will not yield very reliable values for the liberation energy, and it is probable that the figure found by the author for 1-Mev beta-rays is somewhat nearer the true value. Finally, as was pointed out by Freeman and van der Velden,[6] pulse counting methods are more likely than current measuring methods to yield accurate results for measurements of kT and the energy per electron-hole pair. Current methods give a value for kT which is the average over all the electrons in the current. If, owing to the presence of a mechanical flaw or some other cause, some electrons are unable to contribute to the current to the extent that they would be able to if in a normal lattice, the measured current will be less than the value it would have in a normal lattice. This in turn would lead to a higher value for the energy per electron-hole pair. In pulse counting methods it is not necessary to assume that *all* the electrons and holes produced by *all* the beta-particles travel in a normal lattice. It is necessary only to assume that all the electrons and holes produced by *some* beta-particles travel in the normal lattice, and this is obviously a much more reasonable assumption. Summarizing, it seems fairly safe to assume that the maximum observed pulse heights do represent the motion of the electrons and holes in the normal diamond lattice.

CONCLUSIONS

It is hoped to extend the foregoing types of investigation to a number of diamond crystals, and in this way it may be possible to correlate the possible variation in trap density or energy per electron-hole pair with some other physical property. In particular, it has sometimes been suggested that the response of the crystal depends on its structure, i.e., whether it is a laminated, mosaic, or perfect crystal lattice. The degree of imperfection of the structure of the crystal can be determined by means of the divergent beam x-ray technique developed by Lonsdale[12] and such studies may yield information as to whether trapping sites mainly exist at the boundaries of mosaic blocks or laminations; i.e., if the traps do occur at the boundaries, then the trap density should be greater for those diamonds which show the better divergent beam photographs.

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Potential and Gradient Distributions in Parallel Plane Diodes at Currents below Space-Charge-Limited Values

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The equations for electron motion in a plane diode under the influence of space charge are integrated. The solution is expressed in terms of four dimensionless numbers, normalized so that their range is from zero to unity. The gradient at the cathode is expressed as a function of the current density as it varies from zero to space-charge-limited values. Space distributions of potential and gradient are given in terms of the location of the plane of interest between the cathode and the anode for a number of specific current densities.

Evidence is shown of the existence of a plane for which the gradient is essentially independent of the current density.

INTRODUCTION

EXPRESSIONS for the complete description of the potential and gradient distributions in parallel plane diodes under conditions of emission-limited currents are given in terms of four dimensionless variables.[1] Others[2–6] have given equations for the same phenomena, but those of which the author is aware all make use of parameters whose physical significance is very obscure. The variables used in this derivation represent current density, voltage, distance, and cathode gradient; normalization is such that these variables range between zero and unity as the current density varies from zero to space-charge-limited values.

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PROCEDURE

We define our variables as

$$\rho=\frac{\text{current density}}{\text{space-charge-limited current density}}=\frac{i}{i_{S-C-L}},$$

$$\varphi=\frac{\text{voltage at distance } x \text{ from cathode}}{\text{voltage at anode, at distance } x_a \text{ from cathode}}=\frac{V_x}{V_a},$$

$$\xi=\frac{\text{distance from cathode}}{\text{anode-cathode spacing}}=\frac{x}{x_a},$$

$$\gamma=\frac{\text{gradient at cathode under influence of space charge}}{\text{gradient at cathode in absence of space charge}}$$

$$=(x_a/V_a)(dV/dx|_{x=0}).$$

The usual assumptions of zero velocity of the particles at the electrode of their origin, motion of the particles in the x-direction and conservation of energy, when combined with Poisson's equation yield

$$d^2V/dx^2=4\pi i(m/2Ve)^{\frac{1}{2}}$$

or

$$d^2\varphi/d\xi^2=4\pi(m/2e)^{\frac{1}{2}}x_a^2V_a^{-\frac{3}{2}}i_{S-C-L}\rho\varphi^{-\frac{1}{2}}.$$

As is well known,

$$i_{S-C-L}=[(2e/m)^{\frac{1}{2}}/9\pi]V_a^{\frac{3}{2}}/x_a^2.$$

Hence

$$d^2\varphi/d\xi^2=(4/9)\rho\varphi^{-\frac{1}{2}}.$$

Integrating once

$$(d\varphi/d\xi)^2=(16/9)\rho\varphi^{\frac{1}{2}}+C_1. \qquad (1)$$

Integrating again

$$(8\rho\varphi^{\frac{1}{2}}-9C_1)[\rho\varphi^{\frac{1}{2}}+(9/16)C_1]^{\frac{1}{2}}=8\rho^2(\xi+C_2). \qquad (2)$$

