partition, the latter has the value it would have if the gas temperature were zero. Its properties were examined in Sec. C.

The foregoing development completely solves the problem of intermediate fields if the assumption of a constant mean free time can be made. There remains the question what to do in other cases, particularly for the model treated in Sec. D. It is true that, in principle, the general problem could be solved by the method developed there. For the gas temperature complicates only the central term in (14), while the method of solution was based on the structure (35) of the higher outside term which remains unaffected. However, the further course of the calculation in Sec. D makes the method less desirable. We would be able to produce a number for the drift velocity for a given numerical ratio of the electric field and the temperature, but we would not gain direct information about the functional relationship. This relationship would only reveal itself indirectly after extended numerical computations. It is to be hoped that a more satisfactory way of proceeding can be found.

In conclusion, I wish to thank Miss C. L. Froelich and the computation staff of the Bell Telephone Laboratories for carrying out the computation mentioned in Sec. D.

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Coherent Scattering Processes Arising from Quantum Correlations

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Quantum statistical forces in ideal symmetric and antisymmetric fluids establish an ordered arrangement of their fluid molecules in space. The coherent scattering properties of these fluids resulting from this spatial order are studied in this paper.

I. INTRODUCTION

IN an earlier paper[1] we studied the incoherent slow neutron scattering by ideal monatomic symmetric, or Bose-Einstein (B.E.), and antisymmetric, or Fermi-Dirac (F.D.), fluids. These were presumed to represent possible asymptotic models of liquid He^4 and He^3, respectively. We should like to complete here the theory of the scattering properties of these fluids by an investigation of their coherent scattering, especially for slow neutrons, resulting from the respective spatial correlations of their atoms. These spatial correlations are caused by the quantum statistical attractive and repulsive forces in the phase space of these systems of ideal dimensionless atoms.

In the wave kinematic approximation the coherent scattering of short wave electromagnetic radiation by atoms is similar to the coherent scattering of slow neutrons with comparable de Broglie wavelengths. The possible, though small, neutron-electron interaction of nonmagnetic origin together with their electromagnetic coupling will be neglected here. Then the linear momentum exchange, with vanishingly small energy exchange, between these fluids and the incident slow neutrons, which is the coherent scattering process, is determined primarily by the specific slow nuclear scattering amplitudes of the fluid atoms. These quantities are, in turn, the specific amplitude structure factors of the nuclei for slow neutrons. These are supposed to be known, at present, only empirically, in contrast to the atomic structure factors for radiation which can be evaluated with sufficient precision from first principles. The additional difference between the two types of radiative and neutron scattering processes consists in the diversity of the nuclear scattering amplitudes for different neutron-nucleus spin configurations. This difference vanishes for nuclei of zero spin angular momentum, for instance, as in the case of He^4.

Our problem is to investigate the statistical or correlation coherent scattering structure factors of ideal symmetric and antisymmetric fluids. The study of the physical characteristics will then bring out a series of remarkable analogies exhibited by B.E. fluids and normal fluids both near and away from their respective critical regions.

The structure factors will be defined in the next section, while their evaluation and discussion will be reserved for the subsequent sections.

II. THE COHERENT SCATTERING OF SLOW NEUTRONS BY IDEAL SYMMETRIC AND ANTISYMMETRIC FLUIDS (ASYMPTOTIC LIQUID He^4 AND He^3 MODELS)

The possible practical interest of these fluids may be associated with He^4 and He^3 atoms with zero or half-unit of spin angular momenta. This justifies the limitation of the study of the scattering to these two spin cases. One of the main differences in the correlation of symmetric and antisymmetric fluids arises from the

[1] Goldstein, Sweeney, and Goldstein, Phys. Rev. **77**, 319 (1950).

fact that all atoms of the ideal symmetric fluid are statistically correlated, while an ideal antisymmetric fluid of atoms with spin $h/4\pi$ can be looked upon as a mixture of two fluids with oppositely directed spin momenta, with correlations existing only within each fluid of atoms of parallel spin directions. To the approximation of the present studies, atoms with opposite spin directions are uncorrelated and cannot give rise to interference of their respective scattered waves.

Let N spinless atoms, occupying volume V at temperature T, form an ideal symmetric fluid. Further, let $P_S(r, T)dv/V$ be the probability of finding an atom in the volume element dv with its center at a distance r from a given atom at the origin of the coordinate system. Here, $P_S(r, T)$ is the spatial probability distribution function. In a classical ideal gas, this quantity reduces to unity. In ideal symmetric and antisymmetric fluids, the departures of the probability distribution from unity are pure quantum effects, and it is justifiable to denote these as quantum correlations.

The coherent scattering cross section per unit solid angle of a symmetric collection of N atoms, for neutrons of initial and final propagation vectors $\mathbf{k}$ and $\mathbf{k}'$, may be written as

$$S_S(\Delta k, T)=V^{-2}\sum_{i=1}^{N}\sum_{j=1}^{N}\int_V \cos(\boldsymbol{\Delta k}\cdot\mathbf{r}_{ij}) \times\sigma_{ij}P_S(r_{ij}, T)dv_i dv_j. \quad (1)$$

This cross-section formula is more general than the one used for the scattering of x-rays by atoms.[2] Here, σ_{ij} is an elementary cross section to be defined below,

$$|\boldsymbol{\Delta k}|=|\mathbf{k}'-\mathbf{k}|=2k\sin\theta=4\pi(\sin\theta)/\lambda \quad (2)$$

is the momentum loss of the neutron on scattering expressed in units of $h/2\pi$, 2θ is the scattering angle, and λ the neutron wavelength. The argument of the cosine is the phase difference between the waves scattered by the atoms i and j of separation r_{ij} or $|\mathbf{r}_i-\mathbf{r}_j|$. Of the N^2 terms in Eq. (1), the N diagonal terms are the individual contributions of the N atoms, with σ_{ii} for all $i(1, 2, \cdots, N)$ being the single atom nuclear coherent cross section per unit solid angle. In order for a single atom to scatter coherently, it is necessary that its elastic cross section be the one associated with an apparently infinitely heavy nucleus. Hence,

$$\sigma_{ii}=\sigma_s(1+A^{-1})^2=\sigma_\infty, \quad (3)$$

where A denotes the mass of the scattering fluid atoms in units of the neutron mass, and σ_s stands for the slow neutron elastic cross section of the free atom per unit solid angle and in the relative motion. Since the slow neutron scattering process involves only neutrons of zero relative angular momentum, the scattering is spherically symmetrical in the relative motion and is characterized by a unique s-wave phase shift or scattering amplitude, $\pm a$, of either sign. At the low neutron energies envisaged here, a may be said to represent the asymptotic infinite wavelength scattering amplitude and should thus be an energy-independent quantity. The $N(N-1)$ off-diagonal mutual interference terms contribute all the same cross section. Thus one finds

$$S_S(\Delta k, T)=N\sigma_\infty\Big[1+V^{-2}(N-1)\int_V \cos(\boldsymbol{\Delta k}\cdot\mathbf{r}_i-\mathbf{r}_j) \times P_S(r_{ij}, T)dv_i dv_j\Big]. \quad (1a)$$

