$s=(\omega+\delta\omega)/k$, as compared with its unperturbed value $s_0=\omega/k$, is given by

$$s=s_0(1-2\nu F+\cdots). \tag{3}$$

Clearly, a rapid convergence of the perturbation expansion requires

$$2\nu F\ll 1,$$

which excludes Fröhlich's cases (1). Taking the second-order approximation literally, one would conclude from (1) and (3): $s<s_0/2$ for all values of ν, and even $s<0$ for $\nu>2^{\frac{1}{2}}$, which is absurd.

What actually happens with the lattice vibrations if $2\nu F\gtrsim 1$ becomes clearer by studying a one-dimensional model of a metal, *viz.*, a linear lattice allowing phonons and electrons to propagate in two opposite directions only Supposing the electron gas to be close to the ground state, in the sense that the interactions affect only electrons with velocities near the Fermi boundary velocities ($\pm v$), it is then possible, according to Tomonaga,[3] to describe the electrons in terms of an equivalent boson gas[4] whose hamiltonian is of the familiar form

$$H_e=\tfrac{1}{2}\,\Sigma_k\,(P_k^*P_k+\Omega_k^2Q_k^*Q_k). \tag{4}$$

Here, Q_k is the amplitude of a density wave:

$$\int dx\,\exp(-ikx)\sum_{\rho=1,2}\psi_\rho^*\psi_\rho,$$

except for a normalization factor, and P_k is the canonically conjugate quantity as defined by Tomonaga [see Eqs. (3.4) and (3.5) in reference 3]. For the phonons (lattice vibrations) we have similarly

$$H_s=\tfrac{1}{2}\,\Sigma_k\,(p_k^*p_k+\omega_k^2q_k^*q_k). \tag{5}$$

The wave number-frequency relations in the two cases are

$$\Omega_k=vk,\quad \omega_k=sk. \tag{6}$$

Now, the electron-lattice coupling can be simply described by adding the interaction term

$$H_i=\tfrac{1}{2}\,\Sigma_k\,c_k(Q_k^*q_k+Q_kq_k^*). \tag{7}$$

Indeed, the operator $Q_k^*q_k$ corresponds to a process in which any electron acquires a momentum $\hbar k$ by absorption or emission of a phonon of momentum $\pm\hbar k$. Disregarding other interactions for the time being, we have for the total hamiltonian

$$H=H_e+H_s+H_i. \tag{8}$$

Because H is bilinear in the canonical variables, rigorous solutions are readily obtained by transforming to normal coordinates. The frequencies of the normal vibrations are

$$2^{-\frac{1}{2}}[\Omega_k^2+\omega_k^2\pm\{(\Omega_k^2-\omega_k^2)^2+4c_k^2\}^{\frac{1}{2}}]^{\frac{1}{2}}. \tag{9}$$

In the limit of weak coupling, $c_k\to 0$, this means that the sound frequencies ω_k are reduced by a factor

$$1-c_k^2(2\omega_k^2|\Omega_k^2-\omega_k^2|)^{-1}+\cdots, \tag{10}$$

corresponding to the factor $(1-2\nu F+\cdots)$ in (3). (The expression (10) is independent of k because $c_k\sim k^2$.) As the coupling strength increases and the factor (10) approaches the value $\frac{1}{2}$, the rigorous expression (9) indicates that the proper frequencies become imaginary for

$$|c_k|>\Omega_k\omega_k. \tag{11}$$

The proper vibrations are then unstable, in the sense that the potential energy of the coupled oscillators no longer has a true minimum value at the equilibrium position ($Q_k=q_k=0$); the corresponding surface becomes saddle-shaped. As a consequence, the lattice would break down.[5]

From this result it is obvious that the hamiltonian (8) is entirely inadequate for the study of the strong coupling case. Other energy terms restoring the stability must be taken into account. One such stabilizing term is the coulomb interaction of the electrons which in our variables is simply[6]

$$H_c=\text{const}\,\Sigma_k\,Q_k^*Q_k\quad(>0)$$

except for an additive constant. Adding H_c to H_e (4), amounts to replacing Ω_k^2 in (4) by $(v^2k^2+\text{const})$ (or attributing a rest-mass to the Bloch-Tomonaga bosons). This insures the stability at least for the low k-values.

A fuller discussion of the one-dimensional model is hardly worth while. For the electron gas in three dimensions no such simple description in terms of bosons is available, but similar complications are to be expected, as even the perturbation theory indicates that the velocity of sound tends toward zero with increasing coupling strength. This makes it imperative to improve the hamiltonian as well as the mathematical technique if the strong coupling assumption is to be taken at all seriously.

