the axis was 1.2 cm. The number of "black" and "gray" tracks (grain density >1.4 times minimum) was used as an index of star size, in order that our results could be more readily compared with those of others.[5]

Figure 2, based on 3565 stars, shows their frequency *versus* depth. Frequencies were corrected for stars accumulated prior to the balloon flight, and during ascent and descent of the equipment. The upper points show the frequency of all stars with ≥3 prongs. The black circles refer to small stars, and the open circles to larger ones. The first circle in each set is plotted just beyond zero, to take account of a bit of overlying aluminum. The three diamond-shaped points give frequencies in "free air" emulsions, suspended far from any dense materials. These three values were obtained from the second flight and normalized to the first; the rest of the data are derived from a single flight. Stars initiated by π^--mesons coming to rest were not included in the data.

The results may be summarized as follows:

(1) The frequency of the larger stars, as well as that of the small ones, increases noticeably between "free air" and the upper surface of the Pb.[6] This suggests that some energetic secondaries (>200 Mev) with an appreciable upward component may be generated in the Pb.

(2) In the first centimeter there is a drop in frequency, associated with the smaller stars. This initial decrease has not been reported before. We have confirmed its existence in other experiments with geometries similar to this one. A possible explanation for this effect is given in the following letter.

(3) A maximum occurs at ≃27 g/cm², as in other experiments. However, it differs from previous results (except Blau's[7]) in that stars with >6 prongs contribute significantly to this effect.

(4) A broader peak at ≃60–70 g/cm², not observed heretofore, probably represents the major contribution of locally generated secondaries to the star population. This becomes more apparent as follows: Let us adopt as a measure of the transition effect at depth x, the ratio $T(x)$ of the observed star frequency $f(x)$ to that which would appear if the incident nucleonic radiation were absorbed exponentially without generating star-producing secondaries; i.e., let

$$T(x)=f(x)/f(\text{free air})e^{-x/\lambda}.$$

For λ, we take[3] the value 310 g/cm². Figure 3 gives $T(x)$ *versus* x for all stars ≥3 prongs.[8] The appearance of $T(x)$ suggests that the second maximum in Fig. 2 is due to a saturation of the star frequency at ≃70 g/cm².

In view of the near isotropy of intensity over a wide range of θ at high altitudes, our data could be plotted as a function of mean path of absorber traversed by the incoming radiation (at various θ), instead of the vertical depth. This procedure would be especially justified if the energy spectrum as a function of θ (not well known at these altitudes) could be taken into account. We are attempting to do this.

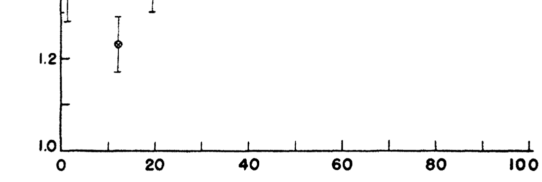

FIG. 3. Ratio $T(x)$ of observed star frequency to that expected if the incident nucleonic radiation were absorbed exponentially.

We are grateful to Mr. E. O. Davis for help with photoplate processing and microscopy; and to Miss K. DeAngelis, Mrs. D. Applyby, Mrs. N. Redfearn, and Mrs. F. Brewster for their assistance in scanning the emulsions. The flights were arranged through the ONR and the General Mills Aeronautical Research Laboratory.

* Reported at the Washington meeting of the American Physical Society, Bull. Am. Phys. Soc. **26**, No. 3, 7 (1951).

† See accompanying letter by M. M. Shapiro, Phys. Rev. **83**, 456 (1951).

[1] Bernardini, Cortini, and Manfredini, Phys. Rev. **74**, 845 (1948); Malaspina, Merlin, Pierucci, and Rostagni, Nuovo cimento **7**, 145 (1950); Schopper, Höcker, and Kuhn, Phys. Rev. **82**, 444 (1951). Earlier, an effect for singly charged particles had been found by Heitler, Powell, and Heitler, Nature **146**, 65 (1940).

[2] J. J. Lord and Marcel Schein, Phys. Rev. **75**, 1956 (1949); Blau, Nafe, and Bramson, Phys. Rev. **78**, 320 (1950). In both experiments, C.2 emulsion were placed horizontally at 4 depths, up to a total of 4 and 3 cm, respectively. Blau *et al.* also investigated the effect in Cu.

[3] George and Jason, Proc. Phys. Soc. (London) **A62**, 243 (1949). This value, $\lambda=310\pm20$ g/cm², is observed at altitude 3.5 km. It may be different in the stratosphere, but is unlikely to differ by as much as a factor 2.

[4] Thus far our data do not extend beyond 100 g/cm², since due allowance for rays at oblique incidence would require an absorber wider at the base. This was precluded by restrictions in flight pay-loads. Despite this limitation, the extra depth should prove useful in evaluating (a) λ for relativistic, star-generating protons incident at small (<25°) zenith angles; (b) attenuation of heavy primary nuclei in the Pb.

