we have the results shown in Table I. It is interesting to note that they fall into a distinct group, i.e., $ft\sim 10^{13}$. In view of this result, one can expect to find some reasons which indicate the interpretation that Sb^{124} is to be classified into the C_{2T} group ($\Delta J=\pm 2$, parity change no).

TABLE I. *ft* values of β-decay in the second forbidden transition.* f_{1T}, f_{2T} are the first and second forbidden *f*-functions, respectively (see reference 6); the coulomb corrections are also made.

Element	t(sec)	$W_0(mc^2)$	$\Delta J=\pm 2$, yes tf_{1T}	$\Delta J=\pm 2$, no tf_{2T}	k
Cl^{36}	1×10^{13}	2.4	6.1×10^{12}	9.8×10^{13}	$\surd 18$
Tc^{99}	7×10^{12}	1.57	1.1×10^{11}	8.4×10^{12}	7.7
Sb^{124}	5×10^{6}	5.5	1.0×10^{10}	2.4×10^{12}	13

* If we take into account the fact that all β-emitters of the "*a*" type group ($\Delta J=\pm 2$, yes) have $ft=10^8$ (see references 7, 9, and 8), it seems more natural to classify Cl^{36}, Tc^{99}, and Sb^{124} into C_{2T} groups ($\Delta J=\pm 2$, no), rather than into "*a*"-type groups. Theoretically, tf_{2T} should be proportional to $\Sigma|T_{ij}|^2$.

In fact, C_{1T} ($\Delta J=\pm 2$, parity change yes) gives an *ft* value for ${}_{51}\mathrm{Sb}_{73}{}^{124}$ (see Table I) which seems too great to be classified in the "*a*" type group, where the corrected *ft* values are about $10^7\sim 10^8$ (see Taketani, *et al.*,[7] also J. Davidson).[8] It is interesting to note that the empirical *ft* values of different transitions, i.e., the allowed ($\int\sigma$), the first forbidden (ΣB_{ij}), and the second forbidden (ΣT_{ij}), fall into distinct groups, i.e., 10^4, $10^8\sim 10^9$, and $10^{12}\sim 10^{13}$, respectively.[9]

In conclusion it is suggested that, when carrying out an analysis of β-decay, one can get useful and effective information from the evaluation of corrected *ft* values in addition to the usual analysis of the spectrum shape and the analysis of the selection rule on the basis of the shell model.

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Third-Order Elastic Coefficients of Crystals

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THE independent nonzero third-order elastic coefficients of crystals $C_{pqr}=C_{prq}=C_{qpr}(p,\ q,\ r=1,\ 2,\ 3,\ 4,\ 5,\ 6)$ (sixth-order polar tensor) have hitherto been known only for cubic groups and for C_3.[1,2] Here they are derived for all the crystal classes S_2; C_{1h}, C_2, C_{2h}; C_{2v}, D_2, D_{2h}; C_4, S_4, C_{4h}, C_{4v}, D_{2d}, D_4, D_{4h}; C_3, S_6, C_{3v}; T, T_h, T_d, O, O_h in which one can find orthogonal cartesian coordinates that do not transform into linear combinations of themselves under the symmetry operations of the crystal,[3] and for $D_{3d}(C_{3v}\times i=D_3\times i)$ and D_3, which are equivalent to C_{3v} for polar properties of even order. The numbers of coefficients agree in all cases with those found group-theoretically by Bhagavantam and Suryanarayana[2] and by Jahn.[2] The coefficients for symmetry C_3, T, O, T_h, O_h agree with those given by Birch.[1]

For symmetry C_1 the number of independent third-order elastic coefficients equals the numbers of combinations with repetition of class 3 of 8 objects: $8!/5!3!=56$. They are listed in Table I. Symmetry $S_2=C_1\times i$ is equivalent to C_1.

For symmetry C_{1h}

$$\sigma_h: x\rightarrow x,\quad y\rightarrow -y,\quad z\rightarrow z, \tag{1}$$

the 24 coefficients with indices odd in y must vanish, since they change sign under σ_h (Table I). Symmetries $C_{2h}(C_{1h}\times i=C_2\times i)$ and $C_2(C_2=C_2(y))$ are equivalent to C_{1h}.