[1] J. Bardeen, Phys. Rev. **79**, 167 (1950).
[2] H. Fröhlich, Phys. Rev. **79**, 845 (1950).
[3] S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950). See also F. Bloch, Helv. Phys. Acta **7**, 385 (1934), Sec. 4.
[4] The specific heat of the one-dimensional boson gas is proportional to T, and it is actually equal to that of the electron gas because

$$\int_0^\infty dx\times(e^x-1)^{-1}=2\int_0^\infty dx\times(e^x+1)^{-1}.$$

[5] As Mrs. M. G. Mayer tells me, this fact was known to her and E. Teller some years ago, and it was for this reason that they gave up an attempt to explain superconductivity on the basis of strong electron-lattice interactions.
[6] See S. Tomonaga, reference 3, Eq. (3.10). Our Q_k is identical with Tomonaga's $\rho_n(L/2\pi n)(m/2n_{\max})^{\frac{1}{2}}(k=2\pi n/L)$. Note that for the coulomb potential $J_n\sim n^{-2}$.

Erratum: Determination of the Photo-Nuclear Cross Sections $S^{32}(\gamma,d)P^{30}$, $S^{32}(\gamma,np)P^{30}$, and $P^{31}(\gamma,n)P^{30}$

[Phys. Rev. **81**, 815 (1951)]

L. KATZ AND A. S. PENFOLD
Department of Physics, University of Saskatchewan, Saskatoon, Saskatchewan, Canada

AN error was discovered in the computation of the $S^{32}(\gamma,\frac{np}{d})P^{30}$ cross section in the paper of the above title. The activation curve as published in Fig. 2 is correct, but the cross section, as well as the integrated cross section calculated from it, should be larger by a factor of 4. (The ordinate of Fig. 5 should be multiplied by 4.)

The larger contribution to the total photoneutron flux from the (γ,np) reaction is now in better agreement with that predicted by Cameron[1] for this region of the periodic table.

[1] A. G. W. Cameron, Phys. Rev. **82**, 272 (1951).

Response of Some Scintillation Crystals to Heavy Particles*

C. J. TAYLOR,† M. E. REMLEY, W. K. JENTSCHKE, AND P. G. KRUGER
Department of Physics, University of Illinois, Urbana, Illinois
(Received May 18, 1951)

A RELATIVELY small amount of information has been reported concerning the response of scintillation crystals to heavy particles,[1–4] but knowledge of this kind is essential to the successful employment of the scintillation counter technique for quantitative energy measurements of heavy particles.

Protons of 1 to 5 Mev, deuterons and molecular hydrogen ions of 1 to 11 Mev, alpha-particles of 4 to 21 Mev from the University of Illinois cyclotron, and polonium alpha-particles were used in the measurements to be described. The response of anthracene, stilbene, and thallium activated sodium iodide to these particles has been investigated.

The collimated beam from the cyclotron was brought out into air through a thin Nylon window and allowed to strike the scintillating crystal which was mounted in optical contact with an RCA 5819 photomultiplier tube and covered with a 0.16 mg/cm^2 aluminum foil for a reflector. Aluminum absorbing foils were interposed in the beam to obtain the various particle energies incident on the crystals. Pulses from the photomultiplier were

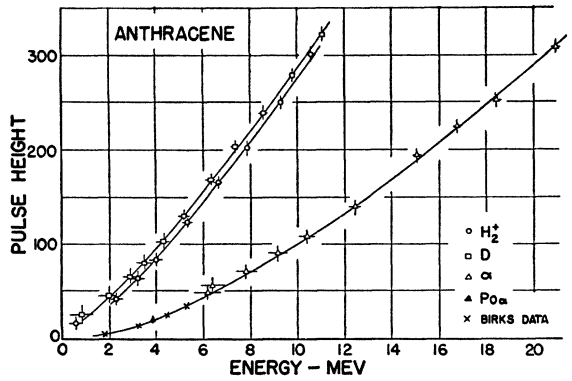

FIG. 1. Response of anthracene to molecular hydrogen ions, deuterons, and alpha-particles. Note that the molecular hydrogen ions are detected as two protons entering the crystal simultaneously.

amplified by a Los Alamos model 100 linear pulse amplifier and analyzed with a twelve-channel pulse amplitude analyzer. Essentially gaussian distributions were obtained with full widths at half-maxima of six to eight percent for the higher energy particles. The determination of the maximum energy of the incident particles accurate to one percent was obtained by measurement of the ranges in air.

The variation of the peak pulse height with energy for the different particles and crystals used is shown in Figs. 1 to 3. The vertical lines indicate estimates of the errors in the pulse heights. As the energy of the particles is reduced by the introduction of absorbers, the initial one percent uncertainty in the maximum energy produces progressively larger uncertainties in the calculated energies of the lower energy particles. The horizontal lines attached to the points show the total variation in energy at a given energy corresponding to a variation of one percent in the initial energy. These are not to be interpreted as errors in the determination of the energies, but rather as the amount by which the entire curve would be translated in energy for a one percent change in the initial energy.