Now let us examine our constants of integration, C_1 and C_2. They are constants in that they are independent of the variable of integration, ξ, and are to be evaluated by the boundary conditions imposed on the variables. It is customary and convenient to set

$$x=V=0 \qquad (3)$$

at the cathode, and our method of normalization requires this. When applied to Eq. (1)

$$C_1=\left(\frac{d\varphi}{d\xi}\bigg|_{\xi=0}\right)^2\equiv\gamma^2 \qquad (4)$$

by our definition. Equations (2), (3), and (4) can be solved for C_2:

$$C_2=-(27/32)(\gamma^3/\rho^2).$$

When we substitute these values of C_1 and C_2 into Eq. (2) and re-arrange the terms, we find

$$16(\rho\varphi^{\frac{3}{2}}-\rho^2\xi^2)=27(\varphi\gamma^2-\xi\gamma^3). \qquad (5)$$

By differentiation of (5)

$$d\varphi/d\xi=(27\gamma^3-32\rho^2\xi)/(27\gamma^2-24\rho\varphi^{\frac{1}{2}}). \qquad (6)$$

This concludes the derivation of our general expressions. From Eqs. (5) and (6) one can learn everything about potentials and gradients in the space between plane parallel diodes.

CATHODE GRADIENT UNDER EMISSION-LIMITED CONDITIONS

One of the interesting relations which follows from (5) is that between the current density, ρ, and the cathode gradient, γ. For all space-charge-limited diodes, $\rho=1$ and $\gamma=0$ by definition. Similarly, when voltage is applied and zero current (emission limited) is being drawn, $\rho=0$ and $\gamma=1$. These limiting values are ob-

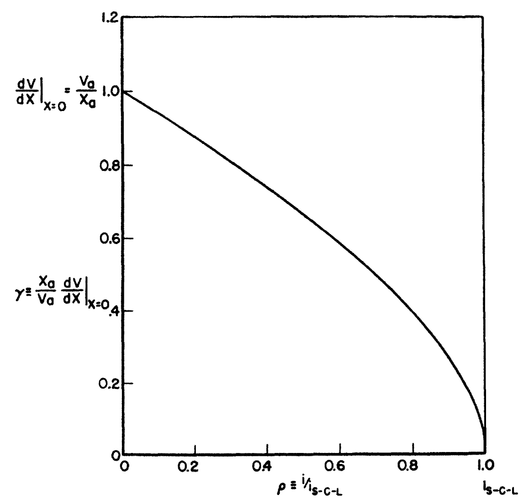

FIG. 1. Cathode gradient as function of current density.

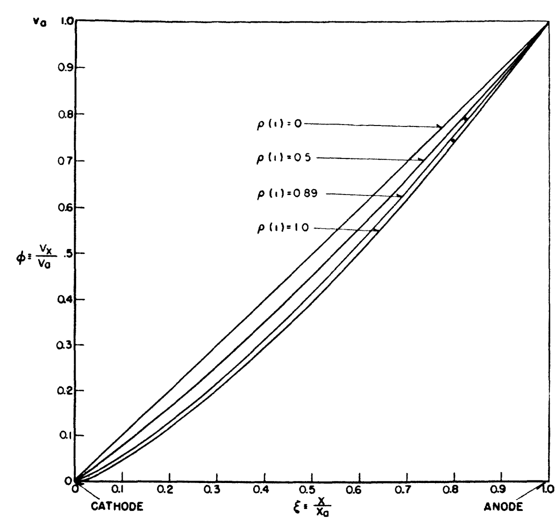

FIG. 2. Potential in space between cathode and anode for several current densities.

