This completes the connection of the solutions at the interior boundary.

VII. CONCLUSION

Here we have given the first complete nonstatic solution, holding from $R=0$ to $R=\infty$, representing an isolated spherically symmetric nonstatic distribution radiating out energy. There are two boundaries, the interior one and the exterior one, but we have been able to give a line-element whose coefficients are continuous throughout from $R=0$ to $R=\infty$. It has already been proved by the author[5] that in the case of a distribution with the line-element

$$ds^2=-e^\lambda dR^2-R^2(d\theta^2+\sin^2\theta d\varphi^2)+e^\nu dT^2,$$

the continuity of $g_{\mu\nu}$ alone is sufficient to insure that the total mass of the isolated distribution is conserved.

The solution discussed in detail in the last section shows that the corresponding material distribution is contracting, dR_i/dT being negative. But m_i/R_i is a constant, m_i being the mass of the distribution as observed from its external gravitational field. So the newtonian gravitational potential energy of the interior distribution remains constant and the contraction therefore is not gravitational. It seems to be a purely relativistic effect. The radiating distribution loses energy and so its mass decreases. But if m_i/R_i is to remain a constant, then R_i must also decrease. This may explain the contraction.

Some of the particular solutions derived here have suggested the existence of a new class of solutions of Einstein's field equation. These solutions would represent non-isolated spherically symmetric distributions absorbing energy from the cosmos. These are at present being investigated.

PHYSICAL REVIEW VOLUME 83, NUMBER 1 JULY 1, 1951

On the Ionization Yields of Fission Fragments*†

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(Received September 13, 1950)

The disagreement between the distributions in fission fragment mass and ionization is attributed to a variation in ionization yield with fragment mass and to a dispersion arising from such effects as neutron recoil and instrumental errors of ionization measurement. After the effects of dispersion in the available data of fragments from U^{235} slow neutron fission are taken into account, the variation in ionization yield is estimated from the remaining disagreement in distributions. For the most probable fission asymmetry, the energy-ionization ratio of light fragments is found to be approximately 3.7 percent less than for heavy fragments.

I. INTRODUCTION

A RECENT analysis[1] of the ionization produced by slow, heavy particles indicates that appreciable kinetic energy is lost to recoiling gas atoms having a reduced ionization efficiency. The resulting increase in the energy-ionization rate dE/dI as the heavy particle is stopped accounts for the ionization defect, the difference between the actual energy of the particle and the energy determined from the total ionization on the basis of $w=dE/dI$ of fast particles. Since the ionization defect increases with the mass of the particle, it is expected that w_H, the energy-ionization ratio obtained for complete stopping of a heavy fission fragment, is greater than w_L of a light fragment.

Any direct measure of w_L and w_H would require measurements of both the ionization and energy of individual fragments. Because fission fragments do not all have the same kinetic energy, mass and effective charge, energy measurements of individual fragments by such means as calorimetry, magnetic deflection or electrostatic deflection have not been feasible. From the momentum condition of fission and the measured mass distribution,[2] however, the distribution in the *relative* kinetic energies of the complementary light and heavy fragments can be determined quite accurately. In this investigation, a comparison of this energy ratio distribution with the corresponding ionization ratio distribution[3,4] is used to determine indirectly w_L/w_H for the slow neutron fission of U^{235}.

II. COMPARISON OF DISTRIBUTIONS

Method

Using double "back-to-back" ionization chambers, both Brunton and Hanna[3] and Deutsch and Ramsey[4] have made coincidence measurements of the number of ion pairs I_L and I_H (subscripts L and H always refer

* Work performed in the Ames Laboratory of the AEC.
† Details of this analysis are contained in ISC 98.
‡ Now at Los Alamos Scientific Laboratory, Los Alamos, New Mexico.
[1] Knipp, Leachman, and Ling, Phys. Rev. 80, 478 (1950).
[2] Plutonium Project, Revs. Modern Phys. 18, 513 (1946).
[3] D. C. Brunton and G. C. Hanna, Can. J. Research A28, 190 (1950).
[4] M. Deutsch and M. Ramsey, MDDC 945 (1946) (unpublished).