For symmetry C_{2v}

$$\sigma_v: x\rightarrow -x,\quad y\rightarrow y,\quad z\rightarrow z;\quad \sigma_v': x\rightarrow x,\quad y\rightarrow -y,\quad z\rightarrow z, \tag{2}$$

σ_v requires that the 12 independent nonzero coefficients for C_{1h} with indices odd in x vanish (Table I). Symmetries $D_{2h}(C_{2v}\times i=D_2\times i)$ and D_2 are equivalent to C_{2v}.

TABLE I. Third-order elastic coefficients of crystals.

C_1, S_2	C_{1h}, C_2, C_{2h}	C_{2v}, D_2, D_{2h}	C_3, S_6	C_{3v}, D_3, D_{3d}	C_4, S_4, C_{4h}	C_{4v}, D_{2d}, D_4, D_{4h}	T, T_h	T_d, O, O_h
xxxxxx	xxxxxx	xxxxxx	xxxxxx	xxxxxx	xxxxxx	xxxxxx	xxxxxx	xxxxxx
xxxxyy	xxxxyy	xxxxyy	xxxxyy	xxxxyy	xxxxyy	xxxxyy	xxxxyy	xxxxyy
xxxxzz	xxxxzz	xxxxzz	xxxxzz	xxxxyy	xxxxzz	xxxxzz	xxxxzz	xxxxyy
xxxxyz	0	0	xxxxyz	xxxxyz	0	0	0	0
xxxxzx	xxxxzx	0	xxxxzx	xxxxzx	0	0	0	0
xxxxxy	0	0	xxxxxy	xxxxzx	xxxxxy	0	0	0
xxyyyy	xxyyyy	xxyyyy	xxxxzz	xxxxyy	xxxxyy	xxxxyy	xxxxzz	xxxxyy
xxyyzz	xxyyzz	xxyyzz	xxyyzz	xxyyzz	xxyyzz	xxyyzz	xxyyzz	xxyyzz
xxyyyz	0	0	xxyyyz	xxyyyz	0	0	0	0
xxyyzx	xxyyzx	0	xxyyzx	xxyyyz	0	0	0	0
xxyyxy	0	0	xxyyxy	xxyyxy	0	0	0	0
xxzzzz	xxzzzz	xxzzzz	xxxxyy	xxxxyy	xxzzzz	xxzzzz	xxxxyy	xxxxyy
xxzzyz	0	0	xxyyzx	xxyyyz	0	0	0	0
xxzzzx	xxzzzx	0	xxyyxy	xxyyxy	0	0	0	0
xxzzxy	0	0	xxyyyz	xxyyyz	xxzzxy	0	0	0
xxyzyz	xxyzyz	xxyzyz	xxyzyz	xxyzyz	xxyzyz	xxyzyz	xxyzyz	xxyzyz
xxyzzx	0	0	xxyzzx	xxyzzx	xxyzzx	0	0	0
xxyzxy	xxyzxy	0	xxyzxy	xxyzzx	0	0	0	0
xxzxzx	xxzxzx	xxzxzx	xxzxzx	xxzxzx	xxzxzx	xxzxzx	xxzxzx	xxzxzx
xxzxxy	0	0	xxzxxy	xxzxxy	0	0	0	0
xxxyxy	xxxyxy	xxxyxy	xxxyxy	xxzxzx	xxxyxy	xxxyxy	xxxyxy	xxzxzx
yyyyyy	yyyyyy	yyyyyy	xxxxxx	xxxxxx	xxxxxx	xxxxxx	xxxxxx	xxxxxx
yyyyzz	yyyyzz	yyyyzz	xxxxyy	xxxxyy	xxxxzz	xxxxzz	xxxxyy	xxxxyy
yyyyyz	0	0	xxxxxy	xxxxzx	0	0	0	0
yyyyzx	yyyyzx	0	xxxxyz	xxxxyz	0	0	0	0
yyyyxy	0	0	xxxxzx	xxxxzx	−xxxxxy	0	0	0
yyzzzz	yyzzzz	yyzzzz	xxxxzz	xxxxyy	xxzzzz	xxzzzz	xxxxzz	xxxxyy
yyzzyz	0	0	xxyyxy	xxyyxy	0	0	0	0
yyzzzx	yyzzzx	0	xxyyyz	xxyyyz	0	0	0	0
yyzzxy	0	0	xxyyzx	xxyyyz	−xxzzxy	0	0	0
yyyzyz	yyyzyz	yyyzyz	xxxyxy	xxzxzx	xxzxzx	xxzxzx	xxxyxy	xxzxzx
yyyzzx	0	0	xxyzxy	xxyzzx	−xxyzzx	0	0	0
yyyzxy	yyyzxy	0	xxzxxy	xxzxxy	0	0	0	0
yyzxzx	yyzxzx	yyzxzx	xxyzyz	xxyzyz	xxyzyz	xxyzyz	xxyzyz	xxyzyz
yyzxxy	0	0	xxyzzx	xxyzzx	0	0	0	0
yyxyxy	yyxyxy	yyxyxy	xxzxzx	xxzxzx	xxxyxy	xxxyxy	xxzxzx	xxzxzx
zzzzzz	zzzzzz	zzzzzz	xxxxxx	xxxxxx	zzzzzz	zzzzzz	xxxxxx	xxxxxx
zzzzyz	0	0	xxxxzx	xxxxzx	0	0	0	0
zzzzzx	zzzzzx	0	xxxxxy	xxxxzx	0	0	0	0
zzzzxy	0	0	xxxxyz	xxxxyz	0	0	0	0
zzyzyz	zzyzyz	zzyzyz	xxzxzx	xxzxzx	zzyzyz	zzyzyz	xxzxzx	xxzxzx
zzyzzx	0	0	xxzxxy	xxzxxy	0	0	0	0
zzyzxy	zzyzxy	0	xxyzzx	xxyzzx	0	0	0	0
zzzxzx	zzzxzx	zzzxzx	xxxyxy	xxzxzx	zzyzyz	zzyzyz	xxxyxy	xxzxzx
zzzxxy	0	0	xxyzxy	xxyzzx	0	0	0	0
zzxyxy	zzxyxy	zzxyxy	xxyzyz	xxyzyz	zzxyxy	zzxyxy	xxyzyz	xxyzyz
yzyzyz	0	0	yzyzyz	yzyzyz	0	0	0	0
yzyzzx	yzyzzx	0	yzyzzx	yzyzzx	0	0	0	0
yzyzxy	0	0	yzyzxy	yzyzzx	yzyzxy	0	0	0
yzzxzx	0	0	yzyzxy	yzyzzx	0	0	0	0
yzzxxy	yzzxxy	yzzxxy	yzzxxy	yzzxxy	yzzxxy	yzzxxy	yzzxxy	yzzxxy
yzxyxy	0	0	yzyzzx	yzyzzx	0	0	0	0
zxzxzx	zxzxzx	0	yzyzyz	yzyzyz	0	0	0	0
zxzxxy	0	0	yzyzzx	yzyzzx	−yzyzxy	0	0	0
zxxyxy	zxxyxy	0	yzyzxy	yzyzzx	0	0	0	0
xyxyxy	0	0	yzyzyz	yzyzyz	0	0	0	0

