To prove this theorem we write

$$\left.\begin{aligned}|\Sigma_e(i|H_1|e)(e|H_2|f)|^2=\Sigma_e|(i|H_1|e)|^2|(e|H_2|f)|^2\\+\sum_{\substack{e_1,e_2\\(e_1\neq e_2)}}(i|H_1|e_1)^*(i|H_1|e_2)(e_1|H_2|f)^*(e_2|H_2|f).\end{aligned}\right\}\quad(4)$$

Now, if ρ is in diagonal form, we have according to (3)

$$S_1(i|H_1|e_1)^*(i|H_2|e_2)=0\quad(e_1\neq e_2).\qquad(5)$$

According to (4) and (5) we see that (1) reduces to (2) if ρ is in diagonal form for the states ψ_e, and the theorem is proved.

We now give an alternative expression for ρ by which the meaning of ρ can more easily be understood than from the definition (3) of ρ. Consider the wave function of an arbitrary intermediate state ψ_{med}, which can be expressed as

$$\psi_{\text{med}}=\Sigma_e c_e\psi_e\qquad(6)$$

by means of the fundamental states ψ_e chosen in (1) and (2). From (6) it follows that

$$\int\psi_{\text{med}}^*\rho\psi_{\text{med}}=\Sigma_{e_1,e_2}c_{e_1}^*c_{e_2}(e_1|\rho|e_2).\qquad(7)$$

If we define the partial transition probabilities $P_{i,\text{med}}^p$ by

$$P_{i,\text{med}}^p=N'S_1|(i|H_1|\text{med})|^2,\qquad(8)$$

where N' is a constant factor, we obtain

$$\int\psi_{\text{med}}^*\rho\psi_{\text{med}}=N''P_{i,\text{med}}^p,\qquad(9)$$

where N'' is a normalization constant. It is easily seen that (9) can be used as a definition of ρ alternative to (3).

We want to stress that the values of $P_{i,\text{med}}^p$, when known only for the fundamental states ψ_e but not for their linear combinations, do not yet supply complete information on the transition probabilities $i\rightarrow e$. For this we must either calculate (3), or $P_{i,\text{med}}^p$ for every ψ_{med}.

To obtain the angular correlation theorem stated by Falkoff and Uhlenbeck[1] (and proved later by Lloyd[2] and Lippmann[3]) as a special case of our theorem, we remark that the z axis is an axis of rotational symmetry of ρ, if the first of two successive particles is emitted along this axis. This leads to a diagonal form for ρ if the ψ_e are eigenfunctions of the z component of the total angular momentum, so that (1) and (2) are equivalent for these ψ_e.

The proof of our theorem has features in common with the considerations of Lippmann,[3] but the use of the density matrix enables us to go further. Our theorem could, namely, also be applied if no axis of rotational symmetry for ρ exists, as in the case of $\gamma-\gamma$ angular correlation of aligned nuclei. We must then try to find states ψ_e, for which ρ is in diagonal form.

We have used the representation of the state of a system after a transition by a density matrix [as in (9)] in an earlier paper,[4] where a partially polarized electron beam was represented by a density matrix.[5] Before that, we used the density matrix to prove another theorem on transition probabilities.[6]

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Intensity of Ultraviolet Radiation from Solar Flares

R. N. Sedra
Faculty of Science, Fouad I University, Cairo, Egypt

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Solar flares are classified[1–3] according to their intensities in the increasing order of magnitudes (1, 2, 3, and 3+), where 3+ represents the most intense flare. It is well established that most solar flares are accompanied by synchronous magnetic disturbances usually called "crochets."[4] It is natural to assume that the change in the earth's magnetic field accompanying the flare is dependent on the intensity of the ultraviolet radiation emitted. The following calculation is based on the assumption that these crochets are intensifications of diurnal magnetic variations.

