$Q_{\min}=\epsilon^2/4\eta$ (with ϵ the energy given to the atomic electron), but he has implicitly assumed $Q_{\min}=\epsilon^2/4\eta\leq\epsilon$ in order to expand the integrand of B_H. This assumption is equivalent to having $\epsilon\leq 2mv^2=4(m/M)(Mv^2/2)$, but it is not always true that the energy given to the atomic electron is such a small fraction of the incident particle's energy, $Mv^2/2$.

It is apparent that these two approximations would hold exactly for a free particle collision of a heavy particle and an electron. In that case the maximum energy which could be taken off by the electron would, indeed, be 4η or $2mv^2$. For bound electrons, however, one should expect a small correction for these approximations. This correction affects the formula for B_H in the coefficients of $1/\eta$ and higher order terms in $1/\eta$. The result of working out the exact correction is that a term $-1/\eta$ should be added to Brown's formula for B_H (which formula is for two K-electrons). This correction should also be added to his formulas for $B_K(\theta,\eta)$ so that they read:

$$B_K(\theta=0.7,\eta)=1.813\ \ln\eta+2.598\ -2.067(1/\eta),$$
$$B_K(\theta=0.75,\eta)=1.7222\ln\eta+2.4954-2.100(1/\eta),$$
$$B_K(\theta=0.8,\eta)=1.6457\ln\eta+2.4017-2.1196(1/\eta),$$
$$B_K(\theta=0.9,\eta)=1.5250\ln\eta+2.2400-2.1309(1/\eta).$$

The details of these corrections will be published later, along with a treatment of the stopping number contribution of L-electrons.

* Now at Los Alamos Scientific Laboratory, Los Alamos, New Mexico.
[1] L. M. Brown, Phys. Rev. **79**, 297 (1950).
[2] H. A. Bethe, Handbuch der Physik **24/1**, 495 (1933).

Domain Boundary Motion in Ferroelectric Crystals and the Dielectric Constant at High Frequency

C. KITTEL*
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received May 31, 1951)

THE domain boundary in a ferroelectric crystal is the region separating domains polarized in different directions. In ferromagnetic crystals the boundary is $\sim 10^2$ lattice constants in thickness, while in paramegnetic crystals[1] at very low temperatures it is probably a single lattice constant thick; in ferroelectric crystals the boundary may be a very few lattice constants thick.[2] The surface energy of the boundary will be increased when it is set in motion because of the inertia of the ions which change position slightly as their dipole moments change direction on passage of the wall. The inertia may give rise to dielectric dispersion at high frequencies.

We calculate the surface inertia of the boundary in barium titanate on a simplified model having a boundary N lattice constants thick, separating domains polarized in opposite directions. We suppose for simplicity that the Ti ion changes position within the unit cell by a distance δ between adjacent domains, and we suppose that this motion is the principal inertial effect in the crystal. In uniform wall motion the average velocity of the Ti ions is related to the wall valocity v_w by the relation $v_{\mathrm{Ti}}=(\delta/Na)v_w$, where a is the lattice constant. The kinetic energy per unit area of wall is therefore $\frac{1}{2}M(N/a^2)(\delta/Na)^2v_w^2$, where M is the reduced mass of the Ti ion. The effective mass per unit area of wall is

$$\rho=M\delta^2/Na.^4 \tag{1}$$

If we take $\delta=0.2$A and $a=4$A, we find $\rho=1.0\times10^{-10}/N$ g/cm^2, and it may not be unreasonable to set $N=1$.

The equation of motion of the wall may be written

$$\rho(d^2x/dt^2)+r(dx/dt)+qx=2P\,E, \tag{2}$$

where r represents damping effects and q the restoring force, associated, for example, with local trapping of the wall. Damping may be occasioned by coupling with lattice vibrations, selective impurity diffusion, local trapping, acoustic radiation, and other causes, but for the present we set $r=0$ purely for convenience. If $\chi_d{}^0$ is that part of the static dielectric susceptibility arising from domain boundary displacements, we see readily that $q=4P_s{}^2/D\chi_d{}^0$, where P_s is the saturation polarization and D is the width of a domain.

In an alternating electric field of angular frequency ω we have

$$\chi_d=\frac{4P_s{}^2/D}{q-\rho\omega^2}, \tag{3}$$

so that there is a resonace at

$$\omega_0=(q/\rho)^{\frac{1}{2}}=[4P_s{}^2/\rho D\chi_d{}^0]^{\frac{1}{2}}. \tag{4}$$

Taking $P_s=50{,}000$ esu, $\chi_d{}^0\approx 100$, and $D\approx 10^{-2}$ cm, we have for barium titanate

$$f_0\approx 2\times10^9 \text{ cps},$$

where we have taken the wall as a single lattice constant thick.