For symmetry C_4

$$C_4: x\rightarrow y,\quad y\rightarrow -x,\quad z\rightarrow z, \tag{3}$$

there are 16 independent nonzero coefficients (listed in column 6 of Table I):

$$\begin{gathered}
C_{xxxxxx}=C_{yyyyyy},\quad C_{xxxxyy}=C_{xxyyyy},\quad C_{xxxxzz}=C_{yyyyzz},\\
C_{xxxxxy}=-C_{yyyyxy},\quad C_{xxyyzz},\quad C_{xxyyxy}=-C_{xxyyxy}=0,\\
C_{xxzzzz}=C_{yyzzzz},\quad C_{xxzzxy}=-C_{yyzzxy},\quad C_{xxyzyz}=C_{yyzxzx},\\
C_{xxyzzx}=-C_{yyyzzx},\quad C_{xxzxzx}=C_{yyyzyz},\quad C_{xxxyxy}=C_{yyxyxy},\quad C_{zzzzzz},\\
C_{zzzzxy}=-C_{zzzzxy}=0,\quad C_{zzyzyz}=C_{zzzxzx},\quad C_{zzyzzx}=-C_{zzyzzx}=0,\\
C_{zzxyxy},\quad C_{yzyzxy}=-C_{zxzxxy},\quad C_{yzzxxy},\quad C_{xyxyxy}=-C_{xyxyxy}=0.
\end{gathered} \tag{4}$$