Now, the two atom probability distribution function can be written as

$$P_S(r_{ij}, T)=1+Q_S(r_{ij}, T), \quad (4)$$

where Q_S is the specific quantum correlation function of ideal symmetric fluids. Replacing Eq. (4) into Eq. (1a) and performing the integration on $\cos[\boldsymbol{\Delta k}\cdot(\mathbf{r}_i-\mathbf{r}_j)]$ leads to the structure factor of ideal fluids in a finite volume V, assumed to be a sphere of radius R. On evaluation of the angular integrals over the quantum correlation term, one obtains

$$\begin{aligned}S_S(\Delta k, T)/N\sigma_\infty &=1+(9\pi/2)(N-1)J^2_{\frac{3}{2}}(x)x^{-3} \\ &\quad+[4\pi(N-1)/V\Delta k]\int_0^\infty \sin(r\Delta k)Q_s(r, T)rdr \\ &=F_S^2(\Delta k, T);\quad x=R\Delta k. \end{aligned} \quad (5)$$

The radial integration in the quantum correlation term is extended to infinity, because the correlation function $Q_S(r, T)$ vanishes, in general, quite rapidly with increasing r. The ratio $S_S/N\sigma_\infty$ is the intensity structure factor per atom of the ideal symmetric fluid.

In an ideal antisymmetric fluid of atoms with spin $h/4\pi$, the two-atom correlations refer only to atoms of parallel spin directions. With

$$P_A(r_{ij}, T)=1-Q_A(r_{ij}, T), \quad (6)$$

where the negative sign in front of Q_A expresses the peculiar spatial repulsion between atoms of parallel spin, one obtains, using Eq. (1) and performing calculations similar to those leading to Eq. (5), the structure factor $F_A^2(\Delta k, T)$. The latter resembles Eq. (5) with $(\frac{1}{2}N)$ replacing N and with a negative sign in front of the quantum correlation integral which includes $Q_A(r, T)$ instead of $Q_S(r, T)$. The incident neutron beam is supposed to be unpolarized. The antisymmetric fluid cross section per unit solid angle $S_A(\Delta k, T)$ or $NF_A^2(\Delta k, T)\sigma_\infty$ is again determined by the infinitely heavy atom elastic cross section per unit solid angle σ_∞, in spite of the fact that the scattering amplitudes in the

[2] A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand Company, Inc., New York, 1935), second edition, pp. 177–181.

triplet and singlet spin configurations are assumed to be different. Here,

$$\sigma_\infty=(1+A^{-1})^2[\tfrac{3}{4}({}^3a)^2+\tfrac{1}{4}({}^1a)^2].$$

The nonoccurrence of the coherent cross section in the $N(N-1)$ mutual interference terms is due to the absence of correlation between atoms of opposite spin directions. This eliminates the interference between the singlet and triplet amplitudes, $({}^1a)$ and $({}^3a)$.

Our problem is to investigate the intensity structure factors $F_S^2(\Delta k, T)$ and $F_A^2(\Delta k, T)$. Beside the evident academic interest attached to the scattering properties of these ideal quantum fluids, the possibility that these fluids might represent asymptotic models of liquid He^4 and He^3 would further justify the study of these processes. Of the two fluids, the symmetric one appears to be of considerably greater practical interest with respect to the scattering of slow neutrons because of the enormous slow neutron absorption cross section of the He^3 nuclei.[1,3] The experimental investigation of slow neutron scattering by liquid He^4 could yield additional information which might prove helpful in a better understanding of this liquid.

III. THE COHERENT SCATTERING STRUCTURE FACTOR OF IDEAL SYMMETRIC FLUIDS

The structure factor is determined by the two-atom probability distribution function $P(r, T)$, defined above. Both in symmetric and antisymmetric fluids the evaluation of the distribution function is straightforward. It is the coordinate space average of the probability density of these fluids over $(N-2)$ of their N atoms and its momentum space average over the momenta of all the atoms. These functions were first obtained in the limit of small degeneration.[4] The rigorous symmetric fluid distribution and correlation functions were also derived.[5] Neglecting terms of the order of $1/N$, these are given by

$$P_S(r, T\geqslant T_0)=1+Q_S(r, T\geqslant T_0)$$
$$=1+\left\{(T/T_0)^{\frac{3}{2}}F^{-1}(0)\sum_{l=1}^{\infty}l^{-\frac{3}{2}}\times\exp[-l\alpha-(\pi r^2/l\Lambda^2)]\right\}^2, \quad (7)$$

$$P_S(r, T\leqslant T_0)=1+Q_S(r, T\leqslant T_0)$$
$$=1+\left[(T/T_0)^{\frac{3}{2}}F^{-1}(0)\sum_{l=1}^{\infty}l^{-\frac{3}{2}}\exp(-\pi r^2/l\Lambda^2)\right]^2$$
$$+2(T/T_0)^{\frac{3}{2}}F^{-1}(0)[1-(T/T_0)^{\frac{3}{2}}]\times\sum_{l=1}^{\infty}l^{-\frac{3}{2}}\exp(-\pi r^2/l\Lambda^2), \quad (8)$$

above and below the condensation temperature T_0, respectively. Here

$$F(0)=\sum_{l=1}^{\infty}l^{-\frac{3}{2}}=2.612, \quad \Lambda=h/(2\pi MkT)^{\frac{1}{2}}, \quad (9)$$

Λ denotes the de Broglie wavelength of the fluid atoms associated with their average thermal motion, M the mass of the atoms; α is the negative Gibbs free energy per atom in units of kT. One of the two atoms of the pair, in Eqs. (7) and (8), is at the origin of the coordinate system. The functions $Q_S(r, T)$ are the characteristic quantum correlation functions. They vanish in the limit $h\to 0$.