Powles and Jackson,[3] and Von Hippel, have observed that at high frequencies the dielectric constant of polycrystalline barium titanate relaxes from about 1500 at 10^8 cycles to about 150 at 3×10^{10} cycles; the center of the relaxation spectrum appears to be at about 3×10^9 cps, in good agreement with our calculated resonance frequency. The fact that the observed curve does not have a resonance character is not in itself disturbing, as this might well be accounted for by a combination of frictional effects and spread in domain widths. Similar situations are encountered in the magnetic properties of the ferrites.

Although the extent of the agreement of theory with experiment is gratifying, it must be remembered that we have at present no independent or direct physical evidence that boundary displacements do in fact contribute to polarization processes in $BaTiO_3$ at high frequencies, and the observed dielectric dispersion at 10^9 cps may still be caused by processes other than that discussed here, although our process must certainly occur when the damping is sufficiently low. Our present experimental knowledge of the origin of polarization processes in ferroelectrics even in static fields is almost negligible.

I am grateful to Professor J. Bardeen for comments on the manuscript, to Dr. P. W. Anderson for several stimulating discussions of ferroelectric problems, and to Dr. E. A. Wood for her patient exposition of experimental data.

* Present address: Department of Physics, University of California, Berkeley, California.
[1] C. Kittel, Phys. Rev. **82**, 965 (1951).
[2] P. W. Anderson (private communication).
[3] J. G. Powles and W. Jackson, Proc. Inst. Elec. Engrs. **96**, III, 383 (1949); see also A. Von Hippel, Revs. Modern Phys. **22**, 228 (1950).

Relative Arrival Times of Air Shower Particles

V. C. OFFICER
Physics Department, University of Melbourne, Melbourne, Australia
(Received June 5, 1951)

THIS note reports the initial results of an experiment on the time relationship between the arrival of air shower particles at an unshielded tray and the arrival of penetrating electrons, photons, or their products at a tray shielded by 10 cm of lead. It appears, after a correction is applied, that the soft and penetrating components arrive within a time $<10^{-8}$ sec. However, preliminary results of a similar experiment[1] in which the lead thickness was 17.5 to 22.5 cm have shown time lags of the order of 0.4 μsec for the penetrating component.

The showers are detected by two 320-cm^2 timing trays of 2-cm diameter, counters 4 m apart, and an intermediate tray, all arranged in a horizontal straight line. Any one of the trays can be shielded with the lead. After wide band amplification and delay the signals from the timing trays are displayed and photographed as radial pulses on a 1-μsec per turn spiral time base triggered by the triple coincidences.[2] An additional delay in one channel insures separation of the pulses. As a result of counter reaction times

alone, the distribution of this separation has a spread of ± 0.20 μsec and a standard deviation of 0.06 μsec.

With the lead on one timing tray 200 showers have been observed, and with it on the other tray 229 showers were observed. In each case a time lag of about 2×10^{-8} sec was found to be associated with the shielded tray, the mean lag for the 429 showers being $2.1\pm0.3_6\times10^{-8}$ sec. As a check, 86 showers were observed with the lead on the intermediate tray. The timing trays should then be fired simultaneously, on the average, and in fact the observed lag of $0.7_4\pm0.6_6\times10^{-8}$ sec is not significantly different from zero. The origin of the time scale was found by feeding the same Geiger pulses into both channels, 47 observations resulting in a distribution with a standard deviation of 0.16×10^{-8} sec. Halfway through each run the counters in the timing trays were interchanged to cancel the effect of unequal average reaction times.

A further check on the performance of the apparatus had been provided by an earlier experiment designed to detect the time of flight of cosmic-ray particles able to penetrate 10 cm of lead. Over a path of 5.45 m the mean time of flight for 288 particles was $1.9\pm0.5\times10^{-8}$ sec, which is not significantly different from the 1.8×10^{-8} sec required at the velocity of light.

Because of a counter reaction time effect it is not necessary to assume that the soft and penetrating shower components arrive at different times. The shower rate of 1.5_4 per hour can be explained if 11 percent of the shower radiation penetrates 10 cm of lead. This means that the shielded tray is usually traversed by only one particle, whereas the unshielded tray is frequently hit by several particles and supplies a pulse having the shortest of the corresponding reaction times. This results in a time lag associated with the shielded tray.

Calculation of this effect[2] has been carried far enough to take into account 75 percent of the showers, and gives a time lag of $2.0_9\times10^{-8}$ sec, in agreement with the observed value. The agreement is rendered uncertain by the presence of two compensating effects. Overlapping the counters reduced their effective diameter and therefore reaction times, but scattering and firing by photons reduced the effectiveness of overlapping. It therefore seems reasonable to conclude only that the majority of the shower particles probably arrive at the apparatus within a time $<10^{-8}$ sec. This is consistent with the theoretical expectation of 10^{-9} sec as an approximate upper limit to this time interval, and, with the thickness of lead so far used, does not necessarily conflict with the results of Mezzetti *et al.* They found many lags of 0.2 μsec and some as high as 0.8 μsec for the 1 to 2 percent of shower particles that penetrate 22.5 cm of lead. These are included in the 11 percent penetrating 10 cm of lead in the present experiment, but the few long lags thus introduced would not affect the mean lag appreciably.