[5] We are now re-examining all the stars to arrive at a distribution based more directly on the energy release, including thin tracks.

[6] This effect is known for very small stars; see reference 2. It has not heretofore been reported for larger ones.

[7] Dr. M. Blau kindly informs us that she has observed a similar peak for stars with ≥6 prongs at ≃23 g/cm² Pb.

[8] The errors are probable errors in the relative values of $T(x)$. The absolute values of $T(x)$ are more uncertain, because they also involve an error in the free-air frequency.

Transition Effect in Pb of the Star-Producing Radiation in the Stratosphere. II

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(Received June 4, 1951)

SOME of the results on frequency of cosmic-ray stars *versus* depth in Pb reported in the preceding letter[1] may be provisionally interpreted as follows:

(1) *Transition effects for larger stars.*—At mountain altitudes, transition effects have been found only for 3- and 4-prong stars. In the stratosphere, on the other hand, they occur for larger stars as well. This is not surprising, since the N-radiation at pressure altitudes of 1 or 2 cm Hg contains a considerable proportion of particles with energies in the Bev region. Some of their secondaries have energies of hundreds of Mev, enough to produce stars with >6 prongs.

Another feature of the star-producing "primaries" in the stratosphere is that they arrive over a wider range of zenith angle than those at mountain altitudes. This may help explain some of the effects found at relatively shallow depths in an absorber. For example, the increase in star frequency between "free air" and the top of an absorber implies the production of rather energetic secondaries with an upward component of velocity. These can be generated by obliquely incident primaries even though the secondaries be emitted predominantly into the forward hemisphere.

(2) *Initial drop in frequency of small stars.*—This appears to be explicable in terms of the energy loss by ionization of relatively "slow" protons arriving obliquely—protons which can initially produce small stars, but are no longer able to do so after losing a large fraction of their energy in the lead. This diminution is closely connected with the type of geometry we employed.[2] The mean path $\bar{p}$ in our absorber for particles at zenith angles θ exceeding, say, 40° and arriving near the axis of the block at a depth of ≃1 cm is 3.6 cm (see Fig. 1). The energy spectrum of the protons incident at $\theta>40°$ contains relatively more slow (≃100–

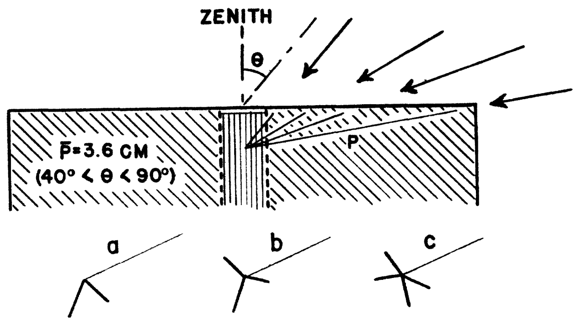

FIG. 1. Schematic section through upper portion of Pb absorber, showing paths of obliquely incident protons, and types of stars whose frequency should decrease as a result of ionization losses by these particles.

200 Mev) particles per steradian than that at smaller zenith angles, since a fuller development of the nucleonic cascade over longer atmospheric paths results in a degradation of this spectrum.

Now consider stars like those shown schematically in Fig. 1. When the upper track is due to an incoming proton, this particle typically possesses an energy of about 90, 150, and 210 Mev, respectively, for stars of type a, b, and c.[3] A proton of type a or b is stopped in less than 3 cm of Pb, and will therefore, on the average, fail to reach the emulsion from directions $\theta>40°$. As for a proton of type c, in traversing 3.6 cm of Pb, its energy is reduced by ionization to roughly 100 Mev. It will then, as a rule, not produce stars like b or c but can generate a star of type a. The net result is a reduction in the small-star frequency f_S at the indicated depth as compared with that above the lead.

The question remains whether the proton flux at this altitude, in the relevant E and θ intervals is such as to account for the observed drop of 14.2±5 percent in f_S. Detailed information on the proton energy spectrum as a function of zenith angle in the stratosphere is meager. However, we have used data obtained with emulsions[4] and counter telescopes[5] to estimate the proton flux in question, and have computed the decrease in f_S expected on the basis of the suggested mechanism. We get values ranging from 10 to 14 percent, in reasonable agreement with the observed effect.

(3) *Nature of the star-generating secondaries.*—Of the particles emitted in nuclear explosions, those most likely to produce further stars are neutrons, protons, and charged pions. It is uncertain, however, as to which of these types of secondaries is mainly responsible for the observed maxima in a Pb absorber.

