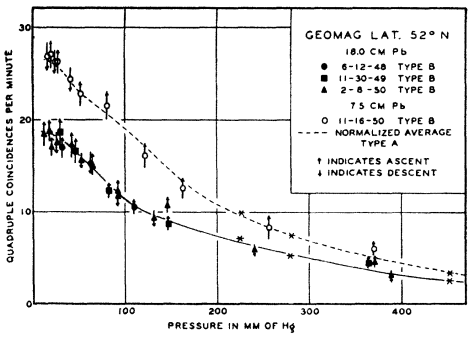

FIG. 1. Intensity *vs* altitude curves obtained with 7.5 cm and 18.0 cm Pb interposed in the counter train. The asterisks represent points obtained in a B-29 aircraft and atop Mt. Evans (see reference 3).

those for larger thicknesses only by applying the customary but nevertheless unsatisfactory normalization procedure to the data obtained by others. A direct measurement was subsequently undertaken, and a detailed investigation of the factors involved in the standardization between two different counter trains was conducted.[3]

The results are plotted in Fig. 1. Details regarding the quadruple-coincidence counter-train utilized in these experiments, ground counting rates, etc., are given in reference 3 under the designation Apparatus B. The dashed curve represents the average of data obtained with a different counter train designated Apparatus A. The normalization of the data is accomplished by the application of the geometrical factors ϕ_B/ϕ_A (Table V, reference 3) to the counting rates obtained with Apparatus A. Direct verification of the validity of this normalization was obtained by simultaneous operation of the two instruments at sea level, at mountain altitude, and in B-29 aircraft. the appropriate value of p in a $\cos^p\theta$ zenith angle distribution law was determined as a function of altitude from experimental data.[4]

Whereas the earlier comparison based upon the usual arbitrary normalization at sea level had indicated an absorption in 18 cm of Pb amounting to 40 percent near the "top of the atmosphere," the present results show that actually 29 percent of the primary particles are stopped in the sense defined above. A comparison of our data with those of Schein and Allen on the basis of a reasonable normalization procedure[3] is not inconsistent with this conclusion.

It is interesting to note the relative effects of (a) the 120 g/cm^2 Pb shield interposed in the counter train at the ceiling altitude and (b) an equivalent mass of superposed atmosphere, alternatively added to the initial 85 g/cm^2 of interposed Pb. The decrease in the observed particle intensity in case (a) is practically the same as that in case (b). This is indeed a striking consequence of a conspiracy of circumstances. Both meson decay and energy-loss considerations make the equivalent atmospheric path a considerably more effective stopping layer. On the other hand, owing to the multiple production of secondaries, geometrical considerations favor the air path from the point of view of the enhancement of the detectable particle intensity. Penetrating secondaries produced within a narrow cone by an incident primary particle are not detected individually in case (a), whereas they can be recorded in case (b). Furthermore, charged secondaries arising from neutral nucleons, which of course constitute a significant fraction of the primary cosmic-ray particles, cannot actuate the counter train in case (a), whereas they are observable in case (b). It is remarkable that in such a complex situation, the various competing processes provide almost exact compensation.

* Assisted by the joint program of the ONR and AEC.
1 M. A. Pomerantz, Phys. Rev. **75**, 69 (1949).
2 M. A. Pomerantz, Phys. Rev. **77**, 830 (1950), and subsequent as yet unpublished results.
3 M. A. Pomerantz, Phys. Rev. **75**, 1721 (1949).
4 M. A. Pomerantz, Phys. Rev. **75**, 1335 (1949).

Erratum: Resistivity and Hall Constant of Semiconductors

[Phys. Rev. **82**, 109 (1951)]

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IN a recent letter with the above title several misprints occurred. The denominator in Eq. (2) should be squared. The *Rnec* value for $\gamma=\xi=0$ should be $315\kappa/512$, in agreement with the value computed for this special case by Johnson and Lark-Horovitz.[1] The symbol ϵ should be used only for the electron or hole energy; other ϵ's should really be ξ's to be consistent with the notation of Fig. 1.

1 V. A. Johnson and K. Lark-Horovitz, Purdue University Semiconductor Research, Sixth Quarterly Report, Signal Corps Project 112B-1 (unpublished), p. 6.

