

$Q_{\text{min}} = e^2/4\eta$  (with  $e$  the energy given to the atomic electron), but he has implicitly assumed  $Q_{\text{min}} = e^2/4\eta \leq \epsilon$  in order to expand the integrand of  $B_H$ . This assumption is equivalent to having  $\epsilon \leq 2mv^2 = 4(m/M)(Mv^2/2)$ , but it is not always true that the energy given to the atomic electron is such a small fraction of the incident particle's energy,  $Mv^2/2$ .

It is apparent that these two approximations would hold exactly for a free particle collision of a heavy particle and an electron. In that case the maximum energy which could be taken off by the electron would, indeed, be  $4\eta$  or  $2mv^2$ . For bound electrons, however, one should expect a small correction for these approximations. This correction affects the formula for  $B_H$  in the coefficients of  $1/\eta$  and higher order terms in  $1/\eta$ . The result of working out the exact correction is that a term  $-1/\eta$  should be added to Brown's formula for  $B_H$  (which formula is for two  $K$ -electrons). This correction should also be added to his formulas for  $B_K(\theta, \eta)$  so that they read:

$$\begin{aligned} B_K(\theta=0.7, \eta) &= 1.813 \ln\eta + 2.598 - 2.067(1/\eta), \\ B_K(\theta=0.75, \eta) &= 1.7222 \ln\eta + 2.4954 - 2.100(1/\eta), \\ B_K(\theta=0.8, \eta) &= 1.6457 \ln\eta + 2.4017 - 2.1196(1/\eta), \\ B_K(\theta=0.9, \eta) &= 1.5250 \ln\eta + 2.2400 - 2.1309(1/\eta). \end{aligned}$$

The details of these corrections will be published later, along with a treatment of the stopping number contribution of  $L$ -electrons.

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## Domain Boundary Motion in Ferroelectric Crystals and the Dielectric Constant at High Frequency

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THE domain boundary in a ferroelectric crystal is the region separating domains polarized in different directions. In ferromagnetic crystals the boundary is  $\sim 10^2$  lattice constants in thickness, while in paramagnetic crystals<sup>1</sup> at very low temperatures it is probably a single lattice constant thick; in ferroelectric crystals the boundary may be a very few lattice constants thick.<sup>2</sup> The surface energy of the boundary will be increased when it is set in motion because of the inertia of the ions which change position slightly as their dipole moments change direction on passage of the wall. The inertia may give rise to dielectric dispersion at high frequencies.

We calculate the surface inertia of the boundary in barium titanate on a simplified model having a boundary  $N$  lattice constants thick, separating domains polarized in opposite directions. We suppose for simplicity that the Ti ion changes position within the unit cell by a distance  $\delta$  between adjacent domains, and we suppose that this motion is the principal inertial effect in the crystal. In uniform wall motion the average velocity of the Ti ions is related to the wall velocity  $v_w$  by the relation  $v_{Ti} = (\delta/Na)v_w$ , where  $a$  is the lattice constant. The kinetic energy per unit area of wall is therefore  $\frac{1}{2}M(N/a^2)(\delta/Na)^2v_w^2$ , where  $M$  is the reduced mass of the Ti ion. The effective mass per unit area of wall is

$$\rho = M\delta^2/Na. \quad (1)$$

If we take  $\delta=0.2A$  and  $a=4A$ , we find  $\rho=1.0 \times 10^{-10}/N$  g/cm<sup>2</sup>, and it may not be unreasonable to set  $N=1$ .

The equation of motion of the wall may be written

$$\rho(d^2x/dt^2) + r(dx/dt) + qx = 2PE, \quad (2)$$

where  $r$  represents damping effects and  $q$  the restoring force, associated, for example, with local trapping of the wall. Damping may be occasioned by coupling with lattice vibrations, selective impurity diffusion, local trapping, acoustic radiation, and other

causes, but for the present we set  $r=0$  purely for convenience. If  $\chi_d^0$  is that part of the static dielectric susceptibility arising from domain boundary displacements, we see readily that  $q=4P_s^2/D\chi_d^0$ , where  $P_s$  is the saturation polarization and  $D$  is the width of a domain.

In an alternating electric field of angular frequency  $\omega$  we have

$$\chi_d = \frac{4P_s^2/D}{q - \rho\omega^2}, \quad (3)$$

so that there is a resonance at

$$\omega_0 = (q/\rho)^{1/2} = [4P_s^2/\rho D\chi_d^0]^{1/2}. \quad (4)$$

Taking  $P_s=50,000$  esu,  $\chi_d^0 \approx 100$ , and  $D \approx 10^{-2}$  cm, we have for barium titanate

$$f_0 \approx 2 \times 10^9 \text{ cps,}$$

where we have taken the wall as a single lattice constant thick.

Powles and Jackson,<sup>3</sup> and Von Hippel, have observed that at high frequencies the dielectric constant of polycrystalline barium titanate relaxes from about 1500 at  $10^8$  cycles to about 150 at  $3 \times 10^{10}$  cycles; the center of the relaxation spectrum appears to be at about  $3 \times 10^9$  cps, in good agreement with our calculated resonance frequency. The fact that the observed curve does not have a resonance character is not in itself disturbing, as this might well be accounted for by a combination of frictional effects and spread in domain widths. Similar situations are encountered in the magnetic properties of the ferrites.

Although the extent of the agreement of theory with experiment is gratifying, it must be remembered that we have at present no independent or direct physical evidence that boundary displacements do in fact contribute to polarization processes in BaTiO<sub>3</sub> at high frequencies, and the observed dielectric dispersion at  $10^9$  cps may still be caused by processes other than that discussed here, although our process must certainly occur when the damping is sufficiently low. Our present experimental knowledge of the origin of polarization processes in ferroelectrics even in static fields is almost negligible.

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## Relative Arrival Times of Air Shower Particles

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THIS note reports the initial results of an experiment on the time relationship between the arrival of air shower particles at an unshielded tray and the arrival of penetrating electrons, photons, or their products at a tray shielded by 10 cm of lead. It appears, after a correction is applied, that the soft and penetrating components arrive within a time  $< 10^{-8}$  sec. However, preliminary results of a similar experiment<sup>1</sup> in which the lead thickness was 17.5 to 22.5 cm have shown time lags of the order of 0.4  $\mu$ sec for the penetrating component.

The showers are detected by two 320-cm<sup>2</sup> timing trays of 2-cm diameter, counters 4 m apart, and an intermediate tray, all arranged in a horizontal straight line. Any one of the trays can be shielded with the lead. After wide band amplification and delay the signals from the timing trays are displayed and photographed as radial pulses on a 1- $\mu$ sec per turn spiral time base triggered by the triple coincidences.<sup>2</sup> An additional delay in one channel insures separation of the pulses. As a result of counter reaction times