In interpreting the transition effect at mountain altitudes, Dallaporta *et al.*[6] ascribe this role to neutrons, which are assumed to be produced by "evaporation" and emitted isotropically. Qualitatively, this theory explains the increase in rate of star production between free air and the upper surface of an absorber, as well as the further increase between the surface and interior. Quantitatively, when applied to a Pb cylinder 15 cm high, exposed at 4550 m, it predicts a peak at a depth of 8 cm. It does not, however, account for the position of the Pb maximum generally observed at mountain altitudes, which lies ≃1 cm deep.[7]

In the stratosphere, the multiplicative effects observed in a Pb block must be due largely to neutrons. However, in view of their long mean free path for star production, it is difficult to account, in terms of neutrons alone, for the observed variations in star frequency over short distances (e.g., the saturation which apparently sets in at ≃70 g/cm², and particularly, the peak at ≃27 g/cm²). Instead, it seems necessary to invoke charged secondaries as well. Since these lose at least part of their energy by ionization, the stars which they produce must occur, on the average, closer to their own point of origin than those due to neutrons. It appears to us, therefore, that an adequate theory of the transition effect must rely upon secondary protons and pions generated in the absorber, as well as upon neutrons.

(4) *Effects of geometry.*—A number of reasons can be given for the differences between results in the several stratosphere experiments. Neither the geometry nor the altitude nor the type of emulsion employed was the same in all cases. In particular, it is reasonable to expect[6] that an effect which depends on the production of secondaries within an absorber may be strongly influenced by its shape, its dimensions, and the arrangement of the emulsions inside it. We have evidence for this from other geometries which we have employed in different flights,[8] with results which differ in some respects from those reported here.[1] Hence we believe that one should not expect detailed agreement between experiments performed by different observers thus far, even at the same altitude. In checking any theory of the transition effect quantitatively against experiment, it will be necessary to take account of the particular geometry employed.

The author has benefited from discussions with his colleagues, Messrs. M. Birnbaum and B. Stiller.

[1] Shapiro, Stiller, Birnbaum, and O'Dell, Phys. Rev. **83**, 455 (1951).
[2] See reference 1, Fig. 1.
[3] Brown, Camerini, Fowler, Heitler, King, and Powell, Phil. Mag. **40**, 862 (1949).
[4] Hornbostel and Salant, private communication; Shapiro, Birnbaum, O'Dell, and Stiller, unpublished data.
[5] J. R. Winckler and W. G. Stroud, Phys. Rev. **76**, 1012 (1950); see especially Fig. 2(b).
[6] Dallaporta, Merlin, and Puppi, Nuovo cimento **7**, 99 (1950).
[7] A slight indication of a rise in frequency at 6 cm appeared in one mountain experiment, but the main peak there, too, was found at 1 cm. See Malaspina *et al.*, Nuovo cimento **7**, 145 (1950). This difficulty in the theory seems inherent in the reliance upon neutrons alone as the star-generating secondaries. It is not attributable to the choice of the evaporation mechanism, with its corollary of isotropic emission (although the use of an evaporation model for the production of neutrons ≳100 Mev is questionable). For the more realistic assumption of predominantly forward emission would displace the calculated peak deeper into the lead, rather than closer to the surface.
[8] See, e.g., M. M. Shapiro and A. F. Gabrysh, Bull. Am. Phys. Soc. **26**, No. 4, 10 (1951).

Erratum: A Re-evaluation of the Fundamental Atomic Constants

[Phys. Rev. **81**, 73 (1951)]

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IN Table VII the value of the Bohr magneton should be changed to read: $(9.27100\pm0.00017)\times10^{-21}$ erg gauss^{-1}.

Asymptotic Formula for Stopping Power of *K*-Electrons

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(Received May 31, 1951)

BROWN[1] has derived an asymptotic formula for the stopping number of K-electrons. This formula is useful for high energies of the incident particle, i.e., for large $\eta=mv^2/2$ (here and later we measure energies in units of Z_{eff}^2 Ry, the "ideal ionization potential" of the K-shell). v is the velocity of the incident particle and m is the electronic mass. Brown's formula for $B_K(\theta,\eta)$, the stopping number contribution of the K-electrons, is of the form

$$B_K(\theta,\eta)=A(\theta)\ln\eta+B(\theta)+C(\theta)\frac{1}{\eta}+\cdots,$$

with θ equal to the observed ionization energy of the K-shell.

It is the purpose of this letter to point out a correction to Brown's work which makes necessary a change in his results for $C(\theta)$ [see his formula (40)]. This correction has its origin in his calculation of the total stopping number of hydrogen, B_H. In that calculation it is necessary to integrate over all allowed $Q=(\mathbf{p}-\mathbf{p}')^2/2m$, where $\mathbf{p}$ and $\mathbf{p}'$ are the momenta of the incident particle before and after collision. For $Q_{\max}$ Brown has used 4η, whereas $Q_{\max}$ is[2] rigorously of the order of magnitude $(M/m)^2$ times 4η with M the mass of the incident particle, i.e., for heavy incident particles $Q_{\max}\approx\infty$. Furthermore, he has used correctly