Substituting Eq. (7) into Eq. (5) and performing the calculations, one finds the following intensity structure factor formula per atom:

$$F_S^2(\Delta k, T>T_0)=1+\Phi_i^2(R\Delta k)+F^{-1}(\alpha)\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}(l+m)^{-\frac{3}{2}}\times\exp[-(l+m)\alpha-(lm/l+m)(\Lambda\Delta k)^2/4\pi]. \quad (10)$$

Here $\Phi_i^2(R\Delta k)$ is the ideal fluid structure factor defined by the second term on the right-hand side of Eq. (5). In the specific quantum statistical term, $(N-1)$ was replaced by N, since $N\gg 1$. Below the condensation temperature one obtains, with Eq. (8),

$$F_S^2(\Delta k, T<T_0)=1+\Phi_i^2(R\Delta k)+(N(T)/N)\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}(l+m)^{-\frac{3}{2}}\times\exp[-(lm/l+m)(\Lambda\Delta k)^2/4\pi]+2(N_c(T)/N)[(\exp[(\Lambda\Delta k)^2/4\pi])-1]^{-1}, \quad (11)$$

where

$$N(T)/N=(T/T_0)^{\frac{3}{2}}, \quad N_c(T)/N=1-(T/T_0)^{\frac{3}{2}} \quad (12)$$

denote the fractions of the excited and condensed atoms, respectively.

The ideal fluid structure factor $\Phi_i^2(R\Delta k)$ is negligibly small for all finite values of Δk or finite scattering angles. It has the limit $(N-1)$ or N in the forward direction associated with Δk or $(4\pi\sin\theta)/\lambda\to 0$. In this ideal limit all atoms scatter in concordance of phase. We shall, however, omit this term from the structure factor formulas (10) and (11). This is justified because all limiting small angle scattering considered below will be associated with directions or propagation vector changes Δk such that for these Φ_i^2 is still negligible.

The preceding symmetric fluid structure factors, with the classical term omitted, have been derived by Galanin[5] in connection with his formal studies on the density fluctuations and scattering of x-rays and visible radiation by ideal B.E. fluids. The independence of the structure factor from the type of waves is, of course, evident. The physical discussion of the scattering phenomenon together with the analogies in the

[3] L. D. P. King and L. Goldstein, Phys. Rev. **75**, 1366 (1949).
[4] G. E. Uhlenbeck and L. Gropper, Phys. Rev. **41**, 79 (1932).
[5] A. D. Galanin, J. Exp. Theoret. Phys. (U.S.S.R.) **10**, 1267 (1940). F. London, J. Chem. Phys. **11**, 203 (1943).

qualitative behavior of ideal symmetric and normal fluids have not been given by this writer. Furthermore, some of the conclusions reached by him in a more recent work on the scattering properties of a modified B.E. fluid model[6] do not appear to be free from objections. We shall return to this point later.

It is possible to obtain a rigorous lower bound for the double sums in Eqs. (10) and (11) and to derive thereby practically rigorous analytically closed structure factor formulas. Indeed, the running factor $lm/(l+m)$ in the exponents of the exponentials in the double sums has its extreme value l. The exponent with the double index will be approximated with the help of the inequality

$$\beta lm/(l+m)\leqslant \beta l;\quad \beta=(\Delta\Delta k)^2/4\pi;\quad l, m=1, 2, \cdots, \infty, \tag{13}$$

$4\pi\beta$ being the square of the neutron momentum change on coherent scattering expressed in units of the momentum associated with the average thermal motion of the atoms. In thus stripping the factor $lm/(l+m)$ of the index m, the double sums can be reduced to a simpler approximate expression, their rigorous lower bound. One thus obtains, after a somewhat lengthy calculation, the following lower limits of the structure factors:[7]

$$F_{S,\,\mathrm{inf}}{}^2=[1-F(\alpha+\beta)F^{-1}(\alpha)](1-e^{-\beta})^{-1},\quad T\geqslant T_0, \tag{14}$$

$$S_{S,\,\mathrm{inf}}{}^2=[1-(T/T_0)^{\frac{3}{2}}](e^{\beta}-1)^{-1}$$
$$+[1-(T/T_0)^{\frac{3}{2}}F(\beta)F^{-1}(0)](1-e^{-\beta})^{-1},\quad T\leqslant T_0, \tag{15}$$

$$F(\alpha)=\sum_{l=1}^{\infty} l^{-\frac{3}{2}}\exp(-l\alpha).$$

Approximate upper limits of the structure factor result from the lower limits on substituting $\beta/2$ for β in the preceding formulas.

The resemblance of the preceding approximate coherent structure factors or coherent symmetric cross sections per atom to the rigorous incoherent or inelastic cross sections obtained previously[1] is evident. It should be remembered, however, that this similarity concerns mainly the incoherent cross section in the relative motion. The coherent cross sections (10) and (11) or (14) and (15) are evaluated, necessarily, in a fixed coordinate system. Also, the incoherent cross sections have been obtained under the condition that the neutron kinetic energy is larger than the kinetic energy of thermal motion of the fluid atoms. No such restrictions have been imposed here.

Using Eqs. (14) and (15), simple expressions can at once be derived for the limiting values of the lower bounds of the structure factors. Keeping the temperature constant, one obtains in the limit of large β, that is, for large angle scattering or large momentum loss of the scattered waves,

$$\lim_{\beta\gg 1} F_{S,\,\mathrm{inf}}{}^2=1-F^{-1}(\alpha)\lim_{\beta\gg 1}F(\alpha+\beta)$$
$$\approx 1-F^{-1}(\alpha)e^{-\alpha-\beta},\quad T>T_0, \tag{16}$$

which is practically unity. Since our discussion will be based on the lower bounds Eqs. (14) or (15), we shall henceforth omit the subscript inf of the structure factors. It is seen that at large angles the angular dependence of the structure factor tends to vanish leading only to the diffuse coherent scattering expressed by the self-interference term or unity on the right-hand side of Eq. (16).

At a constant value of the scattering angle, for given incident waves, the structure factor increases with decreasing temperature because of the increasing range of the correlation function $Q_S(r, T)$. The structure factor reaches a maximum at the condensation temperature, where its value is

$$F_S{}^2(\Delta k, T_0)=[1-F(\beta)F^{-1}(0)](1-e^{-\beta})^{-1}. \tag{17}$$

In the limit of very small momentum changes or $\beta\ll 1$, one obtains, expanding both the numerator and denominator,

$$\lim_{\beta\ll 1} F_S{}^2(\Delta k, T_0)=\lim_{\beta\ll 1}[-dF/d\beta]F^{-1}(0)$$
$$=\pi^{\frac{1}{2}}\beta^{-\frac{1}{2}}F^{-1}(0), \tag{18}$$

which may become very large for small β-values. One might object here, on analytical grounds, to the passage to the limit of small β-values in Eq. (17). This, however, could be obviated by writing $F(\frac{1}{2}\beta+\frac{1}{2}\beta)$ for $F(\beta)$ and expanding around $(\beta/2)$. Or one might divide the interval β into two parts and expand around the larger value. This would only introduce a numerical constant in the limiting structure factor (18), while the characteristic $\beta^{-\frac{1}{2}}$ behavior would, of course, be unchanged.

