Inelastic Scattering of π-Mesons

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The inelastic scattering of a π-meson is considered the result of an elastic collision between a meson and a free nucleon. The nuclear model determines the initial nucleon momentum distribution and limits the possible final states. The energy and angular distribution of scattered mesons is obtained in terms of the scattering probability in relative coordinates, which depends only on the momenta of the meson relative to the struck nucleon. Energy distributions of the scattered mesons, calculated for several cases from the differential cross section, indicate that collisions in which the meson loses most of its kinetic energy are relatively infrequent.

THE continuation of recent experiments[1,2] on the scattering of π^- mesons by nuclei should empirically determine the probability that a scattered meson be within prescribed angular and energy intervals. We have therefore examined the theoretical distribution for moderately energetic mesons incident on free nucleons. We assume that the probability of scattering is completely determined by the meson's momenta relative to the struck nucleon. The nuclear model influences the scattering only by fixing the nucleon's initial momentum distribution and by suppressing final states incompatible with nuclear structure. We shall see that the predominant collisions are those in which the meson does not lose a major fraction of its initial energy.

The meson's momenta[3] in the center-of-mass reference frame are given by

$$\mathbf{p}=\mathbf{p}_1-E_1(E_1+E_2)^{-1}(\mathbf{p}_1+\mathbf{p}_2), \tag{1a}$$

$$\mathbf{p}'=\mathbf{p}_1'-E_1(E_1+E_2)^{-1}(\mathbf{p}_1'+\mathbf{p}_2'). \tag{1b}$$

The velocity in the Lorentz transformation to the center-of-mass reference frame is $c^2(E_1+E_2)^{-1}(\mathbf{p}_1+\mathbf{p}_2)$. Terms containing the square of the velocity have been omitted from Eq. (1); the greatest error is 5 percent of p_1 for a 100-Mev meson. Conservation of energy and momentum require that $p'=p$ and that θ, the angle between $\mathbf{p}'$ and $\mathbf{p}$, be given by

$$\cos\theta=1-|\mathbf{p}_1'-\mathbf{p}_1|^2(2p^2)^{-1}=1-q^2(2p^2)^{-1}. \tag{2}$$

The meson's change in momentum, $\mathbf{p}_1'-\mathbf{p}_1$, is designated by $\mathbf{q}$.

The differential scattering cross section may be written

$$d\Phi(\mathbf{p}')=\sigma(p,\cos\theta)\delta(p'-p)(4\pi p'^2)^{-1}d\mathbf{p}', \tag{3}$$

where σ depends only on p and θ. The restriction of $\mathbf{p}'$ to the surface $p'=p$ has been expressed by the Dirac delta-function. If $g(p_2)d\mathbf{p}_2$ is the probability that the nucleon's initial momentum is in the volume element $d\mathbf{p}_2$, the differential cross section for collisions in which $\mathbf{p}_1'$ lies in the volume element $d\mathbf{p}_1'$ can be obtained by averaging the reaction rate over $g(p_2)$. Thus

$$d\Phi(\mathbf{p}_1')=(E_1+E_2)(4\pi E_2p_1)^{-1}d\mathbf{p}_1' \times\int p^{-1}g(p_2)\sigma(p,\cos\theta)\delta(p'-p)d\mathbf{p}_2. \tag{4}$$

By changing the variable of integration from $\mathbf{p}_2$ to $\mathbf{p}$ through the relation

$$\mathbf{p}_2=-\mathbf{p}+E_2(E_1+E_2)^{-1}(\mathbf{p}_1+\mathbf{p}_2), \tag{5}$$

and by using the defining properties of the delta-function, Eq. (4) can be reduced to

$$d\Phi(\mathbf{p}_1')=(E_1+E_2)^4(4\pi E_1^3E_2p_1q)^{-1}d\mathbf{p}_1' \times\int g(p_2)\sigma(p,\cos\theta)dS. \tag{6}$$

In Eq. (6) dS is an element of area in the surface $p'=p$. Certain values of the nucleon's final momentum $\mathbf{p}_{2e}'$, may be incompatible with the possible nuclear states. The corresponding values of $\mathbf{p}$,

$$\mathbf{p}_e=\mathbf{p}_1-E_1(E_1+E_2)^{-1}(\mathbf{p}_1'+\mathbf{p}_{2e}'), \tag{7}$$

must be omitted from the integration in Eq. (6).