The figures show a distinct difference in the behavior of the organic crystals and sodium iodide. The organic crystals have a nonlinear response of pulse height to energy in the entire region investigated. In sodium iodide protons and deuterons give linear response, while alpha-particles show a nonlinear relation over a low energy region with a linear response above an energy of about 10 Mev. This nonlinear response of the sodium iodide is certainly not due to deterioration of the crystal surface as has been proved by experiments with crystals freshly cleaved in a vacuum.

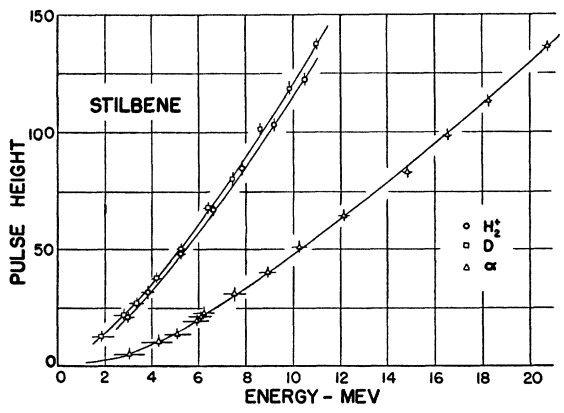

FIG. 2. Response of stilbene to molecular hydrogen ions, deuterons, and alpha-particles. The arbitrary pulse-height scale is the same as in Fig. 1.

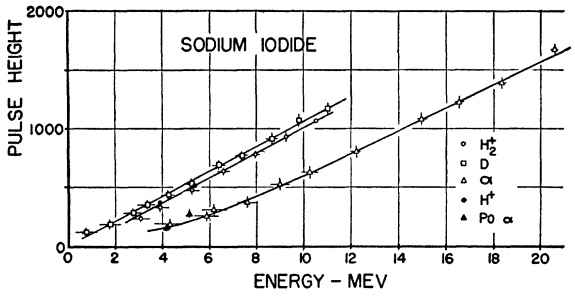

FIG. 3. Response of thallium activated sodium iodide to molecular hydrogen ions, deuterons, and alpha-particles. The arbitrary pulse-height scale is the same as in Figs. 1 and 2.

A detailed analysis shows the nonlinearity to be dependent on the ionization density produced by the incident particle. For high specific energy losses, as for example for alpha-particles below 9 Mev, the specific fluorescence (the number of light quanta emitted per centimeter of air equivalent) attains a constant value and is then independent of the specific energy loss.[5]

While this letter was being written, the work of Frey, *et al.*,[6] was reported, which agrees with the results described here.

The authors wish to thank A. L. Atkins, R. O. Kerman, and W. E. Kreger, who determined the cyclotron beam energy.

A more detailed analysis of the data presented here will be published later.

* Assisted by the joint program of the ONR and AEC.
† Now at North American Aviation, Inc.
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[2] E. G. Michaelis, Helv. Phys. Acta **23**, Sup. 3, 155 (1950).
[3] J. B. Birks, Proc. Phys. Soc. (London) **63**, 1294 (1950).
[4] Franzen, Peele, and Sherr, Phys. Rev. **79**, 742 (1950).
[5] W. Cross of Chalk River Laboratory, in a private communication, has reported results in essential agreement with those reported here.
[6] Frey, Grim, Preston, and Gray, Phys. Rev. **82**, 372 (1951).

The Fluorescence Efficiencies of Some Crystals for Electrons and Heavy Particles*

W. K. JENTSCHKE, F. S. EBY, C. J. TAYLOR,† M. E. REMLEY, AND P. G. KRUGER
Department of Physics, University of Illinois, Urbana, Illinois
(Received May 18, 1951)

THE investigation described in the preceding paper was extended to a study of the fluorescence of anthracene, stilbene, and thallium activated sodium iodide crystals with incident monoenergetic electrons. This extension was particularly directed toward an investigation of the fluorescence efficiency of crystals to electrons as compared to heavy particles with the same specific energy loss. For this reason, electrons with energies of 500 to 5000 ev were used. A pulsed monoenergetic electron beam was provided by a specially constructed cathode-ray gun for examination of this region. The electron beam was focused on the crystal which was mounted on the end of a charge collector. This collector was connected to a vibrating reed electrometer.[1]

The electron beam was pulsed at a rate of ten thousand per second with pulse durations of 0.5 and 2.0 microseconds. Identical results were obtained with both types of pulses, indicating that no long-lived fluorescence of appreciable intensity could have been present. The maximum electron beam current was less than 5×10^{-13} ampere, so that no more than 300 electrons were incident on the crystal during any single pulse. Since the electron beam was defocused over an area of 4 mm² on the crystal, the mechanism for producing fluorescence was the same as for single electrons. The secondary emission coefficients of the crystals used are greater than one for the electron energies investigated, so that no sig-