At temperatures $T<T_0$, the structure factor (15) leads to the following limiting behavior:

$$\lim_{\beta\gg 1} F_S{}^2(\Delta k, T<T_0)=1+e^{-\beta}[1-(T/T_0)^{\frac{3}{2}}(1+F^{-1}(0))], \tag{19}$$

which is similar to the large β-limit above T_0, Eq. (16), in so far as the angular dependence or β-dependence vanishes exponentially. In the opposite limit of small β, one finds, using the expressions (12) of the condensed and excited fractions of the atomos,

$$\lim_{\beta\ll 1} F_S{}^2\approx(N_c(T)/N\beta)+(\Gamma(\tfrac{1}{2})N(T)/NF(0)\beta^{\frac{1}{2}}), \tag{20}$$

where the first term on the right-hand side is the dominant term. This shows that in the condensation region the coherent scattering process at small angles is due mainly to the condensed atoms. As the temperature increases toward the condensation temperature T_0, the importance of the second term on the right-hand side associated with the excited atoms increases; and at T_0,

[6] A. Galanin, J. Exp. Theoret. Phys. (U.S.S.R.) **19**, 175 (1949).
[7] The function $F(x)$ should not be confused with the structure factors which we always write in the form of squares.

expression (20) becomes identical with (18), the latter limit being reached from above T_0. This proves the continuity of the structure factor at the condensation temperature, a result which was seen to be valid in the incoherent scattering process[1] also. It can be shown, furthermore, that the temperature derivative of the structure factor is discontinuous at the condensation temperature. This discontinuity is due to the interference between the waves scattered by condensed and excited atoms. The interference term is given by the first term on the right-hand side of Eq. (15). It does not appear in the incoherent process whose cross section is continuous together with its first temperature derivative across the condensation line.

Since in the scattering of visible light the characteristic quantity β is very small, one would expect, in the He II or condensation range, the structure factor (20) to reduce to its first term which is then quite large. This result for visible radiation was obtained recently by Galanin[6] using a modified B.E. fluid model. However, in the limit $\beta \gg 1$, this model still leads to a relatively large structure factor, varying essentially as $\beta^{-\frac{1}{2}}$. Physically, in this limit the coherent scattering should become independent of the momentum change and the mean correlation distance characteristic of the scattering fluid, which, in the ideal symmetric fluid, is Λ or $h/(2\pi MkT)^{\frac{1}{2}}$. This behavior is clearly exhibited by our formulas (16) and (19), which reduce essentially to the diffuse scattering term corresponding to unity on the right-hand side of these relations. It is worth noticing that Galanin's model[6] resembles somewhat the one used by Brillouin[8] in his investigations of the coherent scattering of electromagnetic radiation of any wavelength by transparent solids and liquids. However, Brillouin's theory leads to the correct physically expected variation of the structure factor, namely, $\exp(-\beta^{\frac{1}{2}})$, if one reinterprets our Λ as the wavelength of the elastic waves in the medium which reflect the incident electromagnetic waves in Brillouin's dynamic model of the scattering medium.

IV. ANALOGIES BETWEEN THE COHERENT SCATTERING OF RADIATION OR SLOW NEUTRONS BY IDEAL SYMMETRIC AND REAL FLUIDS

In the preceding discussion of the B.E. coherent structure factor we have limited ourselves purposely to the condensation region and the vicinity of the condensation temperature or the saturation line in the single phase modification of this fluid. We should like to study now the coherent scattering at temperatures $T > T_0$, e.g., throughout the single phase region.

Let the fluid be at some temperature distinctly higher than its condensation temperature T_0. The parameter α is then large; i.e., it is of the order of unity. It is to be remembered that near T_0, this parameter is extremely small, of the order of N^{-1}, N being the total number of atoms of the fluid. At $T > T_0$, one finds with Eq. (14), for small values of β,

$$\lim_{\beta \ll 1} F_S^2(\beta, T > T_0) = -[F(\alpha)]^{-1}(dF/d\alpha) = \sum_{l=1}^{\infty} l^{-\frac{1}{2}} e^{-l\alpha} \Big/ \sum_{l=1}^{\infty} l^{-\frac{1}{2}} e^{-l\alpha}. \quad (21)$$

In this limit the scattering structure factor becomes independent of β. This result is the same as the one obtained with the center-of-gravity coordinate system incoherent structure factor.[1] The structure factor (21), although derived by using the lower bound (14), turns out to be exact, the lower bound becoming identical with the rigorous structure factor in this limit of small β. The proof of this is simple and will be omitted here. Recalling now the expression (13) of the scattering parameter β, it is seen that small β means:

(a) $\lambda > \Lambda$; or $\lambda \gg \Lambda$, θ finite,

(b) $\lambda \approx \Lambda$; or $\lambda < \Lambda$, θ small.

In case (a), the wavelength of the incident radiation is large in comparison with the mean correlation range Λ of the correlation function Q_S, Eqs. (7) and (8). Since the B.E. fluid might be considered to represent some asymptotic model of liquid He^4, numerically, the approximate correlation range Λ is about 10^{-7} cm at 3°K. Hence, in liquid He, case (a) is realized for visible light at all scattering angles. Case (b) corresponds to the small angle scattering of slow neutrons, thermal or subthermal, and to that of x-rays. Equation (21) expresses the result that both for radiation of the visible region and of short wavelength and slow neutrons, but with small momentum change on scattering, the structure factor becomes independent of the details of the scattering process, that is, of λ and θ as well as of the molecular characteristics of the fluid, such as the correlation distance. Under these conditions, the structure factor depends only on the over-all thermal properties of the fluid. Indeed, with the equation of state of B.E. fluids, one obtains, without difficulty,

$$-F^{-1}(\alpha)dF/d\alpha = -NkTV^{-2}(\partial V/\partial p)_T = NkT\chi_T/V, \quad (22)$$

where χ_T stands for the isothermal compressibility of the fluid. Now, the right-hand side of Eq. (22) is N times the relative mean square fluctuation of the particle concentration (N/V) or n, or that of the number of particles N,

$$-F^{-1}(\alpha)dF/d\alpha = N\langle \Delta n^2 \rangle_{Av}/n^2 = \langle \Delta N^2 \rangle_{Av}/N. \quad (23)$$

Consequently, the coherent intensity structure factor in the limit of $\beta \ll 1$ becomes

$$\lim_{\beta \ll 1} F_S^2(\beta, T > T_0) = NkT\chi_T/V = \langle \Delta N^2 \rangle_{Av}/N. \quad (24)$$

[8] L. Brillouin, Ann. phys. (9), **17**, 88 (1922).