TABLE I. Range of scattering angles for various energy losses. Meson scattering with an energy change from 100 Mev to the value shown in the first column takes place at scattering angles greater than ψ_l. The number of collisions is reduced by the exclusion principle at scattering angles between ψ_{el} and ψ_{eu}.

Scattered meson energy	cp_1'	ψ_e	ψ_{eu}	ψ_{el}
100 Mev	195 Mev	0°	—	0°
95	189	6	150°	7
90	182	12	130	16
85	176	18	110	28
80	169	23	87	47
70	156	33	—	—
60	143	43	—	—
50	128	53	—	—
40	113	63	—	—
30	96	75	—	—
20	77	90	—	—
10	54	117	—	—

[1] H. Bradner and B. Rankins, Phys. Rev. **80**, 916 (1950).

[2] Bernardini, Booth, Lederman, and Tinlot, Phys. Rev. **80**, 924 (1950); **82**, 105 (1951). G. Bernardini, Phys. Rev. **82**, 313(T) (1951).

[3] Notation: E, m, and $\mathbf{p}$ are total energy, mass, and momentum; subscripts 1 and 2 designate meson and nucleon; unprimed and primed letters refer to quantities before and after collision; $d\mathbf{p}$ stands for the volume element $dp_xdp_ydp_z$.

Consider a nuclear model in which A nucleons within a sphere of radius $1.5\times10^{-13}A^{\frac{1}{3}}$ cm constitute a degenerate Fermi gas occupying a volume $Ah^3/4$ in phase space. Then

$$g(p_2)=3(4\pi p_{2m}^3)^{-1} \qquad p_2<p_{2m}, \tag{8a}$$

$$g(p_2)=0 \qquad p_2>p_{2m}, \tag{8b}$$

where $p_{2m}=1.05\times10^{-14}$ g cm/sec. Moreover, all final states with $p_2'<p_{2m}$ are excluded by the Pauli principle; for example, since $p_1'>p_1$ implies $p_2'<p_2$, all collisions with $p_1'>p_1$ are suppressed.

In the space of the relative momentum, $\mathbf{p}$, the surface $p'=p$ intersects the volume $p_2<p_{2m}$ in a circle A. The intersection is real only if

$$p_1'\geqslant[(E_2-E_1)p_1-2E_1p_{2m}](E_1+E_2)^{-1}, \tag{9}$$

which defines the least energy a scattered meson can have. Moreover, with a value of p_1' satisfying Eq. (9) the interesection is real only if q lies in the interval

TABLE II. Comparison of energy distributions.

		Most probable energy loss	Fraction losing more than 50 Mev	Integrated cross section[a]
$\psi=90°$	Constant cross section	20 Mev	0.24	0.80
	π^- on neutrons	20	0.22	0.97
	π^- on protons	20	0.23	0.62
$\psi=180°$	Constant cross section	42	0.42	0.58
	π^- on neutrons	40	0.41	0.75
	π^- on protons	44	0.43	0.42

[a] The quantity tabulated is $4\pi\sigma_0^{-1}\int\frac{d\Phi(E_1',\psi)}{d\Omega dE_1'}dE_1'$.

$q_u\geqslant q\geqslant q_l$, where

$$q_u=[p_{2m}^2+E_2E_1^{-1}(p_1^2-p_1'^2)]^{\frac{1}{2}}+p_{2m}, \tag{10a}$$

$$q_l=[p_{2m}^2+E_2E_1^{-1}(p_1^2-p_1'^2)]^{\frac{1}{2}}-p_{2m}. \tag{10b}$$

The limiting scattering angles, ψ_u and ψ_l, are obtained by inserting q_u and q_l into the relation

$$\sin\tfrac{1}{2}\psi=\tfrac{1}{2}(p_1p_1')^{-\frac{1}{2}}[q^2-(p_1'-p_1)^2]^{\frac{1}{2}}. \tag{11}$$

The angles ψ_l are shown in Table I for a 100-Mev meson; ψ_u does not exist for a 100-Mev meson.