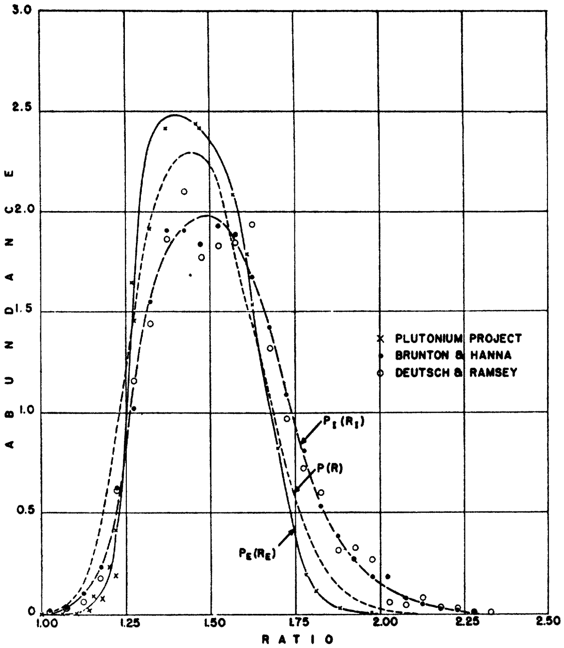

FIG. 1. Ratio distributions for fragments from U^{235} slow neutron fission. The ionization and energy ratio distributions, $P_I(R_I)$ and $P_E(R_E)$, are based on experimental data; the ionization energy ratio distribution $P(R)$ is calculated.

to the complementary light and heavy fragments) formed as fission fragments are stopped in a gas mixture of argon plus a few percent of carbon dioxide. From these data is obtained the distribution $P_I(R_I)$ in the ionization ratio $R_I = I_L/I_H$ shown in Fig. 1. Ionization data in this form can be compared directly with the only available energy data, the distribution $P_E(R_E)$ obtained from the fission momentum relation $R_E = E_L/E_H = m_H/m_L$. Although m_L and m_H are the fragment masses before neutron emission, the distribution in m_H/m_L can be obtained from the distribution in the measured[2] final masses (masses after neutron emission) by correcting for the mass of the emitted neutrons. Using the assumption that 1.25 neutrons are emitted from each fragment, an assumption that is later seen to give only a small error, together with representative points from the mass yield curve, we obtain the approximate $P_E(R_E)$ shown in Fig. 1.

The considerable difference between the ionization ratio data $P_I(R_I)$ and the corresponding energy ratio data $P_E(R_E)$, a difference much larger than the experimental uncertainties of the measurements,[5] is assumed to be due to, first, a broadening of $P_E(R_E)$ by the energy dispersion arising from instrumental errors of ionization measurement and from recoil due to the emission of prompt neutrons and, second, an expansion of this dispersed energy ratio distribution into an ionization ratio distribution of higher ratios due to a decrease of w_L/w_H with mass ratio.

In the quantitative determinations of the dispersion and of the decrease of w_L/w_H with mass, it is important to distinguish between the energy distributions and the ionization distribution being compared. The *ionizations* I_L and I_H are experimental data and so include the effects of dispersion. As such, they are representative of the *ionization energies* $w_L I_L$ and $w_H I_H$. The ionization energies are distinguished from the *initial energies* E_L and E_H by the effect of dispersion, an initial energy being deviated into a larger or smaller ionization energy by the dispersion. Correspondingly, the distribution $P(R)$ in the ionization energy ratio $R = w_L I_L / w_H I_H$ differs from $P_E(R_E)$ by the effect of dispersion and from $P_I(R_I)$ by the variation in w_L/w_H. Since in this paper all distributions and dispersions are normalized to unity, the relation between $P_E(R_E)$ and $P(R)$ is

$$P(R) = \int_0^\infty dR_E D_r(R_E, R) P_E(R_E), \tag{1}$$

and between $P(R)$ and $P_I(R_I)$ is

$$P_I(R_I) = P(R)[(w_L/w_H) + \alpha R_I] \tag{2}$$

or

$$\int_1^{R_I} dR_I P_I(R_I) = \int_1^R dR P(R), \tag{3}$$

where in Eq. (1) $D_r(R_E, R)$ is the distribution in the energy ratio deviation $(R - R_E)$ and in Eq. (2) $\alpha = [d(w_L/w_H)/dR_I]$. Equation (2) follows from the relations $P_I(R_I)dR_I = P(R)dR$ and $R = (w_L/w_H)R_I$.