The diurnal magnetic variations have been successfully explained by the diamagnetic theory.[5] For a magnetized long free path ion gas, the intensity of magnetization I is given by

$$I=-3nkT/2H,$$

where T is the absolute temperature, k is Boltzmann's constant, n is the ion density, and H is the magnetic field. Assuming that the solar ultraviolet flash produces an ion cloud, we may calculate its magnetic effect by replacing it by a bar magnet. The magnetic pole strength per unit area, σ, is equal to $I/4\pi$. Therefore,

$$\sigma=-3nkT/8\pi H.$$

Following the Maris and Hulburt[6] assumption in taking the end, S, of the magnet to be 300 km thick and 1000 km wide, the magnetic strength ρ of S is given by

$$\rho=-(9nkT/8\pi H)\times10^{15}.$$

Taking the distance of the ionized layer from the earth to be 200 km and neglecting the effect of the north pole N, the field F due to S is given by:

$$F=-(nkT/\pi H)\times2.81.$$

Substituting $T=1000°$, $k=1.372\times10^{-16}$ erg per degree Kelvin, and $H=0.5$ gauss, one gets

$$n=-F\cdot4\times10^{12}.\qquad(1)$$

The relation between the rate of ion production and the intensity of incident radiation is given by[7]

$$q=\beta n'i/w,$$

where q is the number of ion pairs produced per cm^3 per sec at height h, w is the energy absorbed in ionizing one molecule, β is the atomic absorption coefficient, n' is the number of molecules per cc at height h, and i is the intensity of the incident radiation at height h. Substituting $w=14$ ev$=14\times1.6\times10^{-12}$ erg, $\beta=3.2\times10^{-17}$ cm^2, and n' at height 200 km[8]$=2.5\times10^8$, one obtains

$$i=q\times2.8\times10^{-3}.\qquad(2)$$

To test the above calculations, we shall take one of the results observed by Newton. He observed, on the 3rd of July, 1941, a solar flare accompanied by a crochet which gives a change in H of -13γ. Calculating the ionic density n from Eq. (1), we find that its value is 5.2×10^8. Also, calculating the intensity of incident radiation from Eq. (2), we find its value to be 8.09×10^2 erg cm^{-2} sec^{-1}. Gledhill and Syendrei[7] obtained an estimate of the normal solar radiation above the earth's atmosphere from ionospheric data. Their value amounts to 0.313 erg cm^{-2} sec^{-1}. Comparing the result obtained from this calculation with that of Gledhill and Syendrei, we notice that our result is greater by a factor of about 2500, which is reasonable with the intense radiation emitted during the solar flare.

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The Effect of Finite Nuclear Size in Beta-Decay*

M. E. Rose and D. K. Holmes
Oak Ridge National Laboratory, Oak Ridge, Tennessee

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In the analysis of the β-spectrum of heavy radioactive nuclei it is necessary to examine the effect of the finite nuclear radius insofar as this effect appreciably influences the behavior of certain of the electronic wave functions at the nuclear radius.[1] In fact, for

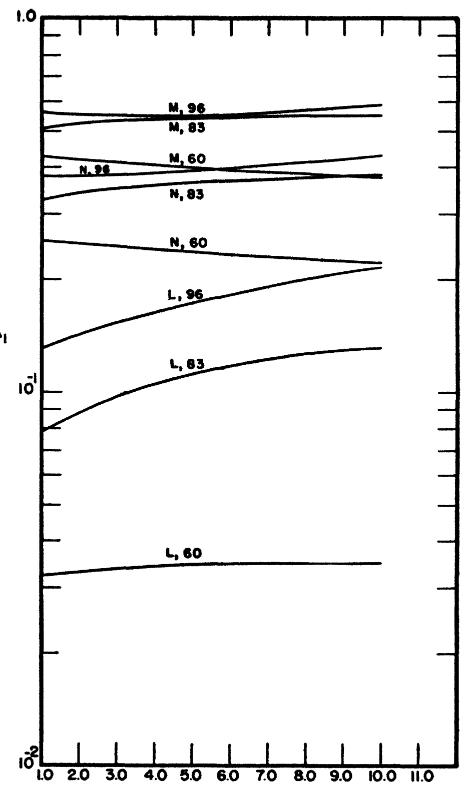

FIG. 1. The correction factor $\Delta_1(j=\frac{1}{2})$ as a function of electron energy W (total energy) in units mc^2. The curves marked L, M, N correspond to $\Delta_1(L)$, $\Delta_1(M)$, $\Delta_1(N)$, respectively, and the numbers affixed to the curves give the value of Z.