The surface $p'=p$ intersects the volume $p_2'<p_{2m}$ in a circle B. The intersection is real only if

$$p_1'^2\geqslant p_1^2-E_1E_2^{-1}p_{2m}^2, \tag{12}$$

which requires that the meson's energy loss be less than the maximum initial kinetic energy of a nucleon. Moreover, with a value of p_1' satisfying Eq. (12) the intersection is real only if q lies in the interval $q_{eu}\geqslant q\geqslant q_{el}$, where

$$q_{eu}=p_{2m}+[p_{2m}^2-E_2E_1^{-1}(p_1^2-p_1'^2)]^{\frac{1}{2}}, \tag{13a}$$

$$q_{el}=p_{2m}-[p_{2m}^2-E_2E_1^{-1}(p_1^2-p_1'^2)]^{\frac{1}{2}}. \tag{13b}$$

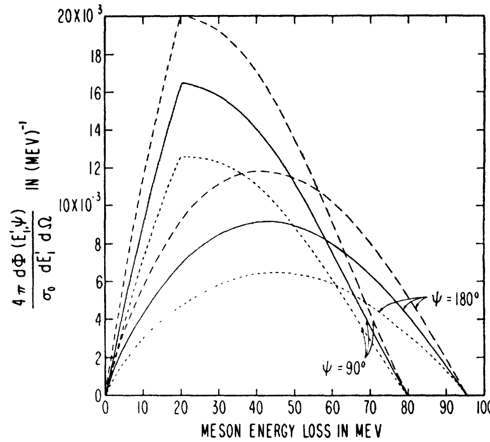

FIG. 1. Energy distribution of scattered mesons: solid curve for a constant scattering cross section; dashed curve for π^- mesons on neutrons or π^+ mesons on protons; dotted curves for π^+ mesons on neutrons or π^- mesons on protons.

The limiting scattering angles, ψ_{eu} and ψ_{el}, found from Eq. (11), are shown in Table I for a 100-Mev meson.

To proceed further, the dependence of σ on p and θ must be specified. The simplest possibility is that σ is a constant σ_0. The integral in Eq. (6) is then proportional to the area between circles A and B. If $\mathbf{p}_1'$ lies within the solid angle $d\Omega$, the differential cross section becomes

$$d\Phi(E_1',\psi)=(\tfrac{3}{4})\sigma_0(E_1+E_2)^2 \times(cE_1E_2p_1^2)^{-1}E_1'UdE_1'd\Omega/4\pi, \tag{14}$$

where

$$U=(qp_{2m}^3)^{-1}p_1'^2[p_{2m}^2 -\tfrac{1}{4}(E_2(E_1q)^{-1}(p_1^2-p_1'^2)-q)^2], \tag{14a}$$

if $\psi_{el}\geqslant\psi\geqslant\psi_l$ or if $\psi_u\geqslant\psi\geqslant\psi_{eu}$, and

$$U=(E_1qp_{2m}^3)^{-1}E_2p_1'^2(p_1^2-p_1'^2), \tag{14b}$$

if $\psi_{eu}\geqslant\psi\geqslant\psi_{el}$. When circle B is not a real intersection, Eq. (14a) applies in the entire interval $\psi_u\geqslant\psi\geqslant\psi_l$. The cross section is always zero outside the last-named interval. The energy distributions of a 100-Mev meson at $\psi=90°$ and at $\psi=180°$ are shown in Fig. 1.

A possible dependence[4] of σ on p and θ is

$$\sigma=\sigma_0[1\mp E_1^{-1}(\tfrac{1}{2}c^2q^2+m_1^2c^4)(c^2p^2+m_1^2c^4)^{-\frac{1}{2}}], \tag{15}$$

where the upper sign applies to collisions of π^- mesons with neutrons or π^+ mesons with protons and the lower sign applies for collisions of π^+ mesons with neutrons or π^- mesons with protons. In the integration in Eq. (6), a negligible error is made if p is replaced by the value it has at the center of circle A. Consequently, the cross section is given by Eq. (14) multiplied by

[4] To the same approximation as Eq. (1), the form of σ in Eq. (15) is that given by weak coupling meson field theory for a pseudoscalar meson. M. Peshkin, Phys. Rev. **81**, 425 (1951); Ashkin, Simon, and Marshak, Prog. Theor. Phys. **5**, 634 (1950).

the factor $1-E_1^{-1}(\frac{1}{2}c^2q^2+m_1^2c^4)(\frac{1}{4}c^2q^2+E_2^2(E_1+E_2)^{-2}c^2p^2+m_1^2c^4)^{-\frac{1}{2}}$. The corresponding energy distributions for a 100-Mev meson at $\psi=90°$ and $\psi=180°$ are shown in Fig. 1.