The 24 components odd in x or in y vanish. Symmetries $C_{4h}(C_4\times i=S_4\times i)$ and S_4 are equivalent to C_4.

For symmetry C_{4v},

$$C_4: x\rightarrow y,\quad y\rightarrow -x,\quad z\rightarrow z;\quad \sigma_v: x\rightarrow y,\quad y\rightarrow x,\quad z\rightarrow z, \tag{5}$$

σ_v imposes four additional relations among the 16 coefficients (4):

$$C_{xxxxxy}=C_{yyyyxy},\quad C_{xxzzxy}=C_{yyzzxy},\quad C_{xxyzzx}=C_{yyyzzx},\quad C_{yzyzxy}=C_{zxzxxy}. \tag{6}$$

Thus, these 4 coefficients vanish (Table I). Symmetries $D_{4h}(C_{4v}\times i=D_4\times i=D_{2d}\times i)$, D_4 and D_{2d} are equivalent to C_{4v}.

For symmetry C_3,

$$C_3: x\rightarrow y,\quad y\rightarrow z,\quad z\rightarrow x, \tag{7}$$

there are 20 independent coefficients (Table I),[1] while for symmetry C_{3v},

$$C_3: x\rightarrow y,\quad y\rightarrow z,\quad z\rightarrow x;\quad \sigma_v: x\rightarrow x,\quad y\rightarrow z,\quad z\rightarrow y, \tag{8}$$

there are 14, since σ_v imposes six additional relations among the coefficients for C_3 (Table I). Symmetries $D_{3d}(C_{3v}\times i=D_3\times i)$ and D_3 are equivalent to C_{3v}.

For symmetry T,

$$C_3': x\rightarrow y,\quad y\rightarrow z,\quad z\rightarrow x;\quad C_3'': -x\rightarrow y,\quad y\rightarrow z,\quad z\rightarrow -x, \tag{9}$$

12 of the coefficients for C_3 vanish (Table I) owing to the additional relations imposed by C_3''. Symmetry $T_h=T\times i$ is equivalent to T.

For symmetry T_d,

$$C_3': x\rightarrow y,\quad y\rightarrow z,\quad z\rightarrow x;\quad C_3'': -x\rightarrow y,\quad y\rightarrow z,\quad z\rightarrow -x;\quad \sigma_d: x\rightarrow x,\quad y\rightarrow z,\quad z\rightarrow y, \tag{10}$$

only 8 of the relations imposed by C_3'' are distinct, owing to the relations imposed by σ_d: 8 of the 14 independent coefficients for C_{3v} vanish (Table I). Symmetries $O_h(T_d\times i=O\times i)$ and O are equivalent to T_d.

The writer wishes to thank Professor Bhagavantam and Mr. Hearmon for correspondence.

[1] F. Birch, Phys. Rev. **71**, 809 (1947).

[2] S. Bhagavantam and D. Suryanarayana, Nature **160**, 750 (1947), have shown group-theoretically that Birch's number of coefficients for symmetry $T_d(=8)$ is in error; the correct number is 6, as was also confirmed by H. A. Jahn, Acta Cryst. **2**(1), 30 (1949). This result is actually obvious, even in more complete form, since symmetries T_d, O, and O_h $(T_d\times i=O\times i)$ are equivalent for polar properties of even order.

[3] F. G. Fumi, Acta Cryst. (1951), to be published.

[4] After the writer had undertaken this work, he was informed by private communication that Mr. R. F. S. Hearmon, Physics Section, Forest Products Research Laboratory, England, had been applying to the third-order elastic tensor the direct method described in C. E. Love, *The Mathematical Theory of Elasticity* (Cambridge University Press, London 1927), Chapter VI.