Energy Levels of a Spheroidal Nuclear Well

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(Received May 31, 1951)

RECENTLY Rainwater[1] has proposed a nuclear model in which the odd nucleon moves in a potential well of uniform depth and oblate spheroidal shape. It is proposed that such a model may explain certain large quadrupole moments. Using a perturbation method Feenberg and Hammack[2] have calculated the displacement of the nuclear energy levels produced by small spheroidal distortions of the potential well.

In the present note we present the results of calculations of somewhat larger distortions than those contemplated by Feenberg and Hammack. Although we have used the proper spheroidal wave functions, our calculation fails to be completely rigorous because we have used the boundary conditions of an infinitely deep well. The results of the calculations are shown in Figs. 1 and 2, where we plot the energy in the dimensionless units $(ka_0)^{\frac{1}{2}}$ against the eccentricity ϵ. Here k is the wave number and a_0 is the radius of the undistorted well. The eccentricity ϵ is here defined as $\epsilon=(a^2-b^2)^{\frac{1}{2}}/a$, where a and b are the semi-axes of the spheroid. One should note that this differs from the quantity $e=(a-b)/a_0$ defined by Rainwater. The curves are labeled with the quantum numbers l, m belonging to the undistorted nucleus. For $\epsilon\leq 0.1$ the results agree with the perturbation calculations of Feenberg and Hammack.

In the future we hope to improve our calculations by using the exact boundary conditions, and to examine the dependence of the

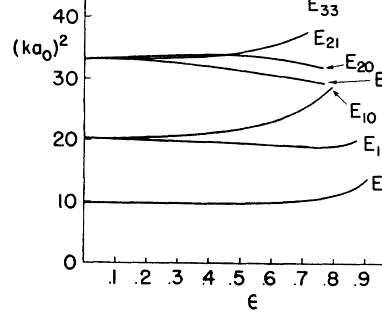

FIG. 1. Energy levels for $n=1$. Levels are labeled with the quantum numbers l, m of the undistorted nucleus.

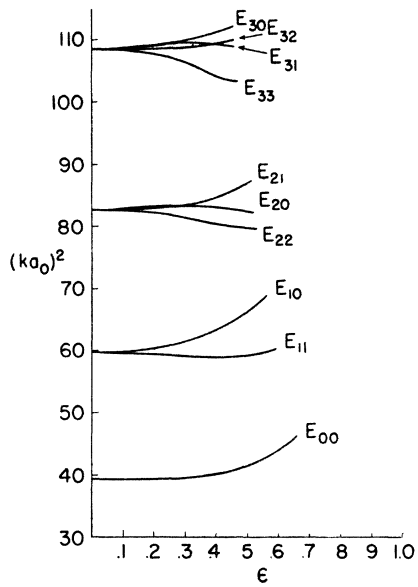

FIG. 2. Energy levels for $n=2$.

quadrupole moment on ϵ. The authors wish to thank Dr. C. Kikuchi for his interest in the present work.

[1] J. Rainwater, Phys. Rev. 79, 432 (1950).
[2] E. Feenberg and K. C. Hammack, Phys. Rev. 81, 285 (1951).

Singular Potentials and the Theory of the Effective Range

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(Received May 28, 1951)

THE theory of the effective range as developed by Schwinger[1] and Bethe[2] has been applied successfully to the analysis of low energy nucleon-nucleon scattering. In particular, for neutron-proton scattering, the phase shift and energy are related by the simple approximate expression,

$$k\cot\delta=-1/a+\tfrac{1}{2}r_0k^2, \qquad (1)$$

where a is the scattering length and r_0 is the effective range. Since the experimental results determine only the two parameters a and r_0, it is possible to determine only general properties of the nucleon-nucleon potentials. It has been shown that for low energies (less than 10 Mev) the depth and range of any nonsingular short range potential such as the square, Yukawa, gaussian, or exponential wells can be adjusted to give satisfactory values for these constants.