[1] Mezzetti, Pancini, and Stoppini, Phys. Rev. **81**, 629 (1951).
[2] Details of the electronics and the interpretation of results will appear in Aust. J. Sci. Research **4**, No. 4 (1951).

The Relative Phase of the Interaction Constants for Mixed Invariants in Beta-Decay*

L. C. Biedenharn and M. E. Rose
Oak Ridge National Laboratory, Oak Ridge, Tennessee
(Received May 14, 1951)

THE most general formulation of the (point) β-interaction involves an arbitrary mixture of the five invariants S, V, T, A, P formed by contracting bilinear covariants of the lepton and nuclear fields.[1] It is customary to assume real mixing (interaction) constants;[2] and, since complex constants can have an effect on the predicted results of the theory, the question arises as to whether the usual assumption of real constants is an unwarranted restriction. We have examined this question and conclude that the relative phases of the mixing constants must be zero (or π) and that the assumption of real constants is in fact no restriction on the theory.

For simplicity let us consider a mixture of scalar and vector interactions. The part of the total (hermitian) interaction energy corresponding to β^- emission then has the form,

$$H_\beta\sim(\Psi^+\Phi)(\psi^+\varphi)+(C_2/C_1)(\Psi^+\gamma^\mu\Phi)(\psi^+\gamma_\mu\varphi). \quad (1)$$

Writing the constant $C_2/C_1=A+iB$ we then consider the interaction (1) as a sum of three terms. Now by construction Eq. (1) is invariant to all continuous Lorentz transformations and, in addition, to space inversion. On the same footing as space-inversion invariance, time-inversion invariance of the physical predictions of the theory should also be required. The significant feature of time reversal is that it involves complex conjugation[3] and is, therefore, sensitive to the phase of the operators in Eq. (1). Using the correct time reversal operator,[4] we find that H_β' (the time-reversed H_β) is

$$H_\beta'\sim[(\Psi^+\Phi)(\psi^+\varphi)]^*+A[(\Psi^+\gamma^\mu\Phi)(\psi^+\gamma_\mu\varphi)]^* -B[(\Psi^+i\gamma^\mu\Phi)(\psi^+\gamma_\mu\varphi)]^*. \quad (2)$$

Since we require that $H_\beta H_\beta^*$ be invariant under time-inversion (or equivalently $|\int H_\beta d\tau|^2$), we conclude that $B\equiv0$ unless all cross terms sensitive to the reversal in sign vanish. These are terms $\sim B$; terms $\sim AB$ vanish identically. The cross terms ($\sim B$) do indeed vanish if we average over all spins, but they do not if we measure the spins of the particles involved. Since such an experiment is in principle meaningful, our conclusion on the relative phase of the interaction constants follows. The generalization to mixtures of all invariants is straightforward.

This result can be made somewhat more transparent by noting that the imaginary unit i is not a scalar under time reversal, and should be regarded strictly as an operator.[4] The covariants obtained with operators involving i, in order for physical results to be invariant under the extended group, must therefore be contracted into operators with similar tensor properties. The relative phase of the contracted covariants is then fixed.

The Tolhoek-de Groot symmetry principle[2] is dependent upon the phase of the mixing constants,[5] and the result that the (VT) and (SAP) invariants cannot mix may be relaxed for complex interaction constants. Our result shows that the Tolhoek-de Groot result on the nonmixing of (VT) and (SAP) is unique.

* This paper is based on work performed for the AEC at the Oak Ridge National Laboratory.
[1] E. J. Konopinski, Revs. Modern Phys. **15**, 213 (1943).
[2] H. A. Tolhoek and S. R. de Groot, Physica **16**, 456 (1950). We follow the notation of this reference.
[3] E. P. Wigner, Nachr. Akad. Wiss. Göttingen, Math.-physik K., 546 (1932).
[4] L. C. Biedenharn, Phys. Rev. **82**, 100 (1951).
[5] G. Trigg and E. Feenberg (private communication).

The Absorption of Cosmic Radiation at High Altitudes*

Martin A. Pomerantz
Bartol Research Foundation of the Franklin Institute, Swarthmore, Pennsylvania
(Received May 23, 1951)

IT has been pointed out previously[1] that small thicknesses of interposed absorber (1 cm of Pb−7.5 cm of Pb) do not effectively absorb primary cosmic-ray particles entering in the vertical direction at geomagnetic latitude 52°N, whereas absorption commences between 7.5 cm and 18 cm of Pb. It must be recalled that absorption is defined here as the reduction below unity of the probability that each primary, or at least one of its progeny created within the apparatus, always has sufficient residual range and is propagated in the proper direction to actuate the coincidence train.

The convergence near the "top of the atmosphere" of the intensity *vs* altitude curves obtained with different small thicknesses of interposed absorber has been confirmed[2] by numerous experiments performed since the initial series of balloon flights. It was originally possible to compare the results for 7.5 cm of Pb with