Experimental Evidence for the Vacancy Mechanism in Diffusion in Metals and Alloys*

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THERE has been much discussion among the workers in the field of metallic diffusion as to the possibility of using an alloy system such as iron-aluminum, nickel-aluminum or cobalt-aluminum, near the fifty-fifty atomic percent composition, to test the vacancy theory of diffusion, but to our knowledge only one previous report[1] on experimental work has been given. FeAl,[2] NiAl,[3] and CoAl[4] are body-centered cubic with ordered structures at room temperature, aluminum atoms going to cube corners and the ferrous atoms to cube centers. The lattice spacings at the ideal composition (50–50 atomic percent) rise to a maximum. On the cobalt-rich side of the ideal composition (to take a specific example), the cobalt replaces aluminum, atom for atom, and a fall in the lattice spacing results from the replacement of large aluminum atoms by the smaller cobalt atoms. On the aluminum-rich side the number of atoms per unit cell is less than two.[4] For the ordered structure the cube corners are filled by aluminum atoms, but there are insufficient cobalt atoms to fill up the cube centers and the vacancies are not made up by the larger aluminum atoms to that there is a contraction in the cell dimensions. These vacancies have then been considered as a possible source of information on the vacancy mechanism of diffusion. Smoluchowski and Burgess[1] have raised doubt as to the ability of these vacancies to play a role in diffusion, due to their relatively small size compared to the size of an aluminum atom, in connection with an experiment on nickel-aluminum alloys. Their experiments seemed to uphold the arguments presented there, in that no increase in the diffusion coefficient was found for compositions on the aluminum-rich side in NiAl.

The arguments raised by Smoluchowski and Burgess are only applicable to a highly ordered structure and rapidly lose validity under the experimental conditions at elevated temperatures where the actual diffusion takes place. At these temperatures there must exist a considerable degree of disorder permitting vacancy diffusion to take place, where vacancies are distributed in a random fashion over lattice sites.

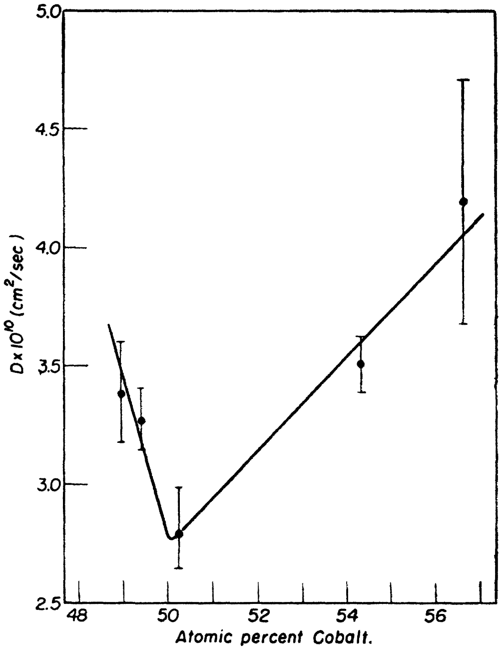

FIG. 1. Values of the diffusion coefficient of cobalt in cobalt-aluminum alloys as a function of cobalt concentration, at 1250°C.

Even though the ordered structure persists at elevated temperatures where the diffusion experiments were carried out, it seems highly unlikely, with such a large number of excess vacancies present, that they would not be of consequence in any process which took place as a result of vacancies. Even admitting that a cobalt or aluminum atom could not move into the cobalt vacancy created, as suggested by Smoluchowski and Burgess,[1] the sheer number of these vacancies would greatly increase the probability of a vacancy being adjacent to a normal lattice defect (e.g. a cobalt atom on a cube corner) or to a vacancy due to thermal excitation, and thus increase the mobility of the vacancies and consequently increase the diffusion coefficient.

We have investigated the diffusion of cobalt into five compositions of cobalt-aluminum alloys near the ideal composition, using radioactive Co[60] as the diffusing atom. The experimental technique used was similar to that employed in our work on self-diffusion in cobalt.[5] The diffusion was carried out at three temperatures, 1050°C, 1150°C, and 1250°C. The preliminary results