The singular potentials of meson theory, however, have not been investigated for consistency with the effective range concept. Pseudoscalar meson theory, which gives qualitatively correct descriptions of many other mesonic phenomena, predicts singular nuclear forces. The nucleon-nucleon potentials given by this theory have the property, at least in the weak and strong coupling limits, that between the meson and nucleon Compton wavelengths the variation with the nucleon-nucleon separation is given by $e^{-\mu r}/r^3$, and, for large r, by $e^{-\mu r}/r$, where μ is the meson mass. Although we cannot rely in detail on the predictions of perturbation methods applied to pseudoscalar theory, it seems probable that the singular nature of the forces is correct, at least for distances larger than $\hbar/Mc$ where relativistic effects are not predominant. We shall, therefore, investigate a potential having only the general features of singularity inside the meson Compton wavelength and a Yukawa tail outside in order to determine the possibility of fitting the binding energy and effective range of the neutron-proton triplet system.

We choose as an example,

$$V(r)=V_0e^{-\mu r}[B/(\mu r)+1/(\mu r)^3] \text{ for } r \text{ greater than } \hbar/Mc, \qquad (2)$$

where B is zero or one, so that the $1/r^3$ force predominates inside the range of the forces. μ is taken to be the π-meson mass (275 electron masses) since it is difficult to see how the intrinsic range ($\hbar/\mu c$) of the potential can be greater than that determined by the mass of the lightest strongly coupled particle, although it certainly may be less.

We shall allow an arbitrariness in the potential for distances less than the nucleon Compton wavelength. Because the nucleons are highly relativistic in this region, we can say nothing about the forces without a solution of the relativistic two-body problem, which has not yet been obtained; in fact, the concept of force becomes fairly meaningless in this range. In addition, the existence of heavy mesons[3] or of the strongly coupled "V-particles" of Anderson[4] will affect the interaction at small distances. We therefore shall consider the effects of modifications of the forces[5] in this range; the three potential strengths for r less than $\hbar/Mc$ which we have investigated are given in column 1 of Table I.

As a further comment on the validity of the theory of the effective range for relativistic problems in general, it can easily be shown that the expression for the effective range for the relativistic one-body problem at low energies is identical with the nonrelativistic expression if the nonrelativistic solutions are replaced by the large components of the relativistic solutions. It must be strongly emphasized that this result does not apply to the two-body problem and only shows that nothing goes wrong with the method in the one-body case. This is in keeping with the spirit of our investigation, which is primarily an attempt to see if one can rule out singular potentials on the basis of low energy scattering experiments.

Calculation of the effective range.—The effective range is given by the expression,[2]

$$\tfrac{1}{2}r_0=\int_0^\infty[\exp(-2\alpha r)-u^2]d^3r, \qquad (3)$$

where u is the solution of the Schrödinger equation for the ground state of the deuteron,

$$d^2u/dr^2=M(E_B+V)u, \qquad (4)$$

which goes asymptotically as $\exp(-\alpha r)$, E_B is the magnitude of the deuteron binding energy, and $\alpha^2=ME_B$. For potentials of the form which we are considering, we cannot obtain closed solutions of Eq. (4). We therefore have used a combination of approximation methods and numerical integration to obtain a solution.

We note that $u(r)$ satisfies the integral equation,

$$u(r)=\exp(-\alpha r)-\int_r^\infty \sinh\alpha(r'-r)/\alpha V(r')u(r')d^3r'. \qquad (5)$$

We have calculated the first Born approximation to this expression [replacing $u(r')$ by $\exp(-\alpha r')$] to give the amplitude, and the second Born approximation for the first derivative. These approximations are valid down to $1.4(\hbar/\mu c)$ for the strongest potentials which we have considered. For smaller r, the integration was carried out numerically, working directly with Eq. (4). Because of the somewhat tedious nature of this work, we have investigated only a very limited choice of the potential for r less than $\hbar/Mc$. The results for the effective range are given in Table I. We see that for attractive or zero forces and for B equal to zero or

TABLE I. Effective range in units of 10^{-13} cm. V' is the potential strength for r less than $\hbar/Mc=0.21(10)^{-13}$ cm. The parameter B is defined in Eq. (2).

	$B=0$	$B=1$
$V'=V(\hbar/Mc)$	<0.80	<0.80
$V'=0$	<0.80	0.80
$V'=+\infty$	1.11	1.38