given angular momentum j, the "small" component f for $\kappa=-j-\frac{1}{2}$ and the "large" component g for $\kappa=j+\frac{1}{2}$ are sensitive to deviations from the coulomb field.[1] Thus, in the correction factors C_{nX} of Greuling[2] which are linearly dependent on the three wave function combinations L, M, N, one would expect the greatest influence on M and the least for L. This is, in fact, exhibited by the results given below

The calculation of the finite size correction is straightforward and is based on methods described elsewhere.[1] The smeared out charge distribution inside the nucleus is represented by a scalar potential

$$V=V_0[1+a_2(r/r_0)^2+a_4(r/r_0)^4], \quad r\leqslant r_0, \tag{1}$$

where the constants are fixed by continuity of V, dV/dr and by charge distortion,[3] although the results are insensitive to the latter. The internal wave functions are joined to the usual regular plus irregular coulomb field solutions[4] at $r=r_0$. Then the effect of finite nuclear size is represented by $\Delta_k(M)$, etc., where

$$M_k=M_k(0)[1+\Delta_k(M)] \tag{2}$$

and similar expressions for L_k and N_k. Here $M_k(0)\cdots$ represents the point nucleus value[2] and $k=|\kappa|=j+\frac{1}{2}$.

The results for electrons[5] are shown in Figs. 1 and 2 for $k=1$ and 2, respectively. For $k\geqslant 3$ the Δ-factors are rather small. As expected the influence of the finite size of the nucleus (a) decreases rapidly with increasing j, (b) is important only for $Z>\sim 60$ and (c) is greatest for M, least for L. The latter fact implies an appreciable effect on the β-decay only for nth forbidden transitions ($n\geqslant 1$) and for spin change equal to n (unfavorable parity change in the case of $G-T$ interactions). The negative sign of Δ_k is due to the depression of $\langle V^2\rangle$, averaged over the nucleus.[1]

Since M_k is the dominant term for heavy nuclei and since the coefficients of M_{n+1} (and N_{n+1}) vanish, the energy insensitivity of $\Delta_k(M)$ and $\Delta_k(N)$ imply an essentially allowed spectrum shape for first-forbidden transitions. However, some change in ft value is to be expected when the finite nuclear size effects are included. On the other hand, for second and higher forbidden transitions, the spectrum shape is changed, because of the rapid k-variation of the Δ-factors. Thus a relatively greater depression of the spectrum is to be expected at the low energy end as compared to the high energy region. The spectrum modification is, of course, greater for the larger maximum energies W_0.

It is clear that the effect of the finite size of the nucleus is unimportant for the allowed transitions and the favorable parity change

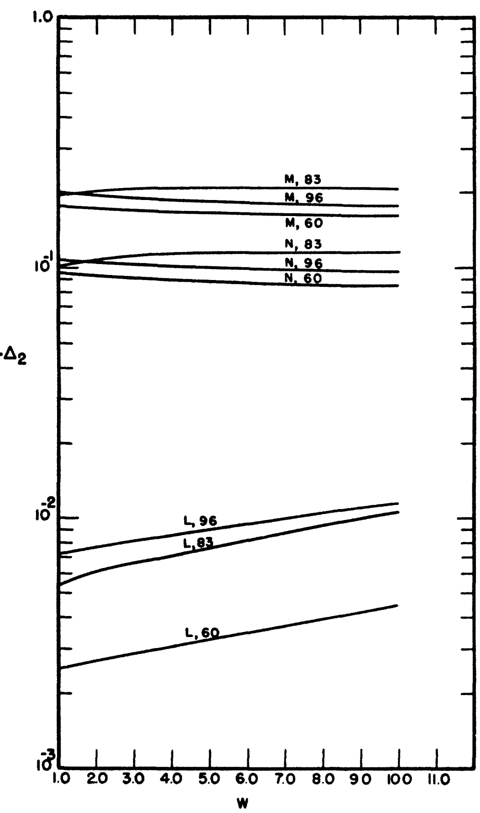

FIG. 2. Same as Fig. 1 but for $k=2$, $j=3/2$. Note change of ordinate scale.

transitions with $G-T$ selection rules. The only well-investigated case for which the finite size corrections should be important is the RaE spectrum. An attempt to fit the observed spectrum with finite size corrections, using pure invariant β-interactions, was unsuccessful. However, recent evidence (made available since the completion of this work) points to a complex spectrum in this case.[6]

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