The treatment of the distributions consists of, first, a calculation of $P(R)$ using an estimated dispersion $D_r(R_E, R)$ in Eq. (1) and, second, a determination of w_L/w_H by a comparison of $P(R)$ and $P_I(R_I)$ by Eqs. (2) or (3).

The Dispersion Width

The determination of the dispersion function $D_r(R_E, R)$ is aided by a comparison of $P_E(R_E)$ and $P_I(R_I)$ in the region of symmetrical fission, where the slight difference in complementary fragment masses minimizes the effect of the variations in w_L and w_H. In this region, it is to be expected[6] that the energy-ionization ratio is a slowly varying function of the fission asymmetry, such that w_L/w_H as a function of R_I linearly approaches unity as R_I approaches unity. On this basis, we require

$$w_L/w_H = 1 + \alpha(R_I - 1), \quad \alpha = \text{constant}, \tag{4}$$

in the region of symmetrical fission, where the dependence of w_L/w_H on such other factors as the total energy $E = E_L + E_H$ and the nuclear charge division is neglected. It is seen that the requirements on $D_r(R_E, R)$ are that it disperse $P_E(R_E)$ by Eq. (1) into a $P(R)$

[5] The points in the peak of the Brunton and Hanna ionization ratio distribution represent $1.2(10^3)$ observations. According to M. S. Freedman and E. P. Steinberg (CC 3420), most of the points of the Plutonium Project yield curve have a precision of about five percent.

[6] N. Bohr, Kgl. Danske Vid. Sels. Math.-Fys. Medd. 18, No. 8 (1948).

which, when compared with $P_I(R_I)$ by Eqs. (2) or (3), satisfies the symmetrical fission requirements on w_L/w_H stated in Eq. (4).

Since the limited available data and the lack of sensitivity of this method make a completely analytical determination of $D_r(R_E, R)$ impractical, the method is used only to estimate the width of an assumed dispersion function. While it is not simple to anticipate directly the form of $D_r(R_E, R)$, it does seem reasonable that the form of the energy dispersions $D_L(E_L, w_LI_L)$ and $D_H(E_H, w_HI_H)$ in the light and heavy fragments should each be nearly gaussian. As illustrated in Fig. 2, these fragment energy dispersions combine into an energy ratio dispersion $D_r(R_E, R)$ and a total energy dispersion $D_T(E, w_LI_L+w_HI_H)$ through the geometric combination of the fragment energy deviations $e_L=w_LI_L-E_L$ and $e_H=w_HI_H-E_H$ into the energy ratio deviation $r=R-R_E$ and the total energy deviation $e=w_LI_L+w_HI_H-E$. By this analysis it can be shown that gaussian energy dispersions having energy deviations e_L and e_H negligible compared to E_L and E_H combine into a gaussian ratio dispersion and a gaussian total energy dispersion.

Various trial values of the ratio dispersion width δr (all widths are full widths at half maximum) have been used in Eq. (1) with the gaussian dispersion

$$D_r(R_E, R) \propto \exp[-(4)(0.69)\{(R_E-R)/\delta r\}^2]$$

to obtain the trial $P(R)$ distributions shown for the region of symmetrical fission in Fig. 3. In order to determine the most suitable dispersion width, these trial ionization energy ratio distributions $P(R)$ were compared with $P_I(R_I)$ by Eq. (2). The width $\delta r=0.10$ was found to be too narrow; α fluctuated from a large positive value near symmetrical fission to negative values at greater ratios. The width $\delta r=0.18$ was found to be too broad; α was a large negative value near symmetrical fission but small at greater ratios. The width $\delta r=0.14$ with its consequent $\alpha\cong-0.10$ was found to give the least fluctuation. The disagreement at 1.025 in Fig. 3 is either due to inadequate data or to the use of a dispersion function with too large a tail.

