for H^1, H^2, He^4, C^{12}, and N^{14} are also listed in Table III. The mass spectroscopic values are consistently larger by more than the probable error. We have examined our experiments in detail for a source of systematic error that would account for this discrepancy. One might suspect some of the energy standards which are used in the calibration of the nuclear measurements. However, an error of this sort would tend to put all of the Q-values in error in the same direction, too high or too low. In this case the error in the masses would be proportional to the mass defect.

We have not found any single Q-value which could be changed to bring the two mass systems into agreement. Because of the interconnection of the Q-values in the second class, it is not possible to change one Q-value without changing a great many others. It should be noted that all of the Q-values in the chain that determine the proton mass, and hence also H^2, H^3, and He^4, are of this second class except $C^{13}(d,\alpha)B^{11}$ and $O^{16}(d,\alpha)N^{14}$. Because of the critical importance of these two reactions and the fact that they cannot be checked by a combination of other reactions at the present time, it would be desirable to have further independent measurements of these two Q-values, as well as precise measurements of other reactions which would form combinations equivalent to these two reactions.

Recent values of the mass spectroscopic doublets are listed in Table IV along with the values of these doublets calculated from the nuclear data. The agreement between the $2H^1-H^2$ and $2H^2-He^4$ and $C^{12}H_2{}^1-N^{14}$ doublets is good, and the disparity between the two mass systems apparently arises from the poor agreement for the $C^{12}H_4{}^1-O^{16}$ and $3H^2-\frac{1}{2}C^{12}$ doublets. Further measurements of these doublets would be desirable.

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