A comparison of the preceding calculations is shown in Table II. The only noteworthy effect introduced by Eq. (15) is the larger integrated cross section for π^- mesons on neutrons. The difference in the scattering of π^- mesons on neutrons and of π^- mesons on protons is most marked in the backward direction.

The experimental results for scattering of π^- mesons[2] indicate that in most inelastic collisions the meson loses 80 percent or more of its initial kinetic energy. It is quite evident that this result cannot be reconciled with the assumptions underlying Eq. (14). The relatively frequent occurrence of large energy losses suggest that a transfer of momentum from the struck nucleon to the rest of the nucleons takes place during the collision. If such a transfer tended to lower the kinetic energy of the struck nucleon before the re-emission of the meson, the qualitative features of the experimental results might be reproduced.

Note added in proof:—I am indebted to Mr. Petschek and Dr. Marshak for completely verifying the derivation of Eq. (14).

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Masses of Light Nuclei from Nuclear Disintegration Energies*

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Values of the atomic masses from n^1 to F^{20} have been derived from the Q-values of nuclear reactions with a procedure of statistical adjustment. Tables are given of several fundamental mass differences, the most probable Q-values, and the atomic masses. Some disparity with the mass spectroscopic results is noted.

I. INTRODUCTION

THE large number of accurate Q-values that have become available in the past two years now make it possible for the first time to calculate the masses of the light nuclei directly in terms of O^{16}, without recourse to mass spectroscopic results. Since there are many more reactions than unknown masses, the masses are considerably overdetermined, and some adjustment procedure must be used to solve for the most probable masses. A general least-squares solution becomes exceedingly complex when so many independent variables are involved, and we have used the simpler but essentially equivalent procedure introduced by Tollestrup, Fowler, and Lauritsen.[1] The large number of reactions which interconnect the light nuclei provide many cross-checks on the internal consistency of the experimental data. By an approximate least-squares adjustment of the experimental Q-values we first obtain a numerically self-consistent set of Q-values which we regard as the most probable Q-values. The results are significant in the sense that the required amounts of adjustment are well within the experimental errors. This consistent set of Q-values determines a unique set of mass values which it seems reasonable to regard as the most probable masses. Probable errors in the masses are calculated by a straightforward compounding of gaussian errors.

II. EXPERIMENTAL Q-VALUES

The experimental Q-values used in deriving the masses are listed in the second column of Table I with a reference to the source of each entry in the last column. We have attempted to include as much data as possible for which high accuracy is claimed. Measurements of many different types are included, but all range measurements have been omitted because of the relatively large experimental uncertainties and the uncertainty of the empirical range-energy relation. The extensive magnetic analysis work by Buechner's group at the Massachusetts Institute of Technology accounts for more than one-fourth of the entries in Table I. The other values come from many different laboratories, and the good consistency is very gratifying.

Only those measurements with the smallest probable error have been included. The dividing line was arbitrarily set at 30 kev; with a few exceptions noted subsequently, all measurements with a probable error less than 30 kev are listed in Table I. With this criterion of selection, it has actually turned out that except for five cases, all of the measurements included have a probable error of 15 kev or less. The error of most of the measurements are much better than 1 percent except for those with Q-values below 1 Mev. But it should be mentioned that the calculation of the nuclear masses from Q-values is a linear and additive operation, and consequently absolute errors and not percentage errors are significant. A low energy reaction should not be excluded because of a large percentage error in its measured Q-value.

Several measurements have been omitted even though a small error was claimed; a list of references to these omitted values is appended to Table I. Many of these measurements, such as the early values for the photodisintegration threshold of deuterium, are known to

* Assisted by the joint program of the ONR and AEC.
[1] Tollestrup, Fowler, and Lauritsen, Phys. Rev. 78, 372 (1950).