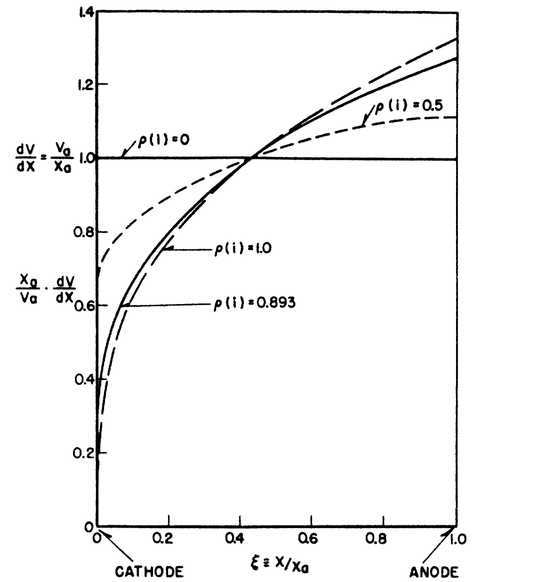

FIG. 3. Gradient in space between cathode and anode for several current densities.

vious; all intermediate values can be obtained by setting $\varphi=\xi=1$ in Eq. (5). This is equivalent to saying that the voltage is anode voltage and that its location is the anode plane. Equation (5) then reduces to

$$16(\rho^2-\rho)=27(\gamma^3-\gamma^2)$$

or

$$\rho=\tfrac{1}{2}+(\tfrac{1}{2}-3\gamma/4)(3\gamma+1)^{\frac{1}{2}}. \qquad (7)$$

Equation (7) is plotted in Fig. 1. This is a general relation which shows the unique dependence of the cathode gradient on the fraction of space-charge-limited current being drawn from the cathode. The rapid rise of cathode gradient as the current is decreased from space-charge-limited values to emission-limited conditions is quite striking. A 10 percent reduction of current from space-charge saturation causes the gradient to increase from zero to more than 25 percent of its value in the absence of space charge.

SPACE POTENTIAL

For a specific current in a given diode the cathode gradient γ, and the current density ρ become constants, γ_0 and ρ_0. If these values are inserted into Eq. (5), a relation between φ and ξ results. Because ξ appears to the second power and φ to the third power, it is easier

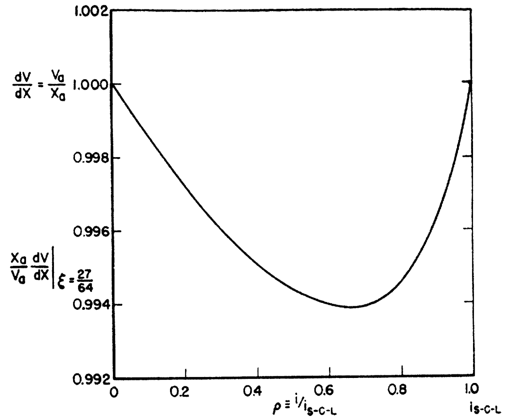

FIG. 4. Gradient at plane 27/64 of cathode-anode spacing from cathode as function of current density.

to solve for ξ as a function of φ. We find:

$$\xi=\frac{27\gamma_0^3}{32\rho_0^2}\left[1+\left(\frac{8\rho_0\varphi^{\frac{1}{2}}}{9\gamma_0^2}-1\right)\left(\frac{16\rho_0\varphi^{\frac{1}{2}}}{9\gamma_0^2}+1\right)^{\frac{1}{2}}\right]. \qquad (8)$$

Plots of Eq. (8) for four conjugate values of ρ_0 and γ_0 are given in Fig. 2. The maximum depression of the space potential for space-charge-limited conditions ($\rho=1$) occurs at $\xi=27/64$, where the normalized slope is unity.

ELECTRIC FIELD

When we have the data of Fig. 1 and have solved Eq. (8) for the selected ρ_0 and γ_0, we may turn to Eq. (6) for the gradient at any point. At the anode, $\varphi=\xi=1$ and the gradient is a function of ρ and γ or, by the use of Eq. (7), a function of either alone.

For any plane between the diode boundaries, Eq. (6) includes all four of our dimensionless variables. Using the data obtained in calculating the curves of Fig. 2, the gradient as a function of position for several current densities is given in Fig. 3.

The data of Fig. 3 suggest that the normalized gradient is equal to unity in the plane $\xi=27/64$, independent of the current density ρ. This is only an approximation, however, as is shown in Fig. 4. Here we have a plot of the gradient in the plane $\xi=27/64$ as a function of ρ. The maximum deviation from unity is only 0.6 percent as the current is varied from zero to space-charge-limited values.

The help of Dr. J. Slepian in the reduction of Eq. (7) to its simplified form is appreciatively acknowledged.