Variation of w_L/w_H

In order to apply $D_r(R_E, R)$ to all degrees of asymmetry, its change in width with fission asymmetry must be determined. An estimate of the possible extremes of this asymmetry dependence is possible by assuming that the dispersion is predominantly due to either (1) instrumental errors or (2) neutron recoil. The characteristics of the true dispersion is expected to be between these extremes. With the extreme (1), the widths of the fragment energy dispersions D_L and D_H are expected to be nearly equal and constant. It can be shown that under these conditions δr varies as $(R_E^2+1)^{1/2}(R_E+1)/E$. On the other hand, the effect of the recoil of an isotropic emission of neutrons from the

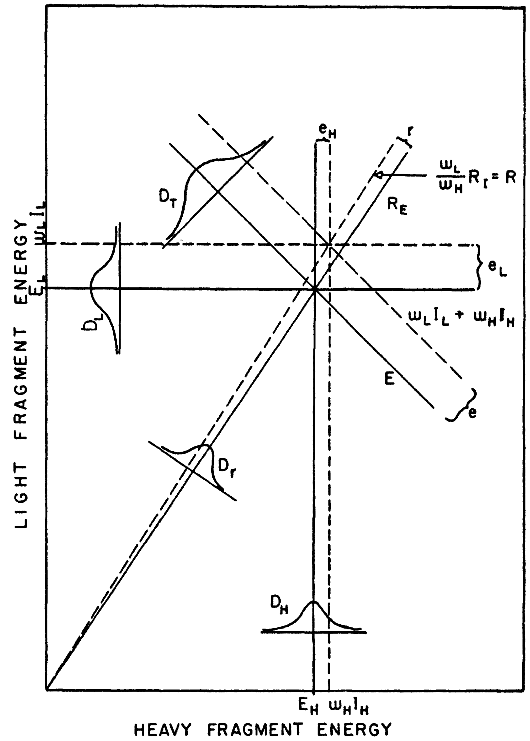

FIG. 2. Illustration of the energy quantities associated with two complementary fragments. Deviations change the initial energy quantities (solid lines) into ionization energy quantities (dotted lines). Possible dispersion functions of deviations are sketched.

moving fragments increases the width of D_L as $(R_E)^{1/2}$ and decreases the width of D_H as $(R_E)^{-1/2}$. On this basis, δr varies as $(2R_E)^{1/2}(R_E+1)/E$ for the extreme (2). These extremes result in $P(R)$ distributions that differ insignificantly.

When the symmetrical fission gaussian dispersion width $\delta r=0.14$ varying within the above extremes is used in Eq. (1), the $P(R)$ distribution in Fig. 1 is obtained. When compared with $P_I(R_I)$ by Eq. (3), this calculated $P(R)$ results in the approximate variation of w_L/w_H shown in Fig. 4.

III. DISCUSSION

The variation of w_L/w_H in Fig. 4 can be used with the assumption that the ionization defects Δ_L and Δ_H vary roughly as the fragment mass to obtain an estimate of the ionization defect. From the definition of the ionization defects

$$\Delta_L=w_LI_L-wI_L,\quad \Delta_H=w_HI_H-wI_H$$

it follows that

$$\frac{w_L}{w_H}=\frac{1+(\Delta_L/wI_L)}{1+(\Delta_H/wI_H)},$$

from which it is found that the total ionization defect

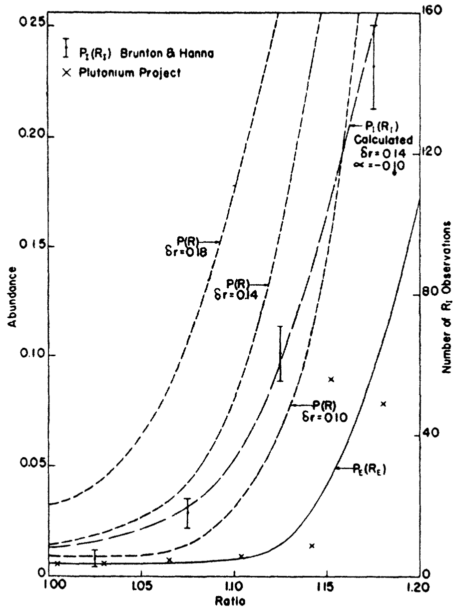

FIG. 3. Comparison of ratio distributions near symmetrical fission of U^{235}. The trial $P(R)$ distributions were obtained by various dispersions of $P_E(R_E)$. Comparison is made between the experimental $P_I(R_I)$ points and the $P_I(R_I)$ distribution calculated from the best determination of $D_r(R_E, R)$ and w_L/w_H.

for the most probable fission asymmetry is about 6 or 7 Mev. Since this essentially is energy unobserved as ionization, it should be added to the 155.8-Mev ionization chamber measurement[3] of the average kinetic energy of fission, bringing it into better agreement with the 165±8-Mev calorimetric measurement.[7] This estimate of the total ionization defect should be compared with the value of 6.7 Mev calculated[1] from properties of the recoil gas atoms.