This shows that in this limit the ideal B.E. fluid scatters coherently as would a normal monatomic fluid far from its critical region, according to Einstein and Smoluchowski.[9] This, of course, is just as it should be physically, because for visible light of wavelength λ far exceeding the molecular correlation length and for x-rays or slow neutrons whose phase change on scattering is also very small at small or moderate scattering angles, the spatial arrangement of the molecules, as described by the distribution or correlation functions, can have no explicit effect whatsoever on the scattering process, since large groups of atoms scatter essentially in phase. A more complete theory valid both for visible light and x-rays in transparent media, solids, or liquids has been proposed by Brillouin.[8] The proportionality of the coherent structure factor of x-rays to the compressibility of the scattering medium is fully contained in Brillouin's work. That this proportionality for x-ray scattering should exist only at small scattering angles was first shown by Zernike and Prins.[10] These writers have outlined the formalism of the coherent scattering phenomena of x-rays by liquids by introducing explicitly, in a formal way, the normal fluid distribution or correlation functions in the theory of these processes. The absence of a satisfactory theory of such correlation functions in normal fluids prevented them from obtaining any explicit liquid structure factor.

Let us consider again the limiting B.E. coherent structure factor (21) or (24). As the temperature of the fluid decreases toward the condensation temperature, the concentration or density fluctuations increase without limit, an anomalous situation quite similar to what obtains in the critical region of normal fluids. A large increase in the intensity of the coherently scattered radiation by ideal B.E. fluids should be expected, the loss of validity of the fluctuation formalism notwithstanding, as pointed out by us some time ago.[11] Ideal B.E. fluids should exhibit a critical opalescence type of effect near their saturation line. It is seen that this conclusion is valid also for the small angle scattering of slow neutrons or x-rays, whose structure factor becomes highly asymmetrical. The scattered slow neutrons or x-rays should appear essentially in the forward direction with considerably reduced relative lateral scattering as the symmetric fluid approaches its transition temperature. It should be noted, in this connection, that the saturation curve of ideal symmetric fluids is the geometrical locus of points of infinite compressibility or vanishing slopes $(\partial p/\partial V)_T$ of their isothermal curves. Since $(\partial^2 p/\partial V^2)_T<0$, the isothermals have their maximum[12] along the saturation or transition line in the pressure-volume diagram. These isothermals, when extrapolated into the two-phase or condensation region from the single phase low density or vapor region, take on the physically excluded behavior of decreasing pressures with decreasing volume. As a result, the ideal B.E. fluids undergo a smooth condensation process[12] whereby supersaturation is prohibited. This type of condensation is quite similar to the condensation of ordinary fluids in their critical region where the isothermals appear also to reach the vapor saturation line with small slopes tending to vanish in the vicinity of the critical point. The similarity in the thermodynamic behavior of ideal B.E. fluids, with attractive statistical forces but no interatomic forces, and of normal fluids, with attractive intermolecular forces but no statistical forces, extends thus into their respective coherent scattering properties near their critical transition regions. However, in ideal symmetric fluids the difficulty of the fluctuation theory of scattering in the vicinity of the saturation line is averted by the rigorous structure factor formulas (10) and (11) or by their lower bounds (14) and (15). Indeed, the preceding discussion of the structure factors (21), (22), and (24) helped to show how the molecular theory of scattering joins with the thermodynamic statistical theory in the limit of small momentum changes of the waves on coherent scattering. In order to obtain the rigorous scattering law near the transition line in this same case of small momentum changes, one has to evaluate $F_S^2(\beta_0, T_0)$ with (14) and then obtain its limit

$$\lim_{\beta_0\ll 1} F_S^2(\beta_0, T_0)\cong\pi^{\frac{1}{2}}\beta_0^{-\frac{1}{2}}F^{-1}(0);\quad \beta_0=\Lambda_0^2(\Delta k)^2/4\pi, \quad (18a)$$

which may be quite large but finite. In the condensation region, as shown already above, the small momentum change structure factor is also finite (Eq. (20)), although it may become quite large. This, then, proves the general validity of the rigorous structure factor formulas or of their simpler lower bounds both as far as the state of the symmetric fluid and the details of the scattering process are concerned.

V. ANALOGIES BETWEEN THE MOLECULAR CORRELATIONS IN IDEAL SYMMETRIC AND REAL FLUIDS

It appears to be of interest, at this juncture, to discuss the classical intuitive solution of the critical opalescence problem for visible light proposed by Ornstein and Zernike.[13] These authors were the first to recognize the fundamentally incomplete character of Einstein and Smoluchowski's[9] statistical theory of light scattering in the vicinity of the critical state of normal fluids. The normal fluid structure factor per atom in the Ornstein-Zernike theory can be written for long wave radiation in the form

$$F_N^2(\beta_N, T)=(NkT\chi_T/V)[1+(NkT\chi_T\beta_N/V)]^{-1},$$

$$\beta_N=(\epsilon\sin\theta/\lambda_N)^2, \quad (25)$$

[9] A. Einstein, Ann. Physik **33**, 1275 (1910); M. v. Smoluchowski, Ann. Physik **25**, 205 (1908).
[10] F. Zernike and J. A. Prins, Z. Physik **41**, 184 (1927).
[11] L. Goldstein, Phys. Rev. **57**, 241, 457 (1940).
[12] L. Goldstein, J. Chem. Phys. **14**, 276 (1946).
[13] L. S. Ornstein and F. Zernike, Amsterdam Proc. **17**, 793 (1914); Physik. Z. **19**, 134 (1918); and **27**, 761 (1926).

where ϵ stands for the short range of the intermolecular forces, and λ_N or λ/n is the wavelength within the scattering fluid, n denoting its refractive index. Clearly, at the critical point the structure factor behaves as β_N^{-1}. This is similar to the structure factor of ideal symmetric fluids below the transition temperature for small values of β (Eq. (20)).

For a brief study of the analogies between normal fluids and the ideal symmetric fluid, let us return to the distribution functions $P_S(r, T)$ or the correlation functions $Q_S(r, T)$ contained in Eqs. (7) and (8). The sums appearing in these correlation functions can be evaluated to a fair approximation by transforming them into integrals. One thus obtains, at once,

$$\bar{Q}_S(r, T \gtrsim T_0)$$
$$= F^{-2}(\alpha)\left(\int_0^\infty l^{-\frac{1}{2}} \exp[-l\alpha - (\pi r^2/l\Lambda^2)]dl\right)^2$$
$$= \Lambda^2 F^{-2}(\alpha) r^{-2} \exp[-r/(\Lambda/4(\pi\alpha)^{\frac{1}{2}})], \quad (26)$$

and

$$\bar{Q}_S(r, T \lesssim T_0) = (2N_T N_c N^{-2} F^{-1}(0)\Lambda) r^{-1} + (N_T N^{-1} F^{-1}(0))^2 \Lambda^2 r^{-2}. \quad (27)$$

It is clear, of course, that these approximate correlation functions lose their validity at small distances r, since the integrals over the summation index l tend to diverge at vanishing separations. However, these functions are probably better than asymptotic in the sense that they should approximate fairly well the rigorous correlation functions (7) and (8) beyond distances $r_c(T)$ which are the roots of the transcendental equations obtained by equating Q_S and $\bar{Q}_S$, in the two temperature regions, respectively. The remarkable differences in the quantum correlations (26) and (27) above and below the transition temperature are more clearly exhibited by the functions $\bar{Q}_S$ than by the rigorous but unwieldy correlations Q_S, (Eqs. (7) and (8)). Above the transition temperature, one is confronted with a rather short-range correlation function varying as $\exp(-r/\bar{r})/r^2$, the range being, at $T > T_0$, where α is of the order of unity, somewhat less than Λ, or

$$\bar{r}(T) = \Lambda/4(\pi\alpha)^{\frac{1}{2}}. \quad (28)$$

As the fluid temperature decreases toward its condensation temperature, the range $\bar{r}(T)$ increases and may be said to become very large, of the order of the linear dimensions of the vessel, at the condensation temperature, since $\alpha \to N^{-1}$. The divergence of the correlation range in the condensation region can be proved rigorously using the exact function $Q_S(r, T < T_0)$ (Eq. (8)). Along the saturation line the correlation function behaves as a long-range function, or

$$\bar{Q}_S(r, T_0) = \Lambda^2 F^{-2}(0) r^{-2}. \quad (29)$$

Below the condensation temperature, in the two-phase region, the correlation function is a very long-range function, since for all significant distances it varies as r^{-1}. It is to be noted that both the rigorous and approximate symmetric fluid correlation functions, though continuous, have a break at the saturation line; i.e., their temperature derivative is discontinuous. It is also of interest to note that the long range, $1/r$, quantum correlation in the condensation region is proportional to the product of condensed N_c and excited atoms N_T and is due to the statistical attraction of these atoms. The statistical forces between excited atoms give rise to spatial correlations decreasing, at best, as $1/r^2$ at or below the saturation line.

In normal liquids, the functional form expressing the correlation between a molecule at the origin and one at a distance r is given, asymptotically, by the correlation density

$$g(r, T) = [3F(T)/2\pi\epsilon^2] \exp(-r/\bar{r})/r;$$
$$F(T, \epsilon) = \int_V f(r, \epsilon, T)dv, \quad (30)$$

where $f(r, \epsilon, T)$ is the local distribution function, whose mean range ϵ has been defined in connection with Eq. (25). It is seen that $g(r, T)$ corresponds to $(N/V)Q_S(r, T)$ in ideal symmetric fluids. The range $\bar{r}$ is given here by

$$(\bar{r}(T, \epsilon)/\epsilon)^2 = (1/6)[1 - F(T, \epsilon)]^{-1}. \quad (31)$$

Assuming further that for very large volumes the classical concentration fluctuation formula (24) remains valid, it can be shown that

$$F(T < T_c) < 1, \quad \lim_{T \to T_c} F(T) \to 1 \text{ and } \lim_{T \to T_c} r(T) \to \infty.$$

As the critical state is approached, the correlation function (30) tends to become the very long range function $3/(2\pi\epsilon^2 r)$.

The analogy between the distance dependence of the asymptotic quantum correlation function $\bar{Q}_S(r, T > T_0)$ and the corresponding real fluid correlation density (30) at liquid temperatures distant from the critical one is thus quite close. The important behavior is the exponential one in both cases, and the difference in the r^{-1} and r^{-2} factors in the two cases is of no importance. However, the asymptotic $1/r$ dependence of the correlation functions in the critical region of normal fluids and in the condensation region of the ideal symmetric fluid is identical.

We should like to conclude this section by noting that with Eqs. (5), (10), (21), (22), and (24) the particle concentration fluctuations in a volume V of the ideal symmetric fluid, in equilibrium with a reservoir, may be written as

$$\langle \Delta N^2 \rangle_{Av} = N\left[1 + (N/V)\int_V Q_S(r, T) 4\pi r^2 dr\right]; \quad (32)$$

a formula which was first derived by Ornstein and

Zernike.[13] Actually, the rigorous expression (32) is due to Zernike and Prins.[10] Again the complete formalism of ideal symmetric fluids accounts fully for this molecular type of formula. It shows further that the fluctuations are finite for $T>T_0$, where the range of the correlation function is small, while at the saturation line or below it, in the condensation region, the fluctuations are also finite, though large, provided only the integration in (32) is extended over finite distances. This condition is necessary, of course, in view of the diverging range of the correlation function. These finite but large fluctuations give rise precisely to the coherent scattering processes, resembling the classical critical opalescence phenomenon at and below the saturation line of ideal B.E. fluids.

VI. THE COHERENT SCATTERING OF SLOW NEUTRONS BY REAL FLUIDS IN THE CRITICAL REGION[14]

The preceding studies of coherent scattering phenomena in ideal symmetric fluids show that the structure factor depends essentially on the scattering parameter β or Δk, the momentum change on scattering. The molecular theory of coherent scattering of normal fluids leads to a similar result. As was pointed out above, these results are valid both for electromagnetic and de Broglie waves. This suggests the possibility that slow neutrons might be used in the experimental investigations of the coherent scattering by normal fluids, in particular, in their critical region.

The normal fluid differential coherent cross section per atom for small momentum changes on scattering in the critical region may be written as

$$\sigma_F(\lambda, \theta, n, T)=\sigma_c(\lambda, \theta, n)F_N^2(\beta, T). \quad (33)$$

For visible radiation, σ_c is essentially the induced dipole scattering cross section, and $F_N^2(\beta, T)$ is the fluid structure factor defined by (25) valid for small β and for any type of radiation. In the case of an atom or molecule one finds, for unpolarized incident radiation,

$$\sigma_c(\lambda, \theta, n)=v^2f(n)(1+\cos^2(2\theta))/\lambda^4, \quad (34)$$

where $v=V/N$ is the volume per molecule, and $f(n)$ is some function of the refractive index of the scattering fluid whose precise form is of no particular interest here. This function originates in the molecular polarizability of the scattering medium. It is seen that the fluid cross section, Eq. (33), is a rather complicated function of both λ and θ in the critical region. As a matter of fact, the experimental data in the critical region seem to confirm neither the wavelength nor the angular dependence of the scattering cross section, Eq. (33). This situation prompted Rocard[15] to elaborate a scattering law in which the fluid structure factor is independent of the scattering angle; therefore, the asymmetry included in Eq. (33) does not appear in his theory. It is interesting that the assumptions of Rocard leading to these results are compatible with the generalized Ornstein-Zernike formalism developed by Klein and Tisza.[16] The Rocard theory yields a structure factor at the critical point which depends on the local or molecular characteristics of the fluid but remains independent of the details of the scattering process, i.e., of λ and θ. It is apparently intermediate between the Einstein-Smoluchowski and Ornstein-Zernike theories.

In view of the rather unsatisfactory status of the experimental verification of the opalescence formula, it seems that the use of slow neutrons, through their small angle coherent scattering by normal fluids near the critical state, might possibly yield further information on the validity or invalidity of Eq. (33) or more exactly the structure factor (25). Here the elementary or nuclear scattering cross section of slow neutrons is independent both of the neutron wavelength and the scattering angle (s-wave scattering). Hence, all wavelength and angular dependence of the fluid cross section per atom should be ascribed to the fluid structure factor. Let us, indeed, assume for a moment that the fluid correlation function is known, and let it be denoted by $Q_N(r, T)$. Then the rigorous molecular theory of coherent scattering leads to the fluid structure factor, per atom or molecule, omitting the ideal fluid term,

$$F_N^2(\Delta k, T)=1+(4\pi N/V\Delta k)\int_0^\infty \sin(r\Delta k)Q_N(r, T)rdr. \quad (35)$$

The intermolecular interference term is omitted throughout this paper; also only zero spin nuclei are considered. For nuclei with spin, Eq. (35) would be modified, because the diffuse coherent term is the bound atom elastic cross section and the mutual interference term is proportional to the bound atom coherent cross section. Equation (35) becomes, rigorously, for small Δk values

$$\lim_{\Delta k\to 0} F_N^2(\Delta k, T) = 1+(4\pi N/V)\int_0^\infty Q_N(r, T)r^2dr=\langle\Delta N^2\rangle_{Av}/N. \quad (36)$$

Again, small Δk means that $\Delta k\bar{r}\ll 1$, $\bar{r}$ denotes the range of the correlation function Q_N. The very general result Eq. (36) proves that the coherent structure factor of normal fluids is independent of the type of waves, i.e., of their momentum change on scattering, or of the local molecular arrangement of the fluid, provided only that this momentum change is small. The structure factor, under these conditions, is determined by the particle or density fluctuations. Also, it is clear from Eq. (35) that at larger angles, with Δk becoming

[14] This problem was reported on at the Chicago meeting of the American Physical Society, Nov. 24–25, 1950.

[15] Y. Rocard, J. phys. radium **4**, 165 (1933). The experimental difficulties in this problem are clearly shown in some recent work on the wavelength variation of the critical opalescence. H. A. Cataldi and H. G. Drickamer, J. Chem. Phys. **18**, 650 (1950); A. L. Rabb and H. G. Drickamer, J. Chem. Phys. **18**, 655 (1950).

[16] M. J. Klein and L. Tisza, Phys. Rev. **76**, 1861 (1949).

large, the high frequency sine factor of the integrand cannot but reduce the integral with a subsequent decrease of the structure factor at these angles. This condition is imposed physically; that is, large momentum exchanges between interacting systems without energy exchange, in coherent scattering or diffraction, become increasingly prohibited when compared with the small momentum exchange processes.

We consider finally the approach to the critical region. First of all, it is seen that the small angle slow neutron coherent scattering should increase with the fluid temperature as the latter increases toward the critical temperature, the compressibility, to which $\langle\Delta N^2\rangle_{Av}$ is proportional, increasing as it approaches the critical region. Now, the compressibility law loses its meaning in the critical region, since the conditions under which Eq. (36) is valid do not exist there. For a rigorous derivation of the structure factor, the explicit expression of $Q_N(r, T)$ is necessary. However, the Ornstein-Zernike theory of the critical opalescence of visible radiation can be considered to be an asymptotic type of solution of the critical state scattering for small momentum changes. This is justified because the modification to be applied to the scattering law far from the critical region in order to extend its validity to the latter region, for small momentum change processes, is valid for any type of wave. Beside the small momentum change limitation, this modification involves only the fundamental relation of the Ornstein-Zernike theory connecting the fluid correlation function $g(r, T)$ with the local distribution function $f(r, T)$. One finds thus with Eq. (25),

$$\lim_{T\to T_c} F_N^2(\beta, T)=\beta^{-1}=\lambda^2/(\epsilon^2\sin^2\theta). \quad (37)$$

Here the quantity ϵ, the mean range of the local distribution function, is not so well defined, since the latter is unknown. But the variation in the neutron wavelength available over the spectrum of the thermal column of fission piles is large enough to allow one to explore a reasonably wide angular range. The structure factor (37) cannot be expected to be valid for large values of β. It is realized, of course, that the incoherently scattered neutrons may complicate the interpretation of the experimental results. However, their effect should be small precisely in the region of validity of the limiting structure factor (37).

It is not without interest to note that the validity of the slow neutron limiting normal fluid structure factor (37) could have been inferred also from the formal analogy, demonstrated above, of the small momentum change scattering processes between ideal symmetric and normal fluids.

VII. THE COHERENT SCATTERING STRUCTURE FACTOR OF IDEAL ANTISYMMETRIC FLUIDS. CONCLUDING REMARKS

The discussion in Sec. I of the coherent scattering by antisymmetric fluids led to the general structure factor formula resulting from Eq. (5) by some modifications outlined above. This will now be studied here briefly. In contrast to the incoherent scattering case,[1] here one must consider separately the two temperature intervals $T\gtrsim T_0$ and $T\lesssim T_0$, T_0 being the degeneration temperature. The distribution functions are given here by the following expressions:[17]

$$P_A(r, T\gtrsim T_0)=1-Q_A(r, T\gtrsim T_0)=1-(gV/N\Lambda^3)^2 \times\left[\sum_1^\infty(-)^{l-1}l^{-\frac{3}{2}}\exp\left(-l\alpha-\frac{\pi r^2}{l\Lambda^2}\right)\right]^2, \quad (38)$$

$$P_A(r, T\ll T_0)=1-Q_A(r, T\ll T_0) =1-(9\pi/2)[\{1-(\pi^2/24)x^2(T/T_0)^2\}J_{\frac{3}{2}}(x)/x^{\frac{3}{2}}]^2, \quad (39)$$

where g is $(2s+1)$ or 2; x stands for Kr, K being the length of the longest propagation vector, i.e., the propagation vector at the top of the Fermi distribution; and the Q_A's are the antisymmetrical correlation functions. In the first of these two relations a distance independent term of the order of N^{-1} has been omitted. The antisymmetric fluid distribution functions express clearly the remarkable repulsion in coordinate space between particles of parallel spin. There is a hole in the particle distribution around any chosen particle.

With Eq. (5) properly modified for antisymmetric fluids and the preceding distribution functions, one obtains, after a somewhat lengthy calculation,

$$F_A^2(\Delta k, T\gtrsim T_0)=S_A/N\sigma_s(1+A^{-1})^2=\sigma_A/\sigma_s(1+A^{-1})^2 =1+\Phi_i^2(R\Delta k)-\phi^{-1}(\alpha)\sum_1^\infty\sum_1^\infty(-)^{l+m}(l+m)^{-\frac{3}{2}} \times\exp[-(l+m)\alpha-\beta lm/(l+m)], \quad (40)$$

for the structure factor per atom, S_A being the cross section of the whole fluid and σ_A the antisymmetric fluid cross section per atom. Here β is defined by Eq. (13) above, and

$$\phi(\alpha)=N\Lambda^3/gV=\sum_1^\infty(-)^{l+1}l^{-\frac{3}{2}}e^{-l\alpha}. \quad (41)$$

Also in the derivation of Eq. (40) $(\frac{1}{2}N-1)$ has been replaced by $(N/2)$. As was the case with the symmetric fluid, the unwieldy double sum may be approximated with its analytically closed lower bound. One again finds, using the method outlined in the symmetric fluid case, omitting the ideal gas structure factor Φ_i^2 in Eq. (40),

$$F_{A,\,\mathrm{inf}}^2(\Delta k, T\gtrsim T_0) =\{1-[\phi(\alpha+\beta)/\phi(\alpha)]\}(1-e^{-\beta})^{-1}. \quad (42)$$

This lower bound is identical with the rigorous inco-

[17] See E. Wigner and F. Seitz, Phys. Rev. **43**, 804 (1933), for absolute zero, and P. L. Bhatnagar and K. S. Singwi, Phil. Mag. **40**, 917 (1949), for all temperatures.

herent structure factor expressed in the center-of-gravity coordinate system of the neutron and target atom.[1] The preceding structure factor, as a result of the repulsion in phase space and coordinate space, is less than unity. The diffuse coherent and mutual interference terms are of opposite sign, the former being necessarily positive. Limiting values of Eq. (42) can be obtained at once. First, the large angle limit is, omitting the subscript inf,

$$\lim_{\beta\gg 1} F_A^2(\beta,\ T\gtrsim T_0)=1-[e^{-(\alpha+\beta)}/\phi(\alpha)]\approx 1, \quad (43)$$

and, in the opposite limit,

$$\lim_{\beta\ll 1} F_A^2(\beta,\ T\gtrsim T_0)=-\phi^{-1}(\alpha)(d\phi/d\alpha), \quad (44)$$

which is rigorous.

Below the degeneration temperature, essentially at $T\ll T_0$, one obtains with the properly modified Eq. (5) and Eq. (39), after a rather long series of elementary integrations,

$$\begin{aligned} F_A^2(\Delta k,\ T\ll T_0)=&\frac{1}{2}+\frac{\gamma}{8}\left(3-\frac{\gamma^2}{4}\right)\\ &+\frac{\pi^2}{16}\left(\frac{T}{T_0}\right)^2\left(\frac{1}{\gamma}-\frac{\gamma}{2}\right), \quad \gamma<2\\ &=1-(\pi^2/64)(T/T_0)^2, \quad \gamma=2\\ &=1. \quad \gamma>2 \end{aligned} \quad (45)$$

Here,

$$\gamma=\Delta k/K=4\pi\sin\theta/K\lambda, \quad (46)$$

where λ is the wavelength of the incident waves, and (2θ) the scattering angle in the coherent process. One notices the slight discontinuities near the limit at which coherent scattering or mutual interference may occur at all. The last of Eqs. (45) shows that coherent scattering through mutual interference is an ideal highly degenerate antisymmetric fluid is only possible as long as the wavelength of the incident waves is smaller than the minimum de Broglie wavelength associated with the fluid, namely, the wavelength at the top of the Fermi distribution. Explicitly, with K being $2\pi/\lambda_{\min}$, one finds that the mutual interference terms vanish for

$$\sin\theta>\lambda/\lambda_{\min},$$

or as soon as λ is equal to $\lambda_{\min}$. This is similar to the optical condition for diffraction or the one associated with the occurrence of Bragg scattering. The coherent scattering structure factor of ideal Fermi-Dirac fluids at complete degeneration is thus characterized by a reduced forward scattering, increasing at larger angles or for larger momentum losses on scattering and tending, from below, toward its diffuse coherent limit. The correction term proportional to $(T/T_0)^2$ considerably modifies this behavior at small γ-values, that is, at values

$$\gamma\leqslant(\pi^2/16)(T/T_0)^2,$$

where the forward scattering becomes more important because the linear extension of the hole in the spatial distribution around a given atom, at the absolute zero, becomes smaller at finite temperatures.

It may be noted here that, in line with the conjecture states in our previous paper,[1] the incoherent and coherent cross sections per atom of the ideal quantum fluids are about the same in an energy or wavelength interval of the incident particles or waves where the theory of the scattering processes is valid. The result concerning the energy and fluid state independence of the slow neutron diffuse coherent cross sections might be questionable. The diffuse coherent term of the structure factors is their high energy limit. In contrast with the scattering of x-rays, which give rise to no difficulties, the preceding result for neutrons would mean that at high neutron energies the total, coherent plus incoherent, neutron scattering cross section is about twice the low density ideal gas scattering cross section. This inconsistent result stems from the fact that the present formalism does not seem to provide any mechanism by which faster neutrons would be prevented from exchanging momentum only with the fluid as a whole through the diffuse coherent scattering process. Physically, one would expect at higher neutron energies to observe only incoherent scattering processes through the ordinary exchange of energy and momentum of the neutrons with the individual fluid atoms. Further studies should clear up this difficulty. According to the present formalism of scattering processes, the same difficulty is present in the theory of coherent scattering of neutrons by normal fluids.

Liquid He^4 and He^3 are the only liquids which may be thought of as being represented, in some asymptotic way, by ideal symmetric and antisymmetric fluids, respectively. The experimental investigation of the slow neutron scattering properties of these fluids, subject to the limitations mentioned above concerning liquid He^3, might disclose certain features which could be interpreted with the help of the scattering properties of the ideal quantum fluids investigated in this work.