Other estimates of the magnitude of the components of the dispersion are in reasonable agreement with the width determined here by distribution comparison. The width $\delta r=0.14$ corresponds to an 8.1-Mev symmetrical fission dispersion width of D_L and D_H. These fragment energy dispersions are composed of the 2.5- and 5-Mev channel widths used by Brunton and Hanna, the several other additional ionization chamber instru-

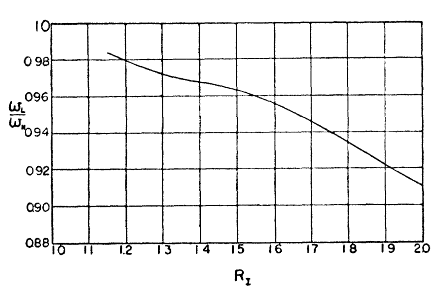

FIG. 4. Curve of the relative energy ionization ratio for light and heavy fragments.

[7] M. C. Henderson, Phys. Rev. **58**, 774 (1940).

TABLE I. Comparison of the energy distribution widths including dispersion with energy widths (without dispersion) calculated on the basis of the estimated dispersion being due predominantly to either (1) instrumental errors or (2) neutron recoil. Data are for the slow neutron fission of U^{235}.

	Width observed by Brunton-Hanna	Calculated energy width from dispersion (1)	Calculated energy width from dispersion (2)
Light fragment energy	12 Mev	8.8 Mev	7.0 Mev
Heavy fragment energy	20	18.3	18.8
Total energy at $R_I=1.2$	30	27.7	27.7
Total energy at $R_I=1.5$	20	16.4	16.1
Total energy at $R_I=2.0$	15	9.6	8.3

mental errors of less than 2-Mev width and the contribution of neutron recoil. On the basis of the average neutron energy found by Wilson,[8] the recoil energies of the fragments are $\pm 2.4(R_E)^{\frac{1}{2}}$ Mev and $\pm 2.4(R_E)^{-\frac{1}{2}}$ Mev for the backward and forward emission of a neutron from the light and heavy fragments, respectively. The combination of these estimated components results in a composite energy dispersion width somewhat less than that obtained by distribution comparison.

Although the determination of the dispersion by distribution comparison is not precise, it is interesting to note that the energy distributions observed by ionization are appreciably narrowed when this dispersion is removed by calculation. The calculated energy widths (widths without dispersion) in Table I are based on the approximation that all dispersions and distributions are gaussian in form.

Any explanation of the difference between $P_E(R_E)$ and $P_I(R_I)$ based on the assumption of a unity value of w_L/w_H for asymmetric fission requires the use of a decidedly asymmetric dispersion function. This possibility seems very unlikely in view of the nature of the instrumental errors and the measurements of DeBenedetti *et al.*,[9] which show that neutron emission from the moving fragments is not strongly preferential in direction. Actually, since any variation in the number of neutrons emitted per fragment tends to broaden the true $P_E(R_E)$ into the approximate distribution based on fragment mass measurements, the true difference between $P_E(R_E)$ and $P_I(R_I)$ can be slightly larger than indicated by the approximate $P_E(R_E)$ in Fig. 1. By taking 0.5 as a reasonable estimate of the dispersion width in the number of neutrons emitted per fragment, we find that the resulting mass ratio dispersion width of 0.01 broadens the true $P_E(R_E)$ by an amount small compared to the estimated energy dispersion width of 0.14.

The author is very grateful to Professor J. K. Knipp for many valuable suggestions and criticisms on this paper.

[8] R. R. Wilson, Phys. Rev. **72**, 189 (1947).

[9] DeBenedetti, Francis, Preston, and Bonner, Phys. Rev. **